Tracking Control of Interconnected Car-like Vehicles using Energy Methods

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Abstract—This article deals with tracking control of car-like vehicles which can communicate with one another. To realize the design of the energy based controller, three steps are involved: firstly, the kinetic energy shaping method is employed; secondly, virtual potential energy is added; and finally, dissipative functions are introduced. These steps are executed in order to control vehicles initially modeled as mass points. The design is later adapted to obtain a control law for the car-like vehicles. These models have inputs that are more realistic, such as steering and accelerating, and not only forces as in the case of point mass models. Moreover, feedforward effects are introduced so as to improve the performance of the controller by using information communicated over a network. Some preliminary simulation results are also presented.

I. INTRODUCTION

Wireless network systems are easily available today due to their wide scope of application that has caused them to decrease in price. These modern means of communication make it possible to connect different elements of a system, such as sensors, actuators or even whole subsystems, to a digital network and thus reveal new potential in the automatic control field.

In the present work the so-called energy methods are applied in order to solve the tracking problem for nonholonomic car-like vehicles moving with constant spacing between them. Firstly, the problem of controlling mass point vehicles moving on the (x,y)-plane is considered. To obtain the desired inter-vehicle spacing on the plane many configurations are possible. As shown in [5], energy methods can be used to generate an appropriate control law for the forces to be applied to point mass vehicles. With this technique the structure of the nonlinear controller together with some free parameters can be obtained.

The idea behind these methods is to modify the mechanical energy of the system so as to obtain the desired behavior. For this purpose, splitting of generalized velocity vectors on the configuration space is necessary, so that it can be distinguished between the velocity components which do alter the spacing, and those which do not. With this information, it is then feasible to differently weight each contribution to the kinetic energy and thus perform the kinetic energy shaping. A further step in the design consists in adding virtual potential energy with the aim of making the desired configuration stable. Introduction of dissipative terms, which are not present in the original system, allows achieving asymptotic stability. The last phase in the feedback design consists in converting the control actions represented in generalized coordinates into real inputs for car-like vehicles, which are restricted in movement by their nonholonomic constraints.

Some of the first contributions to the control theory using energy methods were made in [2] and [4] where kinetic energy shaping, potential energy inclusion and dissipation addition were used to stabilize a pendulum on a chart in the upright position. Apart from this, it is shown in [1] how the splitting of generalized velocity vectors into vertical and horizontal components can be accomplished for a multirobot system. In [15] and [14] Zambou uses the method of Controlled Lagrangians for controlling the longitudinal dynamics of a platoon of trucks. The lateral dynamics is considered in that paper separately and controlled by means of a different method.

The nonholonomy concept is not new and has its origins in the classical mechanics [3]. Nonholonomic models have been used for a long time mainly in the robotic motion area. Today they are also employed in the automotive field, e.g., to design parking assistance systems. Nonholonomic models are characterized by their non-integrable constraints restricting types of motion (velocities) but not positions. In [13] the author presents the analysis of tracking control laws for many different nonholonomic vehicle models and also investigates the behavior of the resulting internal dynamics. This work has been further extended in [9] so that tracking reference-vehicles ahead and behind the rear axle of the follower for the case of car-like models is possible. In both last mentioned papers the authors fix a reference point which should track the leader. In the present paper the problem scope is not restricted to a fixed reference point so that this degree of freedom is left to the energy based controller. A feedforward controller is also introduced to benefit from the information that can be transmitted over the wireless network to improve the system performance.

The present paper is organized as follows: In section II the design of the controller for mass point vehicles using kinetic energy shaping, virtual potential energy and dissipative functions is reviewed. Section III presents the car-like model which is employed to extend the controller and describes its main characteristics. In section IV a feedforward control law
is obtained which maintains the current configuration and uses information received from the network. Subsequently, the energy based controller designed previously is adapted in section V to apply to the car-like models. In section VI some simulation results are presented and analyzed. Finally, conclusions are drawn in section VII and an outlook of the future work is given.

II. CONTROL USING ENERGY METHODS

Three different techniques within the framework of energy methods will be applied in this section: kinetic energy shaping, inclusion of virtual potential energy and addition of dissipative functions. Firstly, the splitting of the velocity vectors, which is necessary to perform the kinetic energy shaping, is carried out. This process allows weighting (shaping) differently the vertical and horizontal contributions to the kinetic energy [5]. Since the obtained kinetic energy-based controller is not capable of correcting position errors, as it will be explained below, it is necessary to add also virtual potential energy terms to make the desired configuration a stable set in the configuration space. Finally, dissipative terms are incorporated into the system for the purpose of achieving an asymptotic behavior.

