A Feature based solution to Forward Problem in Electrical Capacitance Tomography

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Abstract—A new feature-based technique is introduced to solve the nonlinear Forward Problem (FP) of the Electrical Capacitance Tomography (ECT) with the target application of monitoring the metal-fill profile in Lost Foam Casting (LFC) process. The new technique to solve the FP is based on key features extracted from the metal distributions and the Correction Factor (CF). The CF is predicted by an Artificial Neural Network (ANN) based on key distribution features. The CF adjusts the linear solution of the FP for nonlinear effects. The data for the ANN training was generated through ANSYS finite element analysis and the codes written in MATLAB. The ANN was implemented using MATLAB Neural Network Toolbox. This approach shows promising results. The ANN was able to learn the effect of these features on the CF with the % RMS error of 2.21 for training data. For the previously unseen test metal distributions, the average RMS error was 2.2%.

Index Terms — Artificial Neural Network (ANN), Forward Problem (FP), Electrical Capacitance Tomography (ECT), Lost Foam Casting (LFC).

I. INTRODUCTION

Traditionally the ECT is a method of imaging the permittivity distribution inside a closed vessel or flask by measuring the variations in mutual electrical capacitance between the sets of electrodes mounted on its periphery [1]. In this research, this technique has been used to monitor the grounded molten metal distribution in LFC process. The motivation and application are introduced in Section II.

There are two major computational problems in ECT; the Forward Problem (FP) and the inverse problem. The FP is to determine the inter-electrode capacitances for a given grounded metal distribution. The inverse problem is to reconstruct the metal distribution image from the given inter-electrode capacitance measurements. The result is usually presented in the form of an image hence inverse problem solution is also known as image reconstruction [2].

The inverse problem is inherently a highly nonlinear problem due to the shielding effect of metal and the small number of independent measurements (typically 28 to 66) compared to the number of unknowns (the number of pixels in the reconstructed metal distribution image, typically 256 to 1024) [1].

The algorithms to solve the inverse problem are divided into two categories: Iterative algorithms and Non-Iterative algorithms [2]. Due to the ill posed and ill conditioned nature of the inverse problem [2], the non-iterative algorithms (such as linear back-projection (LBP), singular value decomposition, Tikhonov regularization etc) cannot provide an accurate solution [2]. Accurate images could be obtained by using iterative algorithms [2]. Fig. 1 depicts a generic iterative algorithm. First it solves the inverse problem using a non-iterative algorithm then it calculates the inter-electrode capacitances for the reconstructed metal image (i.e. solving the FP), and then modifying the image in order to minimize the discrepancy between the measured capacitances and the calculated capacitances. This process is repeated until a satisfactorily low level of discrepancy is achieved. So it is necessary to solve the FP at least once per iteration for iterative algorithms. The accuracy of the inverse problem solution critically depends on the accuracy of the FP solution.

The linear approach to solving the FP is based on the sensitivity matrix. The sensitivity matrix represents the response of the sensors by changing individually each element inside the imaging area from foam to the grounded metal. Based on that, the inter-electrode capacitance is given by Equation (1):

$$C_{i(m,n)} = S_{max} \times I_{act}$$

where $S_{max}$ is the sensitivity matrix, $C_{i(m,n)}$ is the capacitance measurements vector or the linear capacitance, $I_{act}$ is the image vector, $m$ is the number of independent mutual capacitance measurements or number of electrode pairs, and $n$ is the image resolution or number of pixels in the reconstructed image. Equation (1) provides very fast and computationally simple solution to the FP. However, they suffer from the lack of accuracy due to the linearization of an inherently nonlinear problem of ECT, especially in the case of conductive materials [1].

Numerical techniques for solving FP can produce accurate solutions for any arbitrary metal distribution. However, the accuracy comes at the cost of excessive computational time. The analytical methods and ANFIS based model can produce fairly accurate solutions with little time consumption. However their application is limited to very simple metal distributions with symmetry.

