Extended Target Tracking using an IMM Based Nonlinear Kalman Filters

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Abstract—The unscented Kalman filter (UKF) and ensemble Kalman filter (EnKF) are developed to extended target tracking problem for high resolution sensors. The nonlinear Kalman filters are based on an ellipsoidal model, which is proposed to exploit sensor measurement of target extent. The ellipsoidal model can provide extra information to enhance tracking accuracy, data association performance, and target identification. In contrast to the most commonly used extended Kalman filter (EKF), the UKF and EnKF provide more accurate and reliable estimation performance, due to the presence of high nonlinearity of the model. Correspondingly, the EnKF has lower computational complexity than the UKF. An interacting multiple model (IMM) technique is combined with the filters to adapt the target maneuver and motion mode switching problem which is vital for nonlinear filtering. The developed IMM-UKF and IMM-EnKF algorithms on extended target tracking problem are validated and evaluated by computer simulations.

I. INTRODUCTION

Most of the conventional target tracking algorithms model the target as a point source and estimate its kinematic states based on the incoming sensor measurement, such as range and bearing. This point source assumption is acceptable for low resolution sensors. However, for high resolution sensors, such as a high resolution radar sensor, or in the near field of the sensor, one or more dimensions of the target extent information could be obtained. For example, a high-resolution radar or infrared (IR) sensor can provide the down-range and cross-range extent measurement, which can help exploiting the information of the target shape to enhance the accuracy of estimation of kinematic states and improve the track retention, especially when two or more targets are close in space [1]. Knowledge of the target shape is especially important for target classification and identification as well.

There exist several ways to model the extended target [1-5]. A simple ellipsoidal model which combines the shape parameters into the state vector is adopted and modified in this paper [1]. The relationship between state vector and sensor measurements for an extended target is highly nonlinear. The most common approach to on-line nonlinear filtering is the extended Kalman filter (EKF) which simply linearizes all nonlinear functions so that the traditional Kalman filter (KF) framework can be applied. However, there are some substantial obstacles preventing the further employment of the EKF, such as difficult to tune, and prone to divergence under high nonlinearity [1]. In this paper, we propose both the ensemble Kalman filter (EnKF) and unscented Kalman filter (UKF) to implement the nonlinear filtering.

Recently, the UKF is widely applied as a relatively new nonlinear filtering algorithm, which has been shown to offer significant improvements in the estimation of nonlinear discrete time models in comparison of the EKF [6]. The UKF which based on unscented transformation (UT) [7,8] deterministically samples sigma points and propagates these points through the nonlinear transformation, so that the mean and covariance of the distribution of transformed random variables are captured to the third order accuracy [9].

In this paper we consider another approach to nonlinear estimation known as the ensemble Kalman filter (EnKF). While EnKF estimation has not been studied outside of specialized applications, its importance to specific problems is worthy of note. In particular, EnKF estimation is widely used in weather forecasting, where the models are of extremely high order and nonlinear, the initial states are highly uncertain, and a large number of measurements are available. A brief overview of the technique is given in [10,11].

The EnKF belongs to a broader category of filters known as particle filters [12, 13], which use neither the Jacobian of the dynamics nor frozen linear dynamics. The starting point for EnKF is choosing a set of sample points, that is, an ensemble of state estimates, which captures the initial probability distribution of the state. These sample points are then propagated through the true nonlinear system and the probability density function of the actual state is approximated by the ensemble of the estimates. In the case of the UKF, the sample points are chosen deterministically. In fact, the number of sample points required is of the same order as the dimension of the system. On the other hand, the number of ensembles required in the EnKF is heuristic. While one would expect that a large ensemble would be needed to obtain useful estimates, the literature on EnKF suggests that an ensemble of size 50 to 100 is often adequate for systems with thousands of states. The accuracy of the state estimates as a function of ensemble size is thus an important research question [14]. In order to overcome the model mismatch problem which is substantial for nonlinear systems, the IMM method is adopted.

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The IMM-UKF and IMM-EnKF algorithms offer good tracking accuracy and model adaptation as illustrated in the computer simulations.

The remaining part of the paper is organized as follows. Section II constructs the system dynamic and measurement model which is partly nonlinear. Section III formulates the UKF and EnKF. Section IV shows the simulation results and performance analysis of the algorithm we propose. The simulation is composed of the non-maneuvering target tracking performance and maneuvering target tracking performance with the model combined with the IMM technique to the nonlinear extended target tracking issue. The simulation results are organized to facilitate a comparison of the UKF and the EnKF, as well as to illustrate the estimation performance of the nonlinear Kalman filters. Finally, a conclusion is given in Section V.

II. Dynamic and Measurement Models

A. Ellipsoidal model for target extent

The elliptical model for the shape of extended target moving on a plane is proposed in [1] and adopted in this paper as a modified version. This is a convenient approach to model a rigid body, especially a ship or a vehicle on the ground observed by a high resolution radar sensor. The down-range and cross-range extent are measured as Fig. 1, with the relationship from the target states given by

\[
L(\phi) = l_1 \sqrt{\cos^2 \phi + \gamma^2 \sin^2 \phi}, \quad W(\phi) = l_2 \sqrt{\sin^2 \phi + \gamma^2 \cos^2 \phi}
\]  

(1)

where \( l_1 \) is the length of major axis of the ellipse, \( \phi \) is the angle between the major axis of the ellipse and the sensor-to-target line-of-sight (LOS), and the aspect ratio of the ellipse is defined by \( \gamma = b/a \), where \( b \) and \( a \) are the length of half minor axis and half major axis of the ellipse respectively.

For CV and CT model, the state transition matrices are

\[
F_{CT} = \begin{bmatrix}
1 & T & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & T & 0 \\
0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1
\end{bmatrix}
\]  

(4)

and

\[
F_{CT} = \begin{bmatrix}
\frac{1}{\omega} \sin(\alpha T) & 0 & \frac{-\cos(\alpha T) - 1}{\omega} & 0 & 0 \\
0 & \frac{1 - \cos(\alpha T)}{\omega} & 1 & \frac{\sin(\alpha T)}{\omega} & 0 \\
0 & \sin(\alpha T) & 0 & \cos(\alpha T) & 0 \\
0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1
\end{bmatrix}
\]  

(5)

respectively, where \( T \) is the time step between measurements and \( w_k \) is a zero-mean white Gaussian noise vector with covariance matrix \( Q = \text{diag}(w_x^2, w_y^2, w_{\psi}^2) \). A small positive value for \( w_x \) and \( w_{\psi} \) allows for some adjustment of the target shape estimate during tracking.

The angular velocity in the tracking algorithm could also be unknown and estimated as part of the state vector so that the dynamic model should be modified [15].

B. Dynamic and measurement model

Dynamic Model. The target extent parameters are included within the target state vector, as the targets concerned here are assumed as rigid bodies. Salmond and Parr [1] estimate the length of major axis with the EKF while assuming that the aspect ratio is known. For more universality and flexibility, we propose a joint estimation algorithm to recursively estimate the values of length and aspect ratio simultaneously. The augmented state vector is of the form

\[
x^i_k = (x, \dot{x}, y, \dot{y}, L, \gamma, \phi, \dot{\phi})
\]  

(2)

Where \((x, y)\) is the position of the target with respect to fixed axes and \( k \) denotes the time step. In this paper, we consider the constant velocity (CV) model and the constant rate coordinated turn (CT) model with the known turn rate. The two models are of the linear form

\[
x_{k+1} = F_{k} \cdot x_k + w_k
\]  

(3)

and

\[
F_{CT} = \begin{bmatrix}
\frac{1}{\omega} \sin(\alpha T) & 0 & \frac{-\cos(\alpha T) - 1}{\omega} & 0 & 0 \\
0 & \frac{1 - \cos(\alpha T)}{\omega} & 1 & \frac{\sin(\alpha T)}{\omega} & 0 \\
0 & \sin(\alpha T) & 0 & \cos(\alpha T) & 0 \\
0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1
\end{bmatrix}
\]  

(5)

Measurement Model. We assume a high resolution sensor provides measurements of range and bearing to the target centroid, as well as the down-range extent \( L \) and the cross-range extent \( W \) of the target. If it is assumed that the target ellipse is oriented that the major axis is parallel to the line-of-sight (LOS), and the LOS provides measurements of range and bearing to the target centroid, as well as the down-range extent \( L \) and the cross-range extent \( W \) of the target. If it is assumed that the target ellipse is oriented that the major axis is parallel to the line-of-sight (LOS), and the LOS provides measurements of range and bearing to the target centroid, as well as the down-range extent \( L \) and the cross-range extent \( W \) of the target.

\[
\tan \phi = \frac{\dot{x}(y - y_0) - \dot{y}(x - x_0)}{x(x - x_0) + \dot{y}(y - y_0)}
\]  

(6)

Then the target extent measurement could be written as

\[
L(\phi) = l_1 \sqrt{\cos^2 \phi + \gamma^2 \sin^2 \phi} = \frac{1}{\omega} \sqrt{(\dot{x}y' + \dot{y}x')^2 + \left(\frac{a}{b}\right)^2 (\dot{y}x' - \dot{x}y')^2}
\]  

(7)

and
The whole process can be done in the Kalman filtering framework, except some midway computations completed by the unscented transformation.

B. The EnKF Formulation

The ensemble Kalman filter (EnKF) is a suboptimal estimator, where the error statistics are predicted by using an ensemble integration to solve the Fokker-Planck equation. The ensemble Kalman filtering method is presented in three stages.

First, to represent the error statistics in the forecast step, we assume that at time $k$, we have an ensemble of $q$ forecasted state estimates with random sample errors. We denote this ensemble as $\chi_1^f \in R^{m \times q}$, where

$$\chi_1^f = (x_1^f, \ldots, x_q^f)$$

and the superscript $f$ refers to the $i$-th forecast ensemble member. Then, the ensemble mean $\bar{x}_1^f \in R^m$ is defined by

$$\bar{x}_1^f = \frac{1}{q} \sum_{i=1}^{q} x_i^f$$

Since the true state $x_1$ is not known, we approximate (14) by using the ensemble members. We define the ensemble error matrix $E_1^a \in R^{m \times q}$ around the ensemble mean by

$$E_1^a = [x_1^f - \bar{x}_1^f \ldots x_q^f - \bar{x}_1^f]$$

and the ensemble of output error by

$$E_1^a = [z_1^f - \bar{z}_1^f \ldots z_q^f - \bar{z}_1^f]$$

We then approximate $P_i^f$ by $\tilde{P}_i^f$, $P_i^e$ by $\tilde{P}_i^e$, and $P_i^f$ by $\tilde{P}_i^f$, respectively, where

$$\tilde{P}_i^f = \frac{1}{q-1} E_i^a (E_i^a)^T, \tilde{P}_i^e = \frac{1}{q-1} E_i^a (E_i^a)^T \tilde{P}_i^f = \frac{1}{q-1} E_i^a (E_i^a)^T$$

Thus, we interpret the forecast ensemble mean as the best forecast estimate of the state, and the spread of the ensemble members around the mean as the error between the best estimate and the actual state.

The second step is the analysis step: To