A. Kinetic energy shaping

As stated above, in order to shape the kinetic energy, it is necessary to be able to distinguish between vertical and horizontal velocities. Therefore, the tangent space is split into its vertical and horizontal subspaces. The vertical space is defined by first writing mathematically the condition under which the spacing is kept constant ($|q_i - q_j|^2 = \text{const.}$) and then differentiating the equation with respect to time so as to obtain the corresponding constraints for the velocities.

$$\frac{d}{dt}||q_i - q_j||^2 = 2(q_i - q_j)^T(v_{q_i} - v_{q_j}) = 0. \quad (1)$$

Here $q_i$ and $q_j$ represent positions of the vehicles on the plane (two coordinates per vehicle). The corresponding velocities are denoted by $v_{q_i}$ and $v_{q_j}$. Following [1], the vertical space is defined as the space of all velocities satisfying the constraint (1) and the horizontal space as its orthogonal complement, where the orthogonality is defined using the inner product given by the metric tensor (or mass matrix), obtained from the expression of the kinetic energy.

$$\text{Ver}[T_q Q] = \{ v_q \in T_q Q | (q_i - q_j)^T(v_{q_i} - v_{q_j}) = 0 \},$$
$$\text{Hor}[T_q Q] = \{ v_q \in T_q Q | \langle v_q, v_q \rangle_M = 0 \}. $$

In these expressions, $Q = \mathbb{R}^{2N}$ is the configuration space for $N$ units and $T_q Q$ is the tangent space at a point $q \in Q$, where $q = [q_1, \ldots, q_N]^T$ is the grouping of all vehicle coordinates. Correspondingly, $v_q$ denotes a generalized velocity vector and $v_{q_i}$ a vertical velocity vector. The metric tensor $M$ of the mechanical system is symmetric and positive definite and depends usually on the configuration coordinates. However, for this special case it is constant due to the given mechanical models and will be therefore denoted simply by $M$ instead of $M(q)$. As mentioned before, this mass matrix is directly obtained from the equation of the kinetic energy:

$$K(v_q) = \langle (v_q, v_q) \rangle_M = v_q^T M v_q. \quad (2)$$

It can be observed in [5] that two projection matrices ($V_p$ and $H_p$) can be computed, which permit to decompose the general velocity vector $v_q$ into its vertical and horizontal components. Once the projection matrices are known, it is also possible to compute the vertical and horizontal contributions to the kinetic energy $K = K_v + K_h$. To now perform the kinetic energy shaping, these contributions are differently weighted by using a weighting factor $\alpha$.

$$K_\alpha = 2(1 - \alpha) K_v + 2\alpha K_h = v_q^T M_\alpha v_q. \quad (3)$$

A new mass matrix $M_\alpha$ is obtained from (3) which depends on $\alpha$ and $q$. It should be noted that the original form of the kinetic energy can be recovered by taking $\alpha$ equal to $\frac{1}{2}$ implying that the computed control action becomes zero, since actually nothing has been changed.

To find the controller outputs (forces), the system with the modified energy (without inputs) and the original system (with inputs) must be matched by comparing their Lagrange equations:

$$\frac{d}{dt} \left( \frac{\partial K}{\partial v_q} \right) - \frac{\partial K}{\partial q} = F_{\text{kin}},$$

$$\frac{d}{dt} \left( \frac{\partial K_\alpha}{\partial v_q} \right) - \frac{\partial K_\alpha}{\partial q} = 0.$$

For the studied mechanical system it is easy to find a closed form for the forces since the matrix $M$ does not depend on the coordinates and the number of inputs coincide with the dimension of the configuration space.

$$F_{\text{kin}} = MM^{-1}_\alpha \left( \left[ \frac{\partial (M_\alpha v_q)}{\partial q} \right]^T - 2 \frac{\partial (M_\alpha v_q)}{\partial q} \right) v_q. \quad (4)$$

Because of the fact that the kinetic energy depends principally on the velocities and that neither natural potential energy nor dissipation properties are present in the original system, the controller is, only with kinetic energy shaping, unable to correct and stabilize position errors. To compensate for this, it is convenient to artificially introduce potential energy and dissipative properties into the system as it will be shown in the next subsections.

