A Gyroscope Control System for Unknown Proof Mass and Interface Circuit Errors

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Abstract—This paper presents a novel approach that can compensate errors resulting from the imperfections of mechanical structures and interface circuits for MEMS gyroscope systems. The mechanical structure errors discussed in this paper contribute to unknown proof mass, spring constants, damping coefficients, and existence of cross-axis resilient/damping forces. The interface circuit errors include: mismatch of differential capacitors, parasitic capacitance, offset voltage of operation amplifiers, and circuit noise. Different from most of existing researches, the proposed method has the following features: (1) the mechanical structure imperfections and interface circuit errors are compensated simultaneously using control techniques; (2) the mass of the proof mass can be unknown. This approach is verified on two types of gyroscope designs by numerical simulations. Simulation results indicate that, under those imperfections, the proposed method can obtain correct angular rate within 80 milliseconds.

I. INTRODUCTION

A MEMS vibratory gyroscope can be briefly grouped into three subsystems, as shown in Fig. 1. The “Mechanical structure” mainly consists of a proof mass suspended in a rigid frame. This mass converts the Coriolis force (angular rate information) into displacements along designated directions. These displacements are then converted into another physical quantity for the ease of sensor measurements (e.g., capacitance variations, resistance variation, etc.). The “Interface circuits” converts these measurements into voltage outputs for the ease of signal processing. The “Control algorithm” processes these signals for the feedback control of vibratory mass and for calculating the angular rate. The imperfections existed in each subsystem would significantly degrade the sensing accuracy of angular rates.

Fig. 1. Block diagram of a vibratory MEMS gyroscope system

The mechanical structure imperfections mainly come from the structure designs and fabrication errors. It is normal to have 10%~20% dimension variations and residual film stress from MEMS fabrication process such as lithography, etching, film deposition, and etc [1][2]. All these errors cause the fabricated gyroscope dynamics (mass, spring constants, damping coefficients) deviated from their designated values. Even worse, they induce cross-axis resilient force and cross-axis damping force, which lead to the serious “quadrature error” in gyrooscope systems [1]. Solutions to mechanical structure imperfections include: advanced micromachining processes, complicated mechanical structure designs, post-micromachining [3]-[5], and etc. In a word, these imperfections are often minimized by expensive tooling processes.

The reactance sensing scheme is attractive to MEMS devices because neither additional processing steps nor materials are required for the fabrication process. When adapting this sensing technique, charge amplifiers are often used as an interface circuit to convert capacitance variations into voltage signals [6]. The imperfections in this circuit includes: offset voltage of operational amplifiers, parasitic capacitances, circuit noises, bias ambiguity, and etc. [7]. Several methods have been proposed to deal with those problems including: auto-zeroing, chopper stabilizations, switched capacitor, correlated double sampling, dynamic element matching, and etc. [6][7]. Those approaches are effective and have been widely used. However, they are complicated in circuit designs. In literatures, we have not found a paper using control algorithms to compensate imperfections for charge amplifiers, except our preliminary work [8].

In most existing MEMS systems, the imperfections from mechanical structures and interface circuits were minimized physically and individually. The disadvantage of that is costly. Since late 1990s, many feedback control methods have been proposed to compensate the effect resulting from the imperfections in MEMS gyroscopes. These approaches improve the gyroscope performance without expensive micromachining processes and intensive calibration work [8]-[11], and thus could be promising for the mass production of MEMS gyroscopes. Unfortunately, in those reports, the interface circuits were all assumed to be ideal and the mass of the proof mass must be known.

In this paper, we proposed a control algorithm that can compensate the effect from imperfections of both mechanical structures and interface circuits. Different from existing approaches and our previous work [8], the mass of the proof mass does not need to known beforehand, which is expect to further reduce the calibration work for MEMS vibratory
gyroscopes. The proposed method is developed from the state estimation techniques. The estimation properties are discussed in detail. Lastly, two types of frequently used gyroscope designs are used to demonstrate the feasibility of the proposed method.