A Neural Network based solution that uses a multi-layer feed-forward ANN to predict the capacitive response of the sensor electrodes was introduced in [7]. This multi-input multi-output ANN was trained with the metal distribution image itself as input and corresponding capacitances as outputs. As a result of that, it requires a huge ANN with number of inputs between 256 and 1024 (depending on the distribution image resolution) and number of outputs between 28 and 66 (depending on number of sensor electrodes.
installed). Consequently it requires huge training data, training time and computer memory.

II. PROBLEM STATEMENT

A. Lost Foam Casting

LFC is a technique of metal casting that uses the expendable foam patterns to create the casting. The hot molten metal evaporates the foam and then replaces it, thus creating a casting. The LFC process is highly attractive among the metal casters because of its low cost, flexibility and improved dimensional accuracy.

One of the main problems with LFC is the formation of the casting related defects such as cold shut, fold, mis-run etc. most of the casting defects are caused by the casting variables (coating permeability, foam density, glue joints, pouring or metal temperature, and gating effect etc). The possible defect type can be obtained by monitoring the metal fill profile [5]. Hence by studying the profile of the metal fill the casting defects can be predicted and minimized. The ECT is a cheaper, easy to install and non-invasive method of monitoring such process.

B. Electrical Capacitance Tomography System

An ECT system is generally composed of three different units [6]:

- Capacitive sensors
- Measurement and processing hardware
- Computer system for image reconstruction process, display and (if possible) control

Fig. 2 shows the ECT configuration considered in this research. It had 12 sensor electrodes installed in circular manner around the area of interest (Imaging area) which provides 66 independent mutual capacitance measurements. The distribution image resolution was 16x16 or 256 pixels.

In order to monitor the metal fill profile, in LFC, accurately and online it is necessary to have a fast and accurate solution to the FP of ECT. This research work introduces a new approach to produce fast and accurate solution of the FP.

\[
C = C_l \times CF
\]

Fig. 3 (a) A metal distribution (b) Corresponding Actual and linear inter-electrode capacitances

Table 1 shows the values of CF for the metal distributions shown in Fig. 4 for six electrode pairs. Note that values of CF are considerably lower for the distribution with higher amount of metal.

<table>
<thead>
<tr>
<th>Electrode Pair</th>
<th>Metal Distribution (a)</th>
<th>Metal Distribution (b)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-2</td>
<td>0.66</td>
<td>0.10</td>
</tr>
<tr>
<td>1-3</td>
<td>0.25</td>
<td>0.06</td>
</tr>
<tr>
<td>1-4</td>
<td>0.12</td>
<td>0.03</td>
</tr>
<tr>
<td>1-5</td>
<td>0.06</td>
<td>0.02</td>
</tr>
<tr>
<td>1-6</td>
<td>0.04</td>
<td>0.02</td>
</tr>
<tr>
<td>1-7</td>
<td>0.01</td>
<td>0.02</td>
</tr>
</tbody>
</table>

III. OVERVIEW OF THE PROPOSED SOLUTION

A. Correction Factor

As given in Equation (1), the linear capacitance is calculated by multiplication of the sensitivity matrix with the metal distribution vector. In other words, it is just the addition of capacitances for each metal element present in the given distribution individually so its calculations do not take nonlinear effects such as shielding effect into account. Simulations based on ANSYS and MATLAB show that linear capacitance does provide qualitative values. Fig. 3 (b) shows the actual capacitances and linear capacitances for all sixty six electrode pairs for the metal distribution shown in Fig. 3 (a). Clearly both are showing the same behavior with some scaling. This scaling is different for each electrode pair. For some electrode pairs it is close to 1 and for some others it is as high as 3. If the scaling factors were known, the multiplication of the inverse of scaling factors (or the correction factors) with corresponding linear capacitances will produce the actual capacitance. Thus the Correction Factor (CF) can adjust the linear solution for the nonlinear effects.

\[
C = C_l \times CF
\]
B. Key Distribution Features

As mentioned earlier, use of ANN for FP solution in ECT is not new. In [7] an ANN-based technique was introduced, which requires the grounded molten metal distribution as input (a vector of ‘0’ s and ‘1’ s, where ‘0’ represents the absence and ‘1’ represents the presence of the metal in corresponding pixel). The output of the ANN was normalized capacitances from the electrode pairs. The main limitation with this technique is use of a huge ANN. The total number of connections in this ANN was in the order of 20000. Consequently it requires a huge amount of training data in order to avoid the over fitting problem and to improve the generalization [8]. Additionally the training time and computer memory requirements will be huge. Another disadvantage of this technique is the fast increase in size of ANN with increase in image resolution. For example if the image resolution changes from 20x20 to 32x32, for same network architecture, the number of connections in the ANN will be 45200.

