I. INTRODUCTION

As applications that involve physical human-robot interaction (pHRI), such as series elastic actuators, should generate the desired torque precisely. However, the resistive factors include nonlinear effects and should be considered in the controller design to generate the desired force accurately. Moreover, the uncertainties in the plant dynamics make the precise torque control difficult. In this paper, nonlinear control algorithms are exploited for a rotary series elastic actuator to generate the desired torque precisely in the presence of nonlinear resistive factors and modeling uncertainty. The sliding mode control smoothed by a boundary layer is applied to enhance the robustness for the modeling uncertainty without chattering phenomenon. In this paper, the rotary series elastic actuator (RSEA) is installed on the knee joint of an orthosis, and the thickness of the boundary layer is changed by gait phases in order to minimize the torque error without the chattering phenomenon. The performance of the proposed controller is verified by experiments with actual walking motions.

II. PRELIMINARY WORKS

A. A Rotary Series Elastic Actuator

A rotary series elastic actuator (RSEA) has been proposed for assisting human motions [5]. The RSEA consists of a DC motor, a torsional spring, and two encoders each on the human side and the motor side. The torsional spring is used to generate the desired torque, and acts as an energy buffer between the actuator and the human joint. The similar approaches are shown in [1]–[4], where they applied a linear spring for the same purpose. Since a linear spring requires torque arms for generating torque, a torsional spring is...
directly installed between the human joint and the motor in the design of the RSEA. The position of the DC motor is controlled to have the proper spring deflection such that the RSEA generates the desired torque precisely. The overall design of the RSEA is shown in Fig. 1, and the details are given [5].

B. Identification of Motor Resistive Torque

The actuating torque is generated by a geared DC motor in the RSEA. Geared motors have been widely used in pHRI applications due to their great controllability and flexibility, but nonlinear resistive torque introduced by a gear reducer makes the accurate generation of the desired torque difficult. The resistive torque is undesirable characteristics for pHRI actuators since a human has to make an additional effort to overcome the resistive torque. Thus the resistive torque should be compensated for improved pHRI.

The measured resistive torque in the RSEA is shown in Fig. 2. In an ideal case, the resistive torque should be zero regardless of the angular velocity when the control input is zero (labeled Target in Fig. 2). However, due to the friction force, there is the resistive torque in actual device (labeled Actual in Fig. 2). Note that the magnitude of the bias force is so large that the motor rotates even under zero control input. This phenomenon often occurs in applications of DC motors. The discontinuity at zero angular velocity represents the Coulomb friction force.

The resistive torque shown Fig. 2 is modeled as follows.

\[ \tau_{\text{resistive}} = a_1 + a_2 \text{sgn}(\dot{\theta}_M) + a_3 \dot{\theta}_M \]  

(1)

where the coefficients \(a_1, a_2,\) and \(a_3\) represent terms due to bias, nonlinear friction, and linear damping, respectively. The values of \(a_1, a_2,\) and \(a_3\) were found using a curve fitting method with given data, and the values are shown in Table I.

### III. NONLINEAR CONTROLLER DESIGN FOR A ROTARY SERIES ELASTIC ACTUATOR

A. System Modeling

A schematic diagram of the RSEA installed on a human joint is depicted in Fig. 3. \(I_M\) is the inertia of the motor, and \(\theta_M\) and \(\theta_H\) are the angles of the motor and the human joint, respectively. \(\tau_M\) represents the motor torque, and \(\tau_{\text{resistive}}\) is the resistive torque. The motor and the human joint are connected via a torsional spring with spring constant \(k,\) and the controlled output is the spring torque which is proportional to the spring deflection, i.e., the difference between the motor and the human joint angle.

The following relations are obtained from Fig. 3 by applying Newton’s law and Hooke’s law,

\[ I_M \ddot{\theta}_M = \tau_M(t) + k(\theta_H - \theta_M) - \tau_{\text{resistive}} \]  

(2)

where \(\tau_{\text{resistive}}\) is defined in (1).