B. Addition of virtual potential energy

The method of introducing virtual potential energy is very well known in the field of mobile robots control (see, for instance, [11], [12], [10] and [16]). The principal idea is to artificially add potential energy into the system, which is similar to the one given by the gravitational field. The main difference between the natural and the virtual fields is that in the last case the minimum is not a point but a manifold describing the desired configuration. For platoon formation tracking with constant spacing policy, the following virtual energy expression can be employed

$$V = \sum_{i < j \leq N} \sigma_{ij} m_i m_j \|e_{ij}q\| \left( \frac{\|e_{ij}q\|}{2} - (j - i)ds \right),$$
where \( d_s \) is the spacing reference, \( i \) and \( j \) are unit indices and \( \epsilon_{ij} \) are matrices that compute the distance vector between two vehicles \( i \) and \( j \) from the generalized coordinates \( q \). To find the force produced by this artificial potential, the differentiation \( F_{\text{pot}} = -\frac{\partial V}{\partial q} \) must be computed. The resulting expression can be found in [5] with slight differences in the notation.

C. Incorporation of dissipative functions

The explained strategies do not allow achieving asymptotic stability, because kinetic and potential energies are of conservative type and therefore can not dissipate the energy of the error signals. In order to compensate for this deficit, artificially dissipative terms in form of usual Rayleigh functions can be included. The chosen function representing the rate of dissipated energy per second is given by

\[
D = \frac{1}{2} \left[ \sum_{i<j \leq N} \gamma_{ij} (\epsilon_{ij} v_q)^T M_{ij} (\epsilon_{ij} v_q) \right].
\]

The \( \epsilon_{ij} \) are the same as in the case of potential energy and \( M_{ij} \) stands for the 2x2-submatrices with the corresponding mass values in the diagonal. Once again the corresponding force can be obtained by differentiation of the chosen function \( F_{\text{diss}} = -\frac{\partial D}{\partial q} \).

D. The resulting control law

Finally, the overall control law for the mass point vehicles is calculated by summing up the three contributions,

\[
F = F_{\text{kin}} + F_{\text{diss}} + F_{\text{pot}}.
\]

The parameters \( \alpha, \sigma_{ij} \) and \( \gamma_{ij} \) from the kinetic energy shaping, the virtual potential energy and dissipative functions respectively are free and can be used for tuning the controller.

Application of these three methods permits to asymptotically stabilize the system in a platoon configuration as it was shown in [5], where some simulations for two and three units were presented. In the next sections the focus will be set on the control of two units and the adaptation of the controller to operate with more realistic car-like models. After that, the feedforward part will be designed as well. By this example it will be shown that the feedforward part and, therefore, the communication has an important influence on the performance of the controller.

E. The closed loop for two units

The following contributions to the control law from the three different strategies are obtained by applying the given formulas to the case of two vehicles moving on the plane.

\[
F_{\text{kin}} = -\beta \left( \frac{m_1 m_2}{m_1 + m_2} \right) [ e_x \ e_y \ -e_x \ -e_y]^T,
\]

\[
F_{\text{pot}} = -\lambda e^T e_q,
\]

\[
F_{\text{diss}} = -\gamma M e^T e_q.
\]

In (7) \( m_i \) denotes the mass value of the unit with the index \( i \) and \( E = [e_x \ e_y]^T \) is the distance vector obtained from the generalized coordinates. Furthermore, its time derivative is denoted by \( \dot{E} = [v_{ex} \ v_{ey}]^T \). The nonlinear factors \( \beta \) and \( \lambda \) are defined as follows:

\[
\beta = \frac{2\alpha - 1}{\alpha} \left( \frac{v_{ex} e_y - v_{ey} e_x}{e_x^2 + e_y^2} \right)^2,
\]

\[
\lambda = \sigma (m_1 + m_2) \left( 1 - \frac{d_s}{\|q\|} \right).
\]

Adding the three control contributions and closing the loop, the following differential equitation is obtained:

\[
\dot{E} + \gamma \dot{E} + (\beta + \lambda) E = 0,
\]

where the vector notation for the second derivative \( \ddot{E} = [a_{ex} \ a_{ey}]^T \) has been used again.

The consequences of choosing the weighting factor \( \alpha \) to be \( \frac{1}{2} \) can be observed in (7) and (8a). With this equality, the contribution \( F_{\text{kin}} \) and the factor \( \beta \) (and with them the corresponding term in (9)) vanish indicating that the kinetic energy shaping has no effect on the controller output.

Besides, it can be observed in (7) that the force produced by the virtual potential energy \( F_{\text{pot}} \) goes to zero when the desired configuration is reached, that is when \( d_s = \|q\| \).