II. SYSTEM MODELING

A. Gyroscope Dynamics

Two types of mechanical structures are frequently used in MEMS vibratory gyroscope designs. In Fig. 2(a), a single proof mass is suspended in a rigid frame by four flexures. Due to its symmetrical design, the proof mass can move in two axes. This gyroscope design is referred to as the single-mass gyroscope in this paper. As shown in many existing papers [1], [9]-[11], the dynamic equations of this gyroscope can be written as:

\[
\begin{align*}
    m\ddot{x} + d_{xx}\dot{x} + d_{xy}\dot{y} + k_{xx}x + k_{xy}y &= u_x + 2m\Omega_z\dot{y} \\
    m\ddot{y} + d_{xy}\dot{x} + d_{yy}\dot{y} + k_{xy}x + k_{yy}y &= u_y - 2m\Omega_z\dot{x}
\end{align*}
\]  

where \( m \) is the mass of the proof mass; \( d_{xx}, d_{yy}, k_{xx}, k_{yy} \) are damping coefficients and spring constants along two principal axes; \( \Omega_z \) is the angular rate to be measured; \( d_{xy} \) and \( k_{xy} \) are the cross-axis damping coefficient and spring constant; \( u_x \) and \( u_y \) are the control input along \( x \) and \( y \) axis, respectively. This dynamic equation is often normalized by the mass \( m \) to obtain the following equation.

\[
\begin{align*}
    \ddot{x} + \frac{d_{xx}}{m}\dot{x} + \frac{k_{xx}}{m}x + \frac{k_{xy}}{m}y &= \frac{u_x}{m} + 2\Omega_z\dot{y} \\
    \ddot{y} + \frac{d_{xy}}{m}\dot{x} + \frac{k_{xy}}{m}x + \frac{k_{yy}}{m}y &= \frac{u_y}{m} - 2\Omega_z\dot{x}
\end{align*}
\]  

On the other hand, Fig. 2(b) shows a mechanically decoupled gyroscope design that the mass \( m_2 \) is constrained to move along \( x \)-axis, while the mass \( m_1 \) is free to move along both axes. This gyroscope design is referred to as the decoupled gyroscope in this paper. The dynamics of this gyroscope can be obtained by combining the results shown in [12], which are:

\[
\begin{align*}
    (m_1 + m_2)\ddot{x} + d_{xx}\dot{x} + k_{xx}x + k_{xy}y &= \frac{u_x}{m_1} + 2m_1\Omega_z\dot{y} \\
    m_1\ddot{y} + d_{yy}\dot{y} + k_{yy}y &= \frac{u_y}{m_1} - 2m_1\Omega_z\dot{x}
\end{align*}
\]  

Similarly, the dynamics in (3) can be normalized by the mass:

\[
\begin{align*}
    \ddot{x} + \frac{d_{xx}}{m_1}\dot{x} + \frac{k_{xx}}{m_1}x + \frac{k_{xy}}{m_1}y &= \frac{u_x}{m_1} + 2\frac{m_1}{m_2}\Omega_z\dot{y} \\
    \ddot{y} + \frac{d_{yy}}{m_1}\dot{y} + \frac{k_{yy}}{m_1}y &= \frac{u_y}{m_1} - 2\frac{m_1}{m_2}\Omega_z\dot{x}
\end{align*}
\]  

where \( m_z = m_1 + m_2 \) and \( m_y = m_1 \). It is noted that two cross-axis spring constants \( k_{xy}, k_{yx} \) are chosen to be different due to the asymmetric mechanical structure.

The imperfections of mechanical structures mainly contribute to the existence of the \( d_{xy} \) and \( k_{xy} \) and uncertain values of all spring constants, damping coefficients and the mass. Therefore, including angular rates, there are eight unknown parameters in a single-mass gyroscope design, while there are nine unknown parameters in a decoupled gyroscope design.

B. Capacitive Position Sensing

The capacitive position sensing is one of the most popular methods to measure the position of the proof mass because it can be fairly accurate and easily implemented with MEMS technologies. Depending on the mechanism of its varying capacitance design, the capacitive position sensing can be divided into the comb drive scheme and parallel plate scheme. Here, the comb drive scheme is used as an example to illustrate the concept.