---

**TABLE I**

<table>
<thead>
<tr>
<th>Coeff. (Unit)</th>
<th>(a_1(\text{Nm}))</th>
<th>(a_2(\text{Nm}))</th>
<th>(a_3(\text{Nm}/(\text{rad/}\text{sec})))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>(-1.509 \times 10^0)</td>
<td>(9.572 \times 10^{-1})</td>
<td>(1.414 \times 10^{-1})</td>
</tr>
</tbody>
</table>
For a state space system model, let \( x_1 = \theta_M, \ x_2 = \dot{\theta}_M, \ u = \tau_M, \) and \( y = x_1. \) Then, the resulting state space system equation is,

\[
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= -\frac{k}{I_M} x_1 + \frac{1}{I_M} u - \frac{\tau_{\text{resistive}}}{I_M} + \frac{k}{I_M} \theta_H \\
y &= x_1
\end{align*}
\]  

(3)

B. Feedback Linearization

The feedback linearization control technique algebraically transforms a nonlinear system into a linear one such that linear control methods can be applied [7]. The nonlinearities in the system is canceled by the feedback linearization control input, thus the closed-loop dynamics is in a linear form. To define the control law for the feedback linearization controller, the output \( y \) is differentiated until the control input \( u \) appears. In case of the system in (3), the control input \( u \) appears in the second derivative of \( y \), which means the relative degree of the system is two.

\[
\begin{align*}
\dot{y} &= \dot{x}_1 = x_2 \\
\ddot{y} &= \dot{x}_2 = -\frac{k}{I_M} x_1 + \frac{1}{I_M} u - \frac{\tau_{\text{resistive}}}{I_M} + \frac{k}{I_M} \theta_H \equiv v
\end{align*}
\]  

(4)

where \( v \) is the synthetic input. To determine the synthetic input, a pole placement method is applied, i.e.

\[
v = -c_0(y - y_d) - c_1(\dot{y} - \dot{y}_d) + \ddot{y}_d
\]  

(5)

The values of \( c_0 \) and \( c_1 \) are chosen such that the equation is asymptotically stable: i.e., \( c_0 > 0 \) and \( c_1 > 0 \) in (5). Thus the overall control input for feedback linearization is

\[
u = I_M \cdot (v + \frac{k}{I_M} x_1 + \frac{\tau_{\text{resistive}}}{I_M} - \frac{k}{I_M} \theta_H)
\]  

(6)

Note that the human joint angle, \( \theta_H \), appears in the feedback linearization control law. This contributes to compensation of the human factors imposed on the motor side. However, there are drawbacks in feedback linearization such as 1) it requires accurate model parameters, and 2) there exists internal dynamics if the relative degree is smaller than the dimension of the system. The internal dynamics is problematic if it is unstable. The internal dynamics does not exist in the present problem, since the relative degree is equal to the dimension of the system. The requirement of an exact model, however, may cause problems in actual implementation since there is always modeling inaccuracy in the nominal model.

C. Robustness Enhancement by Sliding Mode Control

The drawback of the feedback linearization control, i.e., the requirement of accurate model parameters, can be overcome by sliding mode control. Sliding mode control deals with modeling uncertainty by applying additional terms to a nominal model [7]. In this section, the design of a typical sliding mode controller for the system in (3) is reviewed briefly.

Given the spring deflection \( E = \theta_M - \theta_H \), the error between the actual and the desired deflection and its derivative are defined as follows.

\[
\begin{align*}
\varepsilon &= E - E_d = (\theta_M - \theta_H) - (\theta_{M_d} - \theta_H) = \theta_M - \theta_{M_d} \\
\dot{\varepsilon} &= \dot{\theta}_M - \dot{\theta}_{M_d}
\end{align*}
\]  

(7)

where \( E_d \) is the desired spring deflection to generate the desired torque.

The sliding surface, \( S \), is defined by

\[
S = (\frac{d}{dt} + \lambda)\varepsilon
\]  

(8)

\[
\begin{align*}
\dot{S} &= \dot{x}_1 + \lambda \dot{x}_2 \\
&= -\frac{k}{I_M} x_1 + \frac{1}{I_M} u - \frac{\tau_{\text{resistive}}}{I_M} + \frac{k}{I_M} \theta_H + \lambda(\theta_M - \theta_{M_d})
\end{align*}
\]  

(9)

where \( \lambda \) is a positive constant.

To assure that the tracking error, \( \varepsilon \), converges to zero, the sliding variable must converge to zero, which takes place if,

\[
S \dot{S} < -\eta |S|
\]  

(10)

where \( \eta \) is a positive constant.