To overcome these problems, this work introduces a technique for solving the FP by training an ANN with key features extracted from the grounded metal distribution instead of training with the distribution itself. The key features are unique numbers associated with a given metal distribution and sensitivity map of the subject electrode pair. They carry the information which directly or indirectly affects the mutual capacitance, such as how metal is physically distributed, separation between electrodes, orientation of electrodes, etc.

IV. EXTRACTION OF KEY FEATURES FROM THE GROUNDED METAL DISTRIBUTION

The proposed ANN uses eight key features extracted from the metal distribution to predict the CF. The aim behind their extraction was to capture the characteristics of the metal distribution which affect the CF. The definitions of these features, their calculation, and the information they carry are presented hereafter. Extraction of some of these features requires the calculation for the center of mass of the metal distribution. This is not same as physical center of gravity but it is the center of mass with respect to the sensitivity map of the subject electrode pair. Equation (3) shows how it is calculated:

\[
\begin{align*}
\bar{x}^j_0 &= \frac{\sum_{i=1}^{N} x_i^j S^j_i}{\sum_{i=1}^{N} S^j_i}, \\
\bar{y}^j_0 &= \frac{\sum_{i=1}^{N} y_i^j S^j_i}{\sum_{i=1}^{N} S^j_i}
\end{align*}
\]  

where \((\bar{x}^j_0, \bar{y}^j_0)\) are the coordinates of the center of mass of the given metal distribution for \(j^{th}\) electrode pair, \((x_i^j, y_i^j)\) are the coordinates of the center of the \(i^{th}\) metal element present in the given distribution, \(S^j_i\) is the sensitivity of the \(j^{th}\) electrode pair for \(i^{th}\) element, and \(N\) is the total number of metal elements present in the distribution.

A. Mean Distance of Metal Elements from the Distribution’s Center of Mass (AD)

\[ AD_j = \frac{1}{N} \sum_{i=1}^{N} \sqrt{(x_i^j - \bar{x}^j_0)^2 + (y_i^j - \bar{y}^j_0)^2} \]  

where \(N\) is the total number of metal elements present in a given distribution, \((x_i^j, y_i^j)\) are the coordinates of \(i^{th}\) pixel having metal, and \((\bar{x}^j_0, \bar{y}^j_0)\) are coordinates of the center of mass of the metal distribution for \(j^{th}\) pair.

B. Scattering among the Sensitivities (SD)

This feature is basically the Standard Deviation (SD) of sensitivity map elements associated with a given metal distribution. Since each electrode pair has a different sensitivity map, depending upon its proximity, location and orientation with respect to the imaging area, this feature depends on both, given metal distribution and the subject electrode pair. Following equation shows how it is calculated:

\[ SD_j = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (S^j_i - S^j_{\text{ave}})^2} \]  

where \(N\) is total number of pixels filled with metal, \(S^j_i\) is sensitivity for \(i^{th}\) pixel in sensitivity map for \(j^{th}\) electrode pair, and \(S^j_{\text{ave}}\) is average of sensitivities for all pixels filled with metal.

C. Weighted Sum of the Metal Distribution (WS)

Weighted Sum (WS) for a given metal distribution is calculated by adding up the sensitivities of the pixels where the metal is present for a given electrode pair. It is similar to the multiplication of the sensitivity matrix with the image vector or calculating the linear capacitances. This feature indicates how the metal is distributed among high and low sensitivity regions of the imaging area. The higher the weighted sum is, the higher is the probability that the metal is distributed in high sensitivity region/s. This feature had no units. Since each electrode pair has a different sensitivity map, this feature depends on both, given metal distribution and the subject electrode pair.

\[ WS_{\text{max1}} = S^j_{\text{max}} \times I_{\text{max1}} \]  

where \(WS_{\text{max1}}\) is the weighted sum vector, \(S_{\text{max}}\) is the linear sensitivity matrix comprising \(m\) sensitivity maps (one map for each electrode pair) arranged in rows with \(n\) columns representing image pixels, \(I_{\text{max1}}\) is the image vector. The image vector contains ‘1’s and ‘0’s (‘1’ represents the presence and ‘0’ represents the absence of
the metal in corresponding pixel), \(m\) is the number of electrode pairs and \(n\) is the total number of elements in image vector.

**D. Distance of the Electrodes from the Distribution’s Center of Mass (AED)**

This feature is the actual distance of the electrodes from the metal distribution’s center of mass for the subject electrode pair. This feature represents the average distance of the metal elements from the electrodes and shows how far metal would be present if it were concentrated at only one point.

This feature depends on both the distribution and the subject electrode pair. The following equation shows the calculation of this feature:

\[
AED_j = \sqrt{(x_e - x_{cm})^2 + (y_e - y_{cm})^2}
\]

where \((x_e, y_e)\) are the coordinates of the electrodes and \((x_{cm}, y_{cm})\) are the coordinates of the metal distribution’s center of mass with respect to the \(j^{th}\) electrode pair.

**E. Ratio of Weighted Sum to the Total Sensitivity (WS/TS)**

Total Sensitivity (TS) is the addition of all elements in the sensitivity map of a given electrode pair. Its calculation is shown in Equation (8). The TS does not depend on the metal distribution as it is constant for any given electrode pair. However, the weighted sum depends on both the metal distribution and electrode pair, so this feature also depends on both the metal distribution and electrode pair. Since it was a ratio, it does not have unit.

\[
TS_j = \sum_{i=1}^{n} s_{ij}
\]

where \(s_{ij}\) is sensitivity of element for \(i^{th}\) pixel in metal image for \(j^{th}\) electrode pair and \(N\) is total number of pixels in the image.

The ratio of weighted sum to total sensitivity represents the ratio of imaging area filled by metal in terms of the sensitivity or how much sensitivity area is occupied by the given metal distribution for that particular electrode pair. The highest value possible for this ratio is 1, because the weighted sum will always be less than the total sensitivity except for the case when whole imaging area is filled with metal where the ratio will be 1.

**F. Distance between the Electrodes (DBE)**

This feature is the actual Distance between the Electrodes (DBE) of the subject electrode pair. It is an important feature because mutual capacitance between two electrodes is inversely proportional to the distance between them and DBE carries the information that how far the electrodes are located from each other. Since it is the only feature which is independent from the given metal distribution and only depends on the subject electrode pair, it helps the ANN to identify the electrode pair.

\[
DBE = \sqrt{(x_{e1} - x_{e2})^2 + (y_{e1} - y_{e2})^2}
\]

where \((x_{e1}, y_{e1})\) and \((x_{e2}, y_{e2})\) are the coordinates of first and second electrode plates, respectively. Fig. 6 shows DBE feature for seven electrode pairs (1-2, 1-7, 1-10, 1-11, 2-3, 4-10, and 8-9). In the ECT system considered in this work, DBE was minimum for electrode pairs 2-3 and 8-9 and was maximum for pairs 1-7 and 4-10. DBE feature was measured in inches.

**G. Angle between Electrode1, Distribution’s Center of Mass, and Electrode2 (MA)**

This feature is the smaller angle (less than or equal to 180 degrees) between the first electrode, metal distribution’s center of mass, and the second electrode of the subject electrode pair. It is calculated as shown in Equation (10). It is called the Mean Angle (MA) because it is also the mean of angles between the first electrode, individual metal elements, and the second electrode. It represents the electrode pair orientation with respect to each other and to the given metal distribution.

\[
MA = \cos^{-1}\left(\frac{E_1 \cdot E_2}{|E_1||E_2|}\right)
\]

where, \(E_1\) is the vector from metal distribution’s center of mass to the first electrode sensor and \(E_2\) is that to the second electrode sensor. The Numerator represents the dot product of these two vectors. This feature was measured in degrees.
H. Mean Weighted Distance of the Metal Elements from the Distribution’s Center of Mass (MD)

This feature is calculated by averaging the weighted distance of the metal elements from the distribution’s center of mass. The distances are weighted by multiplication with corresponding sensitivity. Equation (11) explains the calculation of this feature:

\[ MD_j = \frac{\sum_{i=1}^{N} (x_i - x_{ij})^2 + (y_i - y_{ij})^2 \times S_i^j}{\sum_{i=1}^{N} S_i^j} \]

where \( N \) is the total number of metal elements present in the distribution, \( (x_i, y_i) \) are the coordinates of the distribution’s center of mass for \( i^{th} \) electrode pair, \( (x_{ij}, y_{ij}) \) are the coordinates of \( j^{th} \) pixel having metal, and \( S_i^j \) is sensitivity of the element for \( i^{th} \) pixel in metal image for \( j^{th} \) electrode pair. This feature was measured in inches. Table II provides the values of key features for six electrode pairs for metal distributions shown in Fig. 4.

<table>
<thead>
<tr>
<th>Electrodes</th>
<th>1-2</th>
<th>1-3</th>
<th>1-4</th>
<th>1-5</th>
<th>1-6</th>
<th>1-7</th>
</tr>
</thead>
<tbody>
<tr>
<td>AD</td>
<td>0.77</td>
<td>0.76</td>
<td>0.75</td>
<td>0.75</td>
<td>0.75</td>
<td>0.75</td>
</tr>
<tr>
<td>SD</td>
<td>0.04</td>
<td>0.07</td>
<td>0.07</td>
<td>0.06</td>
<td>0.03</td>
<td>0.03</td>
</tr>
<tr>
<td>WS</td>
<td>1.39</td>
<td>3.51</td>
<td>6.99</td>
<td>8.95</td>
<td>9.61</td>
<td>9.65</td>
</tr>
<tr>
<td>AED 1</td>
<td>3.68</td>
<td>3.77</td>
<td>3.85</td>
<td>3.87</td>
<td>3.89</td>
<td>3.89</td>
</tr>
<tr>
<td>AED 2</td>
<td>4.17</td>
<td>5.04</td>
<td>6.42</td>
<td>7.19</td>
<td>7.95</td>
<td>8.11</td>
</tr>
<tr>
<td>WS/TS</td>
<td>0.10</td>
<td>0.12</td>
<td>0.11</td>
<td>0.11</td>
<td>0.10</td>
<td>0.10</td>
</tr>
<tr>
<td>DBE</td>
<td>3.69</td>
<td>6.01</td>
<td>8.91</td>
<td>10.4</td>
<td>11.7</td>
<td>12</td>
</tr>
<tr>
<td>MA</td>
<td>55.7</td>
<td>84.9</td>
<td>118.4</td>
<td>137.6</td>
<td>164.8</td>
<td>178.1</td>
</tr>
<tr>
<td>MD</td>
<td>0.06</td>
<td>0.16</td>
<td>0.32</td>
<td>0.41</td>
<td>0.45</td>
<td>0.45</td>
</tr>
</tbody>
</table>

As explained in this section, these features carry very vital information about the metal distribution, the electrode pair, separation between the electrode pair, their orientation with respect to each other, etc.

V. ARTIFICIAL NEURAL NETWORK

In general, a neural network is used when exact nature of the relationship between the inputs and the outputs is unknown [9]. If the relationship was known, it could have been implemented directly. Another essential characteristic of the ANNs is that they learn the input/output relationship through the training [9].

An ANN is a system based on the operation of biological neural networks. The ANNs are composed of simple processing elements, known as neurons, organized in different layers and communicating with each other. Each interconnection between two neurons is associated with a weight that specifies the strength of the connection. ANNs play an important role in various applications. They have the property of being a universal approximator, i.e., for any function of arbitrary degree, there is a feed-forward ANN able to approximate it [10]. ANNs are considered an attractive choice for modeling nonlinear and complex problems because of their robustness, ability to withstand noise, their universal approximation property. ANNs have unique ability of predicting and extrapolating information hidden in the training data during the learning or training [11].

A. Architecture

Eight different features are extracted from the metal distributions and used as inputs of ANN. One feature (AED) has two separate values for each metal distribution and the subject electrode pair. Hence input layer of the ANN has nine neurons (one neuron for each key feature). There is no general and systematic rule for constructing an optimal ANN to fit a given application [12]. So the ANN architecture selection was experimental and based on trial and error method. After the input layer, the final ANN had two hidden layers both with 50 neurons. The output layer consists of only one neuron because for any given set of features, it generates only one output (CF).

B. Training Algorithm

The ability of the ANN to provide useful data manipulation lies in the proper selection of the weights. The Training of the ANNs is the process of selecting and modifying those weight vectors. The ANN was trained proposed ANN was trained with SCG algorithm.

C. Data Generation

In this work, the training data were generated through computer simulations based on ANSYS finite element analysis. This analysis is a brute force numeric technique to solve the FP. It calculates the mutual capacitances by explicitly solving the partial differential equations which govern the sensing domain. It provides most accurate results but it is a time consuming process.

To train the ANN effectively and to maximize the generalization, the training data set was assembled from a variety of different metal distributions.

VI. TRAINING DATA SET

The training data set consists of four types of metal distributions. Each type had two or three subtypes. The training data are optimized for maximum generalization with minimum computational cost. That is why a variety of training distributions with big metal pieces, small metal piece, both combined as well as distributions, with random metal filling, were used during learning process.

VII. ARTIFICIAL NEURAL NETWORK TRAINING

The ANN was created in MATLAB 2008 using Neural Network Toolbox. At the start of the training process the initial connection weights and biases were chosen randomly. The network was trained on a Windows XP machine with 2 GB memory. It took about six and one-half hour, and 1711 iterations to train the
network by SCG training algorithm with the RMS error performance of 2.21% (MSE = 0.000489) of over the training data set.

VIII. TESTING & PERFORMANCE

After the training of ANN, it was tested on different sets of arbitrary metal distributions not used in training. Since in actual LFC metal always flows and fills the casting pattern, the test distributions were also of similar nature, i.e. in each test set metal was entering from one or two entry points and then it was expanding inside the imaging area.

A. Test Distributions

Depending on the starting point of the metal flow, three sets of test distributions were generated using ANSYS finite element analysis. Further, the proposed method was tested on some complex (Alphabetical) patterns as shown in Fig 15. In order to test the generalization ability of ANN, it is necessary to test network with completely different metal distributions from training distributions. The alphabetical patterns shown in Fig. 15 represent significantly different test distributions. In first test set, the starting point for metal filling was the middle of the imaging area and then metal was flowing in all possible directions. This set consists of eight distributions. In test distribution set 2, the metal filling was started from middle of the lower side and then it was expanding inside the imaging area. In test distribution set 3, the metal filling started from lower right corner. This set is also consists of 16 distributions. For further investigation on performance of ANN, it was tested with some alphabetical metal distributions.

B. Performance

The performance of proposed technique measured in terms of two parameters. First parameter was correlation coefficient between actual capacitance and calculated capacitance using proposed method and second parameter was % RMS error between actual correction factor and estimated correction factor (Output) by ANN.

Table V provides average values of two performance parameters for all test metal distribution sets. The overall RMS error for all test distributions was 2.2% and the average correlation coefficient was 0.95

<table>
<thead>
<tr>
<th>Distribution</th>
<th>Correlation</th>
<th>% RMS Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Set 1</td>
<td>0.97</td>
<td>0.73</td>
</tr>
<tr>
<td>Set 2</td>
<td>0.91</td>
<td>1.85</td>
</tr>
<tr>
<td>Set 3</td>
<td>0.97</td>
<td>3.44</td>
</tr>
<tr>
<td>Set 4</td>
<td>0.97</td>
<td>1.34</td>
</tr>
</tbody>
</table>

REFERENCES