As shown in Fig. 3, \( C_{o1} \) and \( C_{o2} \) are the capacitances when the proof mass is at its nominal position; \( \Delta C_1 \) and \( \Delta C_2 \) are the capacitance variations induced by motions of the proof mass. Due to fabrication imperfections, \( C_{o1} \) may not equal to \( C_{o2} \), neither do \( \Delta C_1 \) and \( \Delta C_2 \). For example, if the nominal position is shifted by a distance \( d \) from the neutral position of the structure, the above capacitances can be calculated as (5).

\[
\begin{align*}
    C_{o1} &= N\epsilon\frac{W}{Z}(x_0 + d) \\
    C_{o2} &= N\epsilon\frac{W}{Z}(x_0 - d) \\
    \Delta C_1 &= \Delta C_2 = N\epsilon\frac{W}{Z}x
\end{align*}
\]

where \( N \) is the number of comb fingers; \( \epsilon \) is the permittivity; \( W \) and \( Z \) are the height and gap of comb fingers; \( x_0 \) is the overlapped length between fingers when the proof mass is at its neutral position.
charge sensing) and no input bias current for the operation amplifier (which is pretty much true for amplifiers made of MOS technology), the output voltage $V_o$ can be obtained as:

$$V_o = -\frac{1}{C_f} \int_0^t i_C \, dt + V_i + V_n = -\left(\Delta C_1 + \Delta C_2 + C_{o1} - C_{o2}\right) \frac{V_i}{C_f} + \nu \left(C_{o1} + C_{o2} + \Delta C_1 - \Delta C_2 + C_P + C_f\right) \frac{V_i}{C_f} + V_{n2}$$

where $V_{n2}$ is the noise at around the modulation frequency and its standard deviation is smaller than that of $V_n$; $\nu$ can be very small depending on the design of the low pass filter shown in Fig. 4. If the operational amplifier functions properly in this feedback configuration, $V_i$ equals to $V_{os}$ due to the “virtual ground effect.” By combining (6) and (5), the output voltage of the interface circuit is:

$$V_o = -\frac{2V_s}{C_f} N e \frac{W}{Z} x + \alpha + \nu \beta + V_n$$

Since the values of fabrication error $d$, parasitic capacitance $C_P$, and bias voltage $V_{os}$ are unknown, the values of $\alpha$ and $\beta$ are unknowns. Furthermore, $\alpha$ is an unknown constant; $\beta$ can either be an unknown constant or a drift depending on $V_{os}$ is drifting or not. From (7), the output voltages of the sensing circuit is linearly proportional to the displacement of the proof mass. However, it is noisy and deviated by biases or drifts. To adapted to the proposed control method, we define a new variable $\Phi$:

$$\Phi = \alpha + \nu \beta$$

### III. STATE OBSERVER DESIGN AND FEEDBACK CONTROL

To compensate the effect from both mechanical structure imperfections and circuit errors using state estimation techniques, the system dynamics and interface circuit are modeled together. To save the length of this paper, the following derivation is done for the single-mass gyroscope design only. Thus, by combining (2) and (7), a gyroscope system can be described by the following equations:

$$\dot{X} = f(X) + BU + N_s$$
$$Z = HX + N_m$$

where $Z$ is the output vector of a system; $X$ is the state vector of a system; $\Phi_x$ and $\Phi_y$ are two bias signals (drifts) at the circuit outputs; $N_m$ models the noise, while $N_s$ models the Brownian motions of the mechanical structures; $X$, $f(X)$, $B$, $U$, and $H$ are shown at the top of the next page. It is noted that, except the scale factor in circuit output, every system parameters are assumed to be unknown. Besides,
three new states \((\Phi_x, \Phi_y, \Omega_z)\) and system parameters are all assumed to be constant for now.