\( S \) can be calculated as follows.

\[
\begin{align*}
S &= (\theta_M - \theta_{M_d}) + \lambda(\theta_M - \theta_{M_d}) \\
&= -\frac{k}{I_M} x_1 + \frac{a_3}{I_M} x_2 - \frac{a_2}{I_M} sgn(x_2) - \frac{a_1}{I_M} \\
&+ \frac{k}{I_M} \theta_H - \theta_{M_d} + \frac{1}{I_M} u + \lambda(\theta_M - \theta_{M_d})
\end{align*}
\]  

(11)

\[
CE(x) = \lambda(\theta_M - \theta_{M_d})
\]  

(12)

\[
b(x) = \frac{1}{I_M}
\]  

(13)

The parameters in \( f(x) \) and \( b(x) \) are to be obtained by system identification. However, the assumed model shown in (3) may not fully reflect the actual dynamics due to modeling uncertainties. Therefore, \( f(x) \) and \( b(x) \) may depend on unmodeled dynamics and time varying dynamics as well as the nominal dynamics. Thus suppose \( f(x) \) and \( b(x) \) are composed of the nominal model \( f_0(x) \) and \( b_0(x) \) and uncertain parts \( \Delta f(x) \) and \( \Delta b(x) \), respectively: i.e.,

\[
\begin{align*}
f(x) &= \tilde{f}(x) + \Delta f(x) \\
b(x) &= \tilde{b}(x) \cdot \Delta b(x)
\end{align*}
\]  

(14)

(15)

It is assumed that \( \Delta f(x) \) and \( \Delta b(x) \) are state dependent, and that their upper bound can be found as follows.

\[
\frac{|\Delta f(x)|}{\tilde{f}(x)} \leq \alpha(x) \leq \frac{b(x)}{b(x)} 
\]  

(16)

(17)
A control input $u$ that satisfies the condition in (9) is
\begin{equation}
    u = \frac{1}{b(x)}[-\hat{f}(x) - CE(x) - K \cdot \text{sgn}(S)]
\end{equation}
where $K$ is chosen to guarantee the condition (9) such that the system is properly controlled even in the presence of modeling uncertainties. To select a proper $K$, the following worst cases for $\Delta f(x)$ and $\Delta b(x)$ are assumed, i.e.
\begin{align}
    \Delta f(x) &= \alpha(x) \\
    \Delta b(x) &= \frac{b(x)_{\text{min}}}{b(x)} \equiv \beta_{\text{min}}(x)
\end{align}
Then $K$ can be given by
\begin{equation}
    K = \frac{(1 - \beta_{\text{min}})(\hat{f}(x) + CE) + \alpha + \eta}{\beta_{\text{min}}}
\end{equation}
The control law in (18) with $K$ in (21) always satisfies the condition in (9). Note that once $S = 0$ is achieved, the error $e(t)$ converges to zero for any $\lambda > 0$ and $1/\lambda$ is the time constant of the error convergence. However, since the control law is discontinuous across the sliding surface ($S = 0$), it introduces chattering, which is not desirable in general. In particular, the noise and vibration caused by the chattering phenomenon are not desirable in the pHRI application.

**D. Gait Phase-Based Smoothed Sliding Mode Control**

To reduce the chattering phenomenon in the sliding mode control, a saturation function shown in Fig. 4 is often applied instead of a signum function. Outside of the boundary layer $\Phi_t$, the control law is chosen to satisfy the condition in (9), which guarantees that the boundary layer is attractive. And trajectories starting inside the boundary layer remain inside the boundary layer [7].

The control input of the smoothed sliding mode control is given by
\begin{equation}
    u = \frac{1}{b(x)}[-\hat{f}(x) - CE(x) - K \cdot \text{sat}(\frac{S}{\Phi})]
\end{equation}
By applying the saturation function instead of the signum function, the chattering phenomenon can be decreased, but the tracking performance is deteriorated. By adjusting the thickness of the boundary layer, the chattering phenomenon and the tracking error can be traded off. That is, if the thickness of the boundary layer is close to zero, then the controller acts like the sliding mode controller with a signum function, which shows more chattering and less tracking error. On the contrary, if the thickness of the boundary layer is large, then the chattering phenomenon disappears but the tracking performance is much deteriorated.

The torque output of the RSEA, $\tau$, is generated by the spring deflection between the motor and the human joint, i.e.
\begin{equation}
    \tau = k(\theta_{md} - \theta_h)
\end{equation}
Given the desired torque, $\tau_d$, the desired spring deflection is $\frac{\tau_d}{k}$. Then the desired motor angle, $\theta_{md}$, is determined by
\begin{equation}
    \theta_{md} = \frac{\tau_d}{k} + \theta_h
\end{equation}
Note that the desired motor angle is dependent on the human joint angle as in (24) since the appropriate spring deflection is required to generate the desired torque. When the human joint moves fast with a large angle change, the desired motor angle trajectory is also large and changes fast, which makes the tracking error large. To decrease the tracking error without chattering phenomenon, the thickness of the boundary layer needs to be adjusted according to human motion.

