Slewing and Vibration Control of a Nonlinear Flexible Spacecraft

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Abstract—In this paper, the problem of attitude control of a 3D nonlinear flexible spacecraft is investigated. Two nonlinear controllers are presented. The first controller is based on dynamic inversion, while the second approach is composed of dynamic inversion and $\mu$-synthesis schemes. The extension of dynamic inversion approach to flexible spacecraft is impeded by the non-minimum phase characteristics when the panel tip position is taken as the output of the system. To overcome this problem, the controllers are designed by utilizing the modified output re-definition approach. It is assumed that only three torques in three directions on the hub are used. In particular, the assumption that all state variables are measurable is not realistic; hence sliding mode observers is used to estimate states. Actuator saturation is also considered in the design of controllers. To evaluate the performance of the proposed controllers, an extensive number of simulations on a nonlinear model of the spacecraft are performed. The performances of the proposed controllers are compared in terms of nominal performance, robustness to uncertainties, vibration suppression of panel, sensitivity to measurement noise, environment disturbance and nonlinearity in large maneuvers. Simulation results confirm the ability of the proposed controller in tracking the attitude trajectory while damping the panel vibration. It is also verified that the perturbations, environment disturbances and measurement errors have only slight effects on the tracking and damping performances.

I. INTRODUCTION

The problem of attitude control of satellites undergoing large angle motion has received an extensive attention in recent years. The large-angle maneuvers are characterized by nonlinear dynamics and hence nonlinear control design is often required.

Nevertheless, the nonlinear control techniques work in general only in a small neighborhood of the operating point where the linear approximation is valid.

In the context of nonlinear systems, the feedback linearization seems a viable choice since the nonlinear system is exactly transformed into a linear system (valid for the entire operating region) and only then the linear controller is applied. However, the classical feedback linearization suffers from the lack of robustness in the presence of uncertainties, disturbances and noise.

In [1], the problem of attitude recovery of flexible spacecraft using feedback linearization approach is investigated. Although, this method achieves good vibration suppression, it does not address the issue of robustness to combined uncertain conditions (several uncertain conditions, i.e. environment disturbance, sensor noise and uncertain parameters exist together or one uncertain condition with larger variations). Moreover, the selected controller bound is large as if the actuator saturation has not been considered.

Recently, considerable efforts have been made to design robust control systems for simultaneous attitude control and vibration suppression of flexible spacecraft. However, most of the works are based on linear control approach which results in a poor performance for large maneuvers.

Various nonlinear robust control algorithms have been proposed on rigid spacecraft such as adaptive fuzzy mixed $H_2/\ H_\infty$ [2] and adaptive mixed $H_2/\ H_\infty$ [3]. The neural networks, fuzzy or adaptive methods are employed to approximate the unknown nonlinear characteristics at the system dynamics. However, other uncertainties such as sensor noise and environment disturbances have not been considered. Consequently, a robust feedback linearization strategy seems promising.

The most common approach to compensate for the nonlinear dynamics of a rigid spacecraft is the so-called inverse dynamics strategy. However, the extension of this approach to flexible spacecraft is impeded by the non-minimum phase characteristics when the panel tip position is taken as the output of the system. To overcome this problem, [4] a re-defined output on the flexible-link manipulators between the joint and the tip was suggested. The new output is defined so that the zero dynamics related to this output are stable.

In this paper, the attitude control of a 3D flexible spacecraft is addressed using two approaches: dynamic inversion and the composition of dynamic inversion and $\mu$-synthesis. The $\mu$-synthesis control law is formulated such that an outer-loop linear controller can be constructed to provide robust stability/performance against the inexact dynamic cancellation arising in the inner-loop feedback linearization design.

The goal is attitude control and panel vibration suppression in the absence of damping and actuators on panels. The controllers are designed by utilizing the modified output re-definition approach. Hence, the sum of the attitude angles and a scaling of the tip elastic deformation are chosen as the output. This scale results in a stable zero dynamics of the system.