A. State Observer Construction

With the system equations shown in (9), a state observer can be constructed as the following:

\[
\begin{align*}
\dot{X} &= f(X) + BU + LH(X - \hat{X}) \\
\dot{Z} &= H\hat{X}
\end{align*}
\]

(10)

where \(L\) is the observer gain and can be chosen from various nonlinear observer algorithms. In this paper, the extended Kalman filtering (EKF) [13] is used for its effectiveness in noise reduction.

When applying EKF to this system, the continuous-time dynamics in (9) are converted into discrete-time difference equations \((f)\) first. And, the so-called “prediction equations” in the EKF can be calculated by the following steps:

\[
\begin{align*}
\hat{X}_{k+1}^- &= f(k, \hat{X}_k) + BU_k \\
P_{k+1}^- &= A_k P_k A_k^T \\
A_k &= \frac{\partial f_k}{\partial X} \Big|_{X = \hat{X}_k}
\end{align*}
\]

(11)

where the subscript \(k\) denotes the values obtained at the \(k\)th sampling time; \(P_k = E((\hat{X}_k - X_k)(\hat{X}_k - X_k)^T)\) is the state covariance matrix. The “correction equations” in the EKF are:

\[
\begin{align*}
L_{k+1} &= P_{k+1}^- H^T (HP_{k+1}^- H^T + R_{k+1})^{-1} \\
P_{k+1} &= (I - L_{k+1} H) P_{k+1}^- \\
\hat{X}_{k+1} &= \hat{X}_{k+1}^- + L_{k+1}
\end{align*}
\]

(12)

where \(R\) is the covariance matrix of the measurement noise.

In the above derivations, all system parameters are assumed to be constant. However, a functional gyroscope needs to measure time-varying angular rates. Also, the bias voltage of amplifiers could be drifting. To cope with these problems, the fading memory technique [13] can be adopted to work with the EKF, as shown in [8].

B. Feedback Control for Gyroscope System

Since all system parameters and dynamics are estimated in real time, the estimated states can be used to implement feedback controls for gyroscopes. Among various controller designs, we choose the one which keeps the total energy transferred between two axes the same. This control method is chosen because it enforces the feedback system to operate at the resonant frequency of the original system. Thus, the control input can be less. To implement this method, the control input is designed as follows:

\[
U = \begin{bmatrix}
\dot{d}_{xx} \dot{x} + \dot{d}_{xy} \dot{y} + \omega_x \\
\dot{d}_{xy} \dot{x} + \dot{d}_{yy} \dot{y} + \omega_y
\end{bmatrix}
\]

(13)

where \(\omega_x\) and \(\omega_y\) can be chosen as proper bounded signals to avoid interfering the system stability.

Once the values of estimated states and parameters converge to their correct values, the trajectory of proof mass can be described by the following equations:

\[
\begin{align*}
\ddot{x} + \frac{k_{xx}}{m} \dot{x} + \frac{k_{xy}}{m} \dot{y} &= 2\Omega \dot{y} + \omega_x \\
\ddot{y} + \frac{k_{xy}}{m} \dot{x} + \frac{k_{yy}}{m} \dot{y} &= -2\Omega \dot{x} + \omega_y
\end{align*}
\]

(14)

IV. Observability Analysis

Since the proposed method uses state estimation techniques to estimate system states and unknown parameters, the feasibility of the estimation can be examined by the rank of the observability matrix. The observability matrix of a nonlinear system is obtained by the following:

\[
W_o = \frac{\partial}{\partial X} \begin{bmatrix}
X \\
\dot{X}
\end{bmatrix}
\]

(15)

For this feedback control gyroscope system, the observability matrix \((W_o)\) is calculated and has the following format:

\[
W_o = \begin{bmatrix}
[W_{ss}]_{6 \times 6} & [0]_{6 \times 8} & [W_{kd}]_{8 \times 8}
\end{bmatrix}_{14 \times 14}
\]

(16)

After tedious derivations, the above \(W_{ss}\) and \(W_{kd}\) matrices can be greatly simplified to the following:

\[
W_{ss} = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & -k_{xx} & -k_{xy} & 0 \\
0 & 0 & 0 & -k_{xy} & -k_{yy} & 0
\end{bmatrix}
\]

(17)

The \(W_{kd}\) is shown at the top of the next page.