III. THE CAR-LIKE MODEL

Modeling vehicles as mass points which can move in any direction is an approach that has been used by many authors (e.g., in [8] and [6]) to develop control strategies for mobile units in a simplified framework. Some works employ this technique to plan trajectories for robots using artificial control inputs for mass point models and then transform these inputs into real system inputs by means of minor control loops, for example. These minor control loops can also be used to compensate for the nonlinear behavior by means of feedback linearization [6]. The nonholonomic model of the car will be used from now on to work with a more realistic system, bearing in mind that movements are restricted due to the velocity constraints. The car-like model is sketched in Fig. 1, where \( v \) and \( v_r \) represent the longitudinal velocities of the front and rear axles, respectively. The distance vector going from the front of the following vehicle to the leading vehicle is shown in the figure too. The steer angle is denoted by \( \delta \) and the yaw angle by \( \psi \).

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**Fig. 1.** Car-like model
As any model, the car-like model is based on some assumptions which are listed below.

- **Bicycle model approximation:** The wheels on both sides of the axles are combined into a single wheel at the midpoint of the axle.
- **Rigid body:** The distance between any two points on the car body is constant.
- **Rolling without slipping:** Nonholonomic constraints are imposed on the wheels. Side forces and traction forces on the tires are hence not considered. This assumption is reasonable for small velocities and steering angles.

Despite all these restrictions for the model it still is a much better approximation than simple mass points and captures the essence of the vehicle behavior permitting to work with real control actions (steering and accelerating) instead of artificial forces that can point in any direction.

Considering these assumptions, two vector equations can be written to describe the constrained velocities at the front and rear axle:

\[
\begin{bmatrix}
\dot{x} \\
\dot{y}
\end{bmatrix} = \mathbf{R}(\delta + \psi) \begin{bmatrix}
v \\
0
\end{bmatrix}, \quad (10a)
\]

\[
\begin{bmatrix}
\dot{x}_r \\
\dot{y}_r
\end{bmatrix} = \mathbf{R}(\psi) \begin{bmatrix}
v_r \\
0
\end{bmatrix}. \quad (10b)
\]

where \(\mathbf{R}(\cdot)\) denotes a two-dimensional rotation matrix.

If the geometrical constraint given by the rigid body assumption is also taken into consideration, the following expression can be written:

\[
\begin{bmatrix}
x_r \\
y_r
\end{bmatrix} = \begin{bmatrix}
x \\
y
\end{bmatrix} - \mathbf{R}(\psi) \begin{bmatrix}
1 \\
0
\end{bmatrix}. \quad (11)
\]

Differentiating (11) with respect to time and combining it with (10a) and (10b), it is easy to find the relationships below:

\[
v_r = v \cos(\delta), \quad (12a)
\]

\[
\psi = \frac{v}{\ell} \sin(\delta). \quad (12b)
\]

As expected, the velocity of the rear axle is directly related to the velocity of the front axle. Due to this fact only \(v\) and the equation (10a) will be used from now on. The yaw rate \(\psi\) is a function of the steering angle and the velocity, as it can be deduced from (12b).

As in [13], the control inputs are regarded to be \(u_v = \dot{v}\) and \(u_\delta = \dot{\delta}\) (or in vector form \(\mathbf{U} = [u_v, u_\delta]^T\)) which are homogeneous inputs to actual motor torques. If (10a) is now derived once again and (12b) is used, the acceleration \((\ddot{x}, \ddot{y})\) of the vehicle on the plane can be expressed as a function of the actual inputs of the model, namely \(u_\delta\) and \(u_v\).

\[
\begin{bmatrix}
\ddot{x} \\
\ddot{y}
\end{bmatrix} = \mathbf{R}(\delta + \psi) \begin{bmatrix}
u_v u_\delta + \frac{v^2}{\ell^2} \sin(\delta) \\
u_v
\end{bmatrix} = \mathbf{R}(\delta + \psi) \mathbf{Z}_1. \quad (13)
\]

This equation will be employed later to adapt the energy based controller to work with car-like models. The vector \(\mathbf{Z}_1\) is introduced only to simplify the notation and depends solely on car-internal variables and control inputs.