In the design of dynamic inversion controller, this summation is considered as the output. To enforce the position and rate saturation limit, a feedback controller structure is used in the inner loop. Moreover, it is often the
case that the linearized model is different from the linear model. Hence, choosing the weighting functions is very challenging. Another important issue in designing the \( \mu \)-synthesis controller is bounding the linear controller term which is different from the bound for the actual control signal \( r \). Hence, it is crucial to find an appropriate weighting function for the linear controller. To evaluate the performance of the proposed controllers, a set of simulations are performed on a 3D stabilized flexible spacecraft. It was our intention that the sensors noises, disturbances and uncertainty be as close as possible to practical situations.

Up to now, we have assumed that all the states are available. However, in general, not all of the states are measurable and the feedback control scheme should be implemented via the estimated states.

The paper is organized as follows: In section II dynamic equations of flexible spacecraft is considered. The output redefinition approach is discussed in section III. The design of two controllers namely, a nonlinear dynamic inversion definition approach is discussed in section III. The design of controllers. Section VIII gives the conclusions.

II. FLEXIBLE SPACECRAFT EQUATION

The system under investigation consists of a rigid hub and \( N \) appendages attached to it. According to figure 1, each appendage has linear density (mass per unit length) \( \rho_i \), length \( l_i \), and is attached at a distance \( a_i \) from the hub.

The kinetic energy of the system is composed of kinetic energies of the hub, and the appendages. This kinetic energy can be written in the form of:

\[
T = \frac{1}{2} \omega^T I \omega + \sum_{i=1}^{N} T_i
\]  

(1)

\[
T_i = \frac{1}{2} \int \rho_i V_i^T V_i \, dy
\]  

(2)

The velocity of a point on the panel is given by

\[
V_i = \dot{r}_i + \omega \times r_i
\]  

where \( \dot{r}_i = [0 \quad a_i + y_i - \delta_i \quad w_i] \).

The displacement in the \( \delta \) direction. Replacing appendage velocity in the kinetic energy results in:

\[
T_i = \frac{1}{2} \int (\omega^T I \omega + \kappa_i \omega + \rho_i \dot{w}_i^2)dy
\]  

(3)

Where \( I_i \) is the inertia tensor of the \( i \)th appendage, \( I_k \) is the inertia tensor of the rigid hub, \( \kappa_i \) is the \( i \)th appendage angular momentum due to the flexibility, and \( w_i \) is the deflection in the \( -z \) direction of the appendage reference frame.

The attitude dynamic equation is given by

\[
\frac{d\theta}{dt} + \omega \times H = \tau
\]

\[
H = \frac{\partial \theta}{\partial w} + h_\omega
\]  

(4)

Where \( \tau \) is the control torque and \( h_\omega \) is the internal angular momentum due to rotating wheels. The final form of the flexible attitude dynamic equation is

\[
I_\theta \dot{\omega} + I_t \omega + \Omega (I_t \omega + \kappa_\theta + h_\omega) + \kappa_\tau = \tau
\]  

(5)

\( \Omega \) is a skew symmetric matrix formed from the angular velocity vector \( \omega \) and subscript \( t \) is defined as total. To simplify, \( k_\theta \) is braked to two term such as: \( \dot{k}_\theta = \dot{k}_\theta \). To derive the dynamic model of the described system, the assumed modes formulation of the flexible appendage dynamics is used. Flexible deflection of the appendages along the body axis is given by:

\[
w_i = w_i(y, t) = \sum_{j=1}^{n} \psi_j(t) \psi_j(y_i) = \psi_1 q_i
\]  

(6)

where \( q_i \) are modal coordinates, \( n \) is the number of assumed modes, and \( \psi_j \) are shape functions of the appendage deformation.

The potential energy does not include a gravity term and is just the usual potential energy of beam bending deformation:

\[
P = \sum_{i=1}^{N} \int E_i (\frac{d^2 y}{dx^2})^2 dy = \sum_{i=1}^{N} \frac{E h_i^3}{12(1-\gamma^2)} \int q_i^T \psi_i^{\text{tr}} \psi_i^{\text{tr}} q_i \, dy
\]

(7)

The vectors and matrices in the expressions above are obtained from integrals of the appendages mode shapes and their spatial derivatives which are given in the Appendix.

The above equation may be rewritten as:

\[
\begin{bmatrix}
I_t & k_{\theta} \\
-k_{\theta}^T & M_{\phi \phi}
\end{bmatrix}
\begin{bmatrix}
\dot{\omega} \\
\dot{q}
\end{bmatrix}
= \left[ -\left(I_t \omega + \Omega (I_t \omega + \kappa_\theta + h_\omega) + k_{\theta} \right) Kq + Dq + C \right] +
\begin{bmatrix}
\tau \\
0
\end{bmatrix}
\]  

(8)

To include structural damping, a viscous damping term is added to equation (7) which results in a diagonal damping matrix \( D \), with entries \( \zeta \) for the damping parameter. More details can be found in reference [1].

III. OUTPUT REDEFINITION APPROACH

It is well known that the zero dynamics of a flexible spacecraft associated with the panel deflection are unstable. The goal is to control attitude angles and panel deflection of flexible spacecraft. \( \theta \) denote the Euler parameters that represent the orientation of body fixed frame with respect to an inertial frame, we have

\[
\dot{\theta} = R \omega
\]

\[
R^{-1} = \begin{bmatrix}
1 & 0 & -\sin \theta_2 \\
0 & \cos \theta_1 & \sin \theta_1 \cos \theta_2 \\
0 & -\sin \theta_1 & \cos \theta_1 \cos \theta_2
\end{bmatrix}
\]  

(9)

In [4] the sum of the joint angle and a scaling of the tip elastic deformation is chosen as the output for control of a flexible link manipulator, namely, \( y_{\text{out}} = \theta_1 + \alpha_1 q_1 \).

Moreover, it was shown that a critical value \( \alpha_1 > \alpha_1^* \) and are stable for \(-1 < \alpha_1 < \alpha_1^* \). Our objective in this section is to show that by using the new output \( y_{\text{out}} \) the dynamics of the flexible spacecraft may
be expressed in such a way that the feedback linearization method is applicable for controlling the system. The dynamic equation for flexible spacecraft with one panel and considering one elastic mode is of 8th order. Let us define the output as $y = \theta + \alpha q$. Now by two times differentiation of $y$, an explicit relationship between the output $y$ and controller input $\tau$ would be obtained. Hence, it is apparent that the system relative degree is $r=6<n=8$.

Therefore, parts of the system dynamics have been rendered ‘unobservable’ in this input-output linearization, the so-called internal dynamics of the system, since it cannot be seen from the external input-output relationship.

Consider the dynamics of the spacecraft (8) expressed in standard state space form $\dot{x} = f(x) + g(x)u$. The new set of states can be defined by $x = [\theta \; \dot{\theta} \; q \; \dot{q}]$. Choosing the state vector as $x$, the corresponding vector fields $f$ and $g$ can be written as:

$$f(x) = \begin{bmatrix} R \omega \\ \dot{R} \omega + R A_\omega (I_\omega - k_A M_{\psi_\phi}^{-1} (K_q + C)) \\ M_{\psi_\phi}^{-1} \left( (k_A A_\omega \tilde{k}_A M_{\psi_\phi}^{-1} + 1) (K_q + C) - \tilde{k}_A A_\omega I_\omega \right) \end{bmatrix}$$

$$g(x) = \begin{bmatrix} 0 \\ R A_\omega \\ 0 \end{bmatrix} - M_{\psi_\phi}^{-1} \tilde{k}_A A_\omega I_\omega$$

The new output can be expressed as $y = \theta + \alpha q$. To find the external dynamics related to this new output, take $\mu_1 = \theta + \alpha q$ and $\mu_2 = \dot{\theta} + \alpha \dot{q}$. Hence we can write:

$$\mu_1 = \mu_2$$

$$\dot{\mu}_1 = \dot{\theta} + \alpha \dot{q} = (f_2 + \alpha f_3) + (g_2 + \alpha g_3)$$

The third function $g(x)$ is required to complete the transformation i.e. brings the dynamics to its normal form. The third function $y(x)$ should satisfy the following equation

$$L_{gy} \psi_k = \frac{\partial g}{\partial x} g_j = 0$$

$$\frac{\partial g}{\partial \theta} R A_\omega + \frac{\partial g}{\partial \omega} (M_{\psi_\phi}^{-1} \tilde{k}_A A_\omega) = 0$$

One solution for this equation is

$$\psi_1 = q \\ \psi_2 = M_{\psi_\phi}^{-1} \tilde{k}_A R^{-1} \theta + \dot{\theta}$$

By differentiating these functions and by using system dynamics, the internal dynamics can be obtained as:

$$\dot{\psi}_1 = \dddot{q} \\ \dot{\psi}_2 = M_{\psi_\phi}^{-1} \left( K (\mu_1) \psi_1 + C (\mu_2) + D (\xi) \dot{\psi}_2 \right)$$

It is shown that local asymptotic stability of zero-dynamics is enough to guarantee the local asymptotic stability of the internal dynamics [5]. The zero dynamics is defined to be the internal dynamics of the system when the system output is kept identical at zero by a proper input.

$$\psi(0, \psi) = w(0, \psi)$$

$$y = 0 \rightarrow \dot{\theta} = -\alpha \dot{q}$$

Using equation (14) and equations (9, 17-18), it follows that:

$$(I - M_{\psi_\phi}^{-1} \tilde{k}_A R^{-1} \alpha) \dot{\psi}_1 + (M_{\psi_\phi}^{-1} \tilde{k}_A R^{-1} \dot{R} R^{-1} \alpha + M_{\psi_\phi}^{-1} D) \dot{\psi}_1 + M_{\psi_\phi}^{-1} K \psi_1 + M_{\psi_\phi}^{-1} C = 0$$

$I$ is $N \times N$ identity matrix. Since $(I - M_{\psi_\phi}^{-1} \tilde{k}_A R^{-1} \alpha)$ could be made positive by a proper selection of $\alpha$, we can conclude that the zero dynamics is asymptotically stable.

### IV. FEEDBACK LINEARIZATION DESIGN

It is assumed that no actuator is available on the flexible beam-type appendages. It is assumed that full state measurement of the system is available through attitude (e.g. sun sensors and gyros) and structural (e.g. strain gauges) sensors.

The successive differentiation process is done on the output (attitude angle) until the control signal appears:

$$y = \theta + \alpha q \rightarrow \dddot{y} = \dddot{\theta} + \dddot{\alpha} \dddot{q} = \nu$$

Using dynamic equations of spacecraft (8), it follows that

$$\dddot{y} = \dddot{\theta} + \dddot{\alpha} \dddot{q} = (f_2 + \alpha f_3) + (g_2 + \alpha g_3)$$

The coefficient of $\nu$, $A_{\nu}$, in special case ($q=0$) is equal to $I_{\nu}$, in other cases, it can be shown that this term is also invertible. Hence, the signal $\nu$ should be controlled to control the linearized system. The system can be controlled by introducing linear controller of the form:

$$v = \alpha_0^2 \epsilon_\nu - 2 \xi_\nu \omega_0 \nu_\epsilon$$

In most modern spacecraft, momentum exchange devices are used as actuators. Due to saturation effect in these actuators, considering saturation is very important. To enforce the position and rate saturation limits, feedback controller structure shown in figure 3, is used [6]. The gain can be chosen depending on the bounds of output response. In appropriate scaling, $\tanh$ can be used to represent saturation behavior: $u_{sat} = \tanh(u/u_{max}) u_{max}$

### V. COMPOSITE CONTROLLER DESIGN (FEEDBACK LINEARIZATION + $\mu$-SYNTHESIS)

Dynamic inversion and structured singular value synthesis are combined to achieve robust control of flexible spacecraft. The controller structure is shown in figure 2. In this method, nonlinear dynamics is linearized by input-output feedback linearization method. By definition output as $y = \theta + \alpha q$, new linear system is in the form of $\dddot{y} = \nu$; so new control signal $v$ should be designed.

Let us define the following parameters:

$$A_1 = f_2 + \alpha f_3 \\ A_2 = g_2 + \alpha g_3$$

Equation (22) can be written as: $\dddot{y} = A_1 \tau + A_2 v_{real}$ (27)

By considering uncertainty on parameters such as $I_c$, equation (27) can be written as:

$$\dddot{y} = (A_1 + \Delta A_1) \tau + (A_2 + \Delta A_2) v_{real} + \Delta A_1 \tau + \Delta A_2 v_{real} + \Delta v$$

By substituting real parameters, equation (28) can be written as:

$$\dddot{y} = v_{real} + \Delta A_1 A_1^{-1} (v_{real} - A_1 \tau - \Delta A_2 v_{real} + A_2 v_{real} + \Delta A_2 A_2^{-1} \Delta A_1 A_1^{-1} A_2)$$

As equation (29) shows, parameter uncertainty results in a multiplicative uncertainty in controller input ($\Delta A_1 A_1^{-1}$) and a disturbance ($-\Delta A_1 A_1^{-1} A_2 + \Delta A_2$). The controller structure is shown in figure 4.

To include the uncertainty in the model, different system parameters such as $I_c$ were perturbed by 20% of their
nominal values. Then, the nominal transfer function \( \left( \frac{1}{s^2} \right) \) of the system was selected as a double integrator, i.e. \( \frac{1}{s^2} \). Then the bode diagram of the actual system and the nominal transfer function plus the multiplicative weighting functions were obtained. The weighting functions were then tuned to get the best possible match which is: \( W_\Delta = \frac{70(s+1)}{s+100} \).

The effect of uncertain parameters on transfer function and bounds of selected weight is shown in figure 5. \( W_p \) weights the error between complementary sensitivity function of the closed loop system and an ideal model of system response. The performance objective can be written as \( |W_p S| \leq 1 \). According to first and third frequency of vibration modes of flexible panel, this function is chosen as:

\[
W_{p\theta} = \frac{0.1(s^3+0.25)}{(s^2+2s+0.01)}.
\]

Therefore, the hub performance weight has a relatively large magnitude at low frequencies.

According to weakness of dynamic inversion method against constant disturbance, a disturbance weight is chosen that has a relatively large magnitude at low frequencies.

The control input and its rate is bounded as:

\[
\alpha, \frac{\alpha}{\alpha}, \frac{\alpha}{\alpha} \leq 0.8.
\]

In this section, simulation results for the closed loop system (8) with the control laws derived in the previous sections are presented using MATLAB and SIMULINK software. In the simulation, the system parameters are chosen the same as those in [1].

\[
E = 5 \times 10^8 N/\text{m}^2, \quad \rho = 8 \text{ kg/m}, \quad l = 10 \text{ m}, \quad \gamma = 0.3, \quad r = [0.5, 0], \quad m = 0.02m
\]

The control input and its rate is bounded as:

\[
|u| < 0.8 N. m \quad |\dot{u}| < 0.8 N. m/s
\]
The environmental disturbances on the spacecraft are obtained from the following equation:

\[
\begin{align*}
    r_d &= 0.005 - 0.05 \sin(\frac{2\pi}{400}t) + \delta(200,0.2) + v_1 \\
    r_d &= 0.005 + 0.05 \sin(\frac{2\pi}{400}t) + \delta(250,0.2) + v_2 \\
    r_d &= 0.005 - 0.03 \sin(\frac{2\pi}{400}t) + \delta(300,0.2) + v_3
\end{align*}
\]

Where \(\delta(T,AT)\) denotes an impulsive disturbance with magnitude 1, period T, and width AT. The terms \(v_1, v_2, v_3\) denote white Gaussian noises with mean values of 0 and variances of 0.005².

It is assumed that the angular velocity and the pitch angle are measured by rate gyro and earth sensor respectively that are corrupted with random measurement noise. Earth sensor noise has Gaussian distribution, zero mean and standard deviation of 0.2 degree. The Gyro noise sources correspond to a random drift rate and a random bias rate. This model is represented by the following Laplace transformed equation:

\[
\omega_m = H_{gyro}\omega + \omega_d + \omega_n
\]

\(\omega_m\) and \(\omega_d\) are the measured and actual spacecraft angular velocity, respectively. Gyro random bias rate \(\omega_n\) and Gyro random drift noise \(\omega_d\) have Gaussian distribution, with zero mean, and standard deviation of 10⁶ rad/s. Gyro transfer function is:

\[
H_{gyro} = \frac{4469s + 89.22}{s^3 + 89.22s^2 + 4469s + 89.22}
\]

The robustness specification is to account for variation on the values of \(I\) and \(M_{\phi\psi}\) in (8) which would represent the model parameter uncertainties in the system up to 20%.

In this paper, the coefficient \(k_{\phi}\) \(M_{\phi\psi}^{-1}\), by considering one elastic mode, is equal to \([6.0804 \ 0 \ 0]\), \(\alpha\) should be chosen less than its inverse. In simulations, this constant is chosen as: \(\alpha = [0.14 \ 0.14 \ 0.14]\). By considering more elastic mode, i.e. three this coefficient is \([6.0804 \ 0 \ 0\] \[1.1247 \ 0 \ 0\] \[0.4514 \ 0 \ 0\]

hence for higher elastic mode, we can chose larger \(\alpha\).

The first three natural frequency in our simulations are \(\omega_1=11.89\), \(\omega_2=74.55\) and \(\omega_3=208.75\) (radian/sec).

The observer gains are selected to be \(P_2 = [-1 \ -1 \ -1]\), \(\gamma = 0.0001\) [7]. It is being used herein to provide on-line estimates for tip and rate of tip position and pitch rate, which are needed for the computation of the control signals.

Also, the gain parameters in feedback linearization method are chosen as \(\omega_y = 0.015\), \(\xi_y = 1\). In this subsection, a comparison of robustness obtained for the nonlinear system with the two proposed controllers (feedback linearization 2-combination of feedback linearization and \(\mu\) synthesis) are presented.

The results for the classical feedback linearization and composite controller are given in figures (6-7) respectively. In all simulations no damping is considered.

**A. Feedback linearization controller**

In normal conditions or in conditions that only one finite uncertain variation (disturbance, noise and uncertainty) exists, this method responds very well. It means the feedback linearization design leads to smaller maximum overshoot and complete suppression of panel deflection. The dynamic inversion controller achieves this decoupling at the cost of larger and faster control effort.

But in a large maneuver or in combined uncertain conditions (several uncertain conditions exist together or one uncertain condition with larger variations), much larger control efforts are necessary (out of maximum acceptable control input) and the attitude rate and position cannot converge.

Simulation results show that system is more robust against sensors noise and uncertainty than against environment disturbance. And also, the system is more sensitive against sine term of disturbance than disturbance constant term, and it has the most robustness against impulse disturbance term. For brevity the relevant figures are omitted.

**B. Composite controller**

Figure 7 shows the simulation results of the composite controller. With the designed composite algorithm, the response of roll, pitch and yaw angles are shown in figure 7a. We can see that each of the three attitude angles approaches reference trajectory at time of 600s. Hence, fast and precise attitude control is achieved for the current design system. As compared to figure 6a, in dynamic inversion method the response has a large steady state error and cannot converge.

Figure 7e shows low frequency oscillation of the appendage in composite method. The maximum tip deflection of the appendage is larger in dynamic inversion method and can be seen to be around 0.16m in figure 6e. Overall, comparing the plots 7f and 6f, composite controller has larger rate of tip deflection caused in faster panel deflection damping.

Requirement for momentum of each RW is illustrated in figure 7c. As compared to figure 6c, Composite method requires larger controller effort. Simulations show composite control algorithm performs well in large maneuvers; however it has larger controller order.

As shown in figures 6-7: b, e, f, the simulation results demonstrate the capability of the nonlinear observer in accurately estimating the state variables in the presence of uncertainties.

**VIII. CONCLUSION**

Vibration attenuation is a challenging control problem due to the stringent requirements on the performance and inherent characteristic of such structures. In this paper, flexible spacecraft attitude has been controlled by two controller designs. The first controller is the dynamic inversion; the second is a combination of dynamic inversion and \(\mu\) synthesis controller. The controllers have been designed by utilizing the modified output re-definition approach. In practical, the assumption that all state variables are measurable is not realistic; hence, a robust nonlinear observer has been designed based on the sliding mode methodology.
It is assumed only three torques in three directions on the hub are used. Actuator saturation is considered in the design of controllers. Simulation results prove composite controller ability in controlling attitude and also suppression vibration of panels with exhibit excellent performance and robustness for a broad range of operating conditions with minimal control effort. The simulations have demonstrated the capability of the observer in yielding accurate estimates of the state variables in the presence of uncertainties.

It is important to note that these controllers damp vibration of panels without considering damping term and without using any filter. It is notable that this composite control method has never been used on spacecraft and rarely, all terms such as disturbance, noise, uncertainty, nonlinearity and saturation, are considered in the simulations of flexible spacecraft simultaneously.

REFERENCES