In this paper, the RSEA is installed on the knee joint and the thickness of the boundary layer is changed according to two major gait phases, i.e., swing and stance phases. In the swing phase, the movement of the knee joint is large and fast, which makes the tracking error large. On the contrary, the knee hardly moves in the stance phase. Thus the boundary layer is set thinner in the swing phase than in the stance phase as shown Fig. 5 to decrease the torque error without the chattering phenomenon. The actual value of the thickness of the boundary layer in each phase, $\Phi_{SW}$ and $\Phi_{ST}$, is adjusted manually since discomfort feeling caused by the chattering phenomenon and tracking error must both be considered. Also the boundary layer is changed smoothly as shown in Fig. 5 to avoid discomfort feeling when the thickness of the boundary layer is changed.

The swing and stance phases are detected by Smart Shoes [8]–[11]. Smart Shoes measure the ground contact forces (GCFs) by four force sensing units embedded under the insole. In the swing phase, all GCFs are zero since the foot is in the air. In the stance phase, the sum of the GCFs indicates the body weight since the whole body weight is transferred to the foot. At the heel strike phase (the first phase of the stance phase), the sum of the GCFs is over the body weight.
due to the impact force at the heel strike. In the experiment in the next section, if the sum of the GCFs is greater than 5% of the body weight (body weight is about 650 N in the experiment), then the human is considered in a stance phase. Otherwise he/she is regarded in a swing phase.

IV. PERFORMANCE ANALYSIS BY EXPERIMENTS

A. Smoothed Sliding Mode Control

In this experiment, the performance of smoothed sliding mode control is verified comparing with that of sliding mode control. The desired spring deflection was set to sine wave with an amplitude of 1 rad at 1 Hz, and arbitrary motions were given to the human joint side. Fig. 6 shows the experimental results with sliding mode control with control input (18) and smoothed sliding mode control with the control input (22). The motor moves around the human joint to make appropriate spring deflection for the generation of the desired joint torque. The generated torque is calculated by the spring deflection multiplied by the spring constant $k$ [5]. As shown in Fig. 6(a), sliding mode control with the signum function shows extremely high performance, but the chattering phenomenon is observed as shown in the tracking error graph in Fig. 6(a). The vibration and noisy sound by the chattering effect can be easily felt from the human side, which is highly undesirable for pHRI. Smoothed sliding mode control shows a slightly worse performance comparing to pure sliding mode control as shown in Fig. 6(b), but the chattering phenomenon is not observed.

B. Gait Phase-Based Smoothed Sliding Mode Control

The experiments in Fig. 6 shows the performance of the smoothed sliding mode controller without actual human motions. To verify the performance of the proposed method, i.e., the smoothed sliding mode controller with a varying boundary layer on an actual human, the experimental setup shown in Fig. 7 was utilized. One RSEA was installed on the knee joint, and controlled by the proposed control algorithm. The subject wearing this experimental setup walked on a treadmill at about 1.5 km/h.

The desired joint torque pattern in one stride is given in Fig. 8. Just after the heel strike, the knee torque to the flexion direction is applied for the subject to bend the knee easily, and the knee torques to the flexion or the extension directions are applied in turn to help the knee motions in a swing phase.

The experimental result with actual walking motion is shown in Fig. 9. The experimental results with a constant boundary layer are shown in Fig. 9(a). The spring is deflected appropriately by the motor angle change. The torque error is large when the leg is in swing phases as shown in Fig. 9(a) due to the large and fast knee motion. The large torque error can be decreased by reducing the thickness of the boundary layer in the swing phase. Fig. 9(b) shows the experimental result with a varying boundary layer. The thickness of the boundary layer is reduced only in the swing phases and remains the same in the stance phase. The root-mean-square (RMS) values of the torque error of each control algorithm in each phase are calculated, and the values are shown in

![Fig. 6. Experimental results: the desired torque is set as a sinusoidal wave](image)

![Fig. 7. The RSEA and Smart Shoes installed on the orthosis](image)

![Fig. 8. Desired knee joint torque profile in one stride](image)
for physical human-robot interaction. Thus smoothed sliding mode control by utilizing a boundary layer is applied. The rotary series elastic actuator is installed on the knee joint on an orthosis, and the thickness of the boundary layer is varied for the swing and stance phases to decrease the torque error without the chattering phenomenon. The experimental results verify the advantage of the sliding mode control with the varying boundary layer based on the gait phases.

REFERENCES