IV. THE FEEDFORWARD CONTROLLER

Hypothetically, if the models were able to reflect the reality perfectly and the current variables of the vehicle ahead were accessible, it would be possible to compute the corresponding feedforward controller outputs in order to exactly replicate the leader’s movements. Thus, the spacing would be obviously kept constant. This ideal situation is clearly impracticable; that is why it is also be to add some feedback to the feedforward controller in order to correct the controller output in case of model discrepancies, external disturbances and mismatching initial conditions. The combination of feedback and feedforward can improve the overall performance by making the system faster and at the same time capable of correcting errors. A feedforward controller will be designed now to be combined with the energy feedback controller.

The feedforward controller must have access to the information of the vehicle ahead, and therefore a communication channel is required. The variables of the leading vehicle are denoted here with an apostrophe ('). The velocity vectors of the two units are then taken to be equal,

\[
\begin{align*}
v \cos(\delta + \psi) &= v' \cos(\delta' + \psi') \\
v \sin(\delta + \psi) &= v' \sin(\delta' + \psi').
\end{align*}
\]

From these equations and their time derivatives it is deduced that it is possible to choose \(\dot{v} = \dot{v}'\) and \(\dot{\delta} + \dot{\psi} = \dot{\delta}' + \dot{\psi}'\) which after a combination with (12b) result in the corresponding controller output:

\[
\mathbf{U}_f = \begin{bmatrix}
\dot{u}_{fv} \\
\dot{u}_{f\delta}
\end{bmatrix} = \begin{bmatrix}
\dot{\delta}' + v' \left(\frac{\sin(\delta')}{v'} - \sin(\delta)\right)
\end{bmatrix}. \quad (14)
\]

where the \(f\) in the subscripts stands for feedforward.

In this control law there is actually feedback (second term in the parenthesis on the second row) but this is given by the model (12b) and not by the external controller. It is easy to show that with this control law the quantity \(\epsilon^2 = \mathbf{E}^T \mathbf{E}\) is indeed constant, i.e. the spacing remains unchanged.

V. ADAPTATION OF THE ENERGY BASED CONTROLLER

In order to adapt the feedback controller obtained in section II to work with car-like vehicles, the actual control outputs \(u_\delta = \dot{\delta}\) and \(u_v = \dot{v}\) should be used instead of the artificial forces given in (7).

A. Transformation of the nonlinear factors

The factors \(\beta\) and \(\lambda\) are nonlinear and depend on the positions and velocities. However, using the relationships in (15) it is possible to express them in terms of variables that are available on the vehicle, either because they are measurable (like the spacing) or because they form part of the unit-internal variables (like the steering angle).

\[
\begin{align*}
\epsilon_x^2 + \epsilon_y^2 &= \mathbf{E}^T \mathbf{E} = \epsilon^2 \\
v_{es} \epsilon_x - v_{es} \epsilon_y &= -\epsilon^2 \left(\dot{\theta} + \dot{\psi}\right). \quad (15)
\end{align*}
\]
The measured spacing in (15) is given by the expression
\( e = \|E\| = \sqrt{e_x^2 + e_y^2} \) and \( \theta \) is the measured angle shown in Fig. 1. This information can be obtained, for example, by using ultrasound sensors (for the spacing) and a camera (for the angle). Using (15) and (12b), the new formulas for the nonlinear factors are as follows:
\[
\beta = \frac{2\alpha - 1}{\alpha} \left( \theta + \frac{v}{l} \sin(\delta) \right)^2, \quad (16a)
\]
\[
\lambda = \sigma(m_1 + m_2) \left( 1 - \frac{ds}{c} \right). \quad (16b)
\]

B. Transformation of the closed loop equation

In order to obtain the control law as a function of the known variables, the distance vector \( E \) and its derivatives are expressed as a function of the measurable variables \( \theta \) and \( e \).
\[
E = R(\theta + \psi) Z_2 = R(\theta + \psi) \begin{bmatrix} e \\ 0 \end{bmatrix},
\]
\[
\dot{E} = R(\theta + \psi) Z_3 = R(\theta + \psi) \begin{bmatrix} e \\ \dot{e} \left( \frac{\dot{\psi}}{\psi} \sin(\delta) + \dot{\theta} \right) \end{bmatrix}.
\]

Here (12b) was used again to get the second equation and \( Z_2 \) and \( Z_3 \) (which only depend on known variables) were introduced to simplify the notation. Replacing this in the closed loop equation (9) and rearranging of the terms yields:
\[
-\dot{E} = \gamma \dot{E} + (\beta + \lambda) E
\]
\[
R(\delta + \psi) Z_1 = (\beta + \lambda) R(\theta + \psi) Z_2 + \gamma R(\theta + \psi) Z_3.
\]