For \(W_{ss}\), as long as \(k_{xx}k_{yy} \neq k_{xy}^2\), its rank is six and thus the associated six states are globally observable. Similarly, it
can be shown that the rank of $W_{kd}$ is eight if the oscillation of the proof mass contain more than one frequency and a proper choice of $\varpi$.

V. SIMULATIONS

In the following simulations, the proof mass is assumed to be actuated around $1.5 \mu m$, $3 \, kHz$. The circuit outputs are assumed to be DC-biased and contaminated by white noise. Those bias values are $15 \, mV$ for x-axis and $11 \, mV$ for y-axis. The standard deviation of the noise is $0.5 \, mV$. The parameters of gyroscope model are listed in Table I. And, the initial guess of those parameters are $10\%$ to $20\%$ off from their correct values. Two control signals $\varpi_x$ and $\varpi_y$ are both chosen to be $\sin(2\pi \times 300t)$. The sampling rate of the control algorithm is $10 \, MHZ$.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m$</td>
<td>$1.8 \times 10^{-3} , kg$</td>
</tr>
<tr>
<td>$\Omega_x$</td>
<td>$1 , rad/sec$</td>
</tr>
<tr>
<td>$k_{xx}$</td>
<td>$63.955 , N/m$</td>
</tr>
<tr>
<td>$k_{yy}$</td>
<td>$95.92 , N/m$</td>
</tr>
<tr>
<td>$k_{xy}$</td>
<td>$12.779 , N/m$</td>
</tr>
<tr>
<td>$d_{xx}$</td>
<td>$1.8 \times 10^{-6} , N \cdot s/m$</td>
</tr>
<tr>
<td>$d_{yy}$</td>
<td>$1.8 \times 10^{-6} , N \cdot s/m$</td>
</tr>
<tr>
<td>$d_{xy}$</td>
<td>$3.6 \times 10^{-7} , N \cdot s/m$</td>
</tr>
<tr>
<td>$\Phi_x$</td>
<td>$15 , mV$</td>
</tr>
<tr>
<td>$\Phi_y$</td>
<td>$11 , mV$</td>
</tr>
</tbody>
</table>

VIBRATORY GYROSCOPE

Fig. 5 shows the circuit output voltages of the gyroscope system, which correspond to the position of the proof mass along x-axis and y-axis. They are dc-biased and contaminated by white noise. Using the proposed method, the estimated proof mass position and velocity are shown in Fig. 6. According to simulation results, the estimated values quickly converge to their correct values. Figure 7 shows the estimated values of unknown system parameters, which includes two bias signals, seven system parameters and one angular rate. The estimated values converge to their correct values within 80 milli-seconds.

The other simulations are shown for the decoupled gyroscope design, which parameters are listed in Table II. According to the simulation results shown in Fig. 8 and Fig. 9. All the estimated state values converge to their correct values within 50 milli-seconds.

VI. CONCLUSIONS

In this paper, the effect of mechanical structure imperfections were accounted as unknown proof mass, sprung constants, and damping coefficients of a dynamic system. The effect of interface circuit imperfections were accounted as unknown signal drifts and measurement noises. The control input is designed to consist of state feedback and one bounded signal. By doing so, the proof mass trajectory is regulated to oscillate at more than one frequency. And, these unknown parameters, along with system dynamics (position and velocity), can be correctly estimated using state estimation techniques. Besides, the estimation properties can be examined by observability matrices.
In simulations, two types of gyroscope designs, single-
mass and decoupled, are both utilized to verify the concept. 
In the single-mass design, there are 10 unknown parame-
ters and four system states, while there are 11 unknown
parameters and four system states in the decoupled design. 
The extended Kalman filter are utilized to estimate those
unknown values, including angular rates, in real time. The
conversion time of angular rates estimation is less than 80
milliseconds.

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