In order to obtain a closed loop equation independent of absolute coordinates, the acceleration of the leader \( \dot{[x' y']^T} \), which would be an external input, is neglected at this point. This is indirectly taken into account by the feedforward controller, although. Note also that the only absolute variable which is unknown is the yaw angle \( \psi \). However, it is easy to check that this variable is canceled out in the equation, since it appears as an argument of the rotational matrices on both sides. In fact, with some help of the software MAPLE, it is possible to calculate the resulting feedback equations:
\[
U_b = \begin{bmatrix} u_{br} \\ u_{bd} \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \frac{\dot{\theta}}{\dot{\psi}} R(\theta - \delta) ((\beta + \lambda) Z_2 + \gamma Z_3) - \begin{bmatrix} 0 \\ \frac{\dot{\psi}}{\psi} \sin(\delta) \end{bmatrix}.
\]

The overall control law in (17) is independent of the absolute and the unknown variables and to compute it, only internal variables (like \( v \) and \( \delta \)) and measured variables (like \( \theta \) and \( e \)) are required. In reality, \( \theta \) is needed and this may cause some difficulties in practice because of the derivation of the camera information. For this reason, it is necessary for the practical implementation to approximate the derivative by including some low pass property in order to avoid undesired effects.

Finally, the control law is obtained by adding the feedforward and feedback contributions \( U = U_f + U_b \). If the communication is unavailable, the feedforward part can not be used because the information is unaccessible.

VI. SIMULATION RESULTS

For the following simulations a model in MATLAB/SIMULINK was created and a reference distance of 2m was chosen. In order to simulate the controller, it is also necessary to simulate the behavior of the leading vehicle, which in reality will be driven by a person. To get feasible trajectories, an analysis scenario was generated by choosing test functions for the inputs of the leading vehicle \( U' = [u_1' \ u_2']^T \). Without loss of generality \( u_2' = 0 \) (constant speed) is taken to put the focus on the steering angle which varies according to the predefined reference input. Since the input \( u_1' \) is actually the velocity and not the steering angle, a simple linear controller is employed to track the proposed reference in a smooth manner and with no stationary error for the selected reference signals. In Fig. 2 the reference steer angle and its tracked value are plotted, and the controller transfer function is given. This controller locates the three poles of the closed loop in \( s = -r \) and includes a model of the sinusoid in order to drive the error to zero.

The above mentioned reference angle consists of a sinusoidal movement followed by a positive pulse. Note that the actual steering angle varies smoothly and tracks the desired reference with no stationary error.

With this completed, the controller in the following vehicle is simulated for some fixed parameters with and without feedforward based on the use of transmitted information. The results are shown in Fig. 3 where, as expected, the performance of the controller is better if it has access to the information of the leading vehicle. A smaller error (discrepancy with respect to 2m) caused by the nonlinear interaction of the controllers is observed in the case with feedforward, where the peaks due to the sinusoidal movement have smaller amplitude. Figure 3 also presents a small box where the first part of the path traveled by the leader is plotted.

Finally, Fig. 4 shows the steer and yaw angles of the leading and following vehicles. Here it can be observed that the yaw angle is tracked very well. Note however that bigger differences in the case of the steering angles are present, but they are only due to the individual geometry of each vehicle (\( l = 1 \) m and \( l' = 1.2 \) m). The only situation in which the
values must coincide is when the leader is driving along a straight line as it is the case before and after the positive pulse in the reference.

VII. CONCLUSIONS AND FUTURE WORK

A. Conclusions

The main advantage of the energy methods is that an insight into the controller is gained to provide knowledge on its operation in terms of energy. In this paper it has been shown how to modify the controller designed with energy methods on the basis of simple vehicle models in order to obtain the nonlinear controller for more realistic car-like models. The controller is then extended by including a feedforward part that makes it possible to achieve a faster reaction but still requires the use of communication network. Because of this faster reaction shorter spacings are possible while maintaining safety (avoidance of collisions).

B. Future Work

A further important step in the investigation is to implement the controller under more realistic conditions. With this aim in mind, a set of four 1:14 truck scale models is being prepared, see Fig. 5. These models will be equipped with distance sensors, a camera system and a WLAN module for exchange of information and monitoring with a computer. With the collaboration of the Institute of Informatics at the RWTH-Aachen University, it is planned to check safety of the nonlinear system by means of hybrid automata [7].

REFERENCES


Fig. 3. Comparison with and without feedforward

Fig. 4. Yaw and steer angles of the two vehicles

Fig. 5. Test 1:14 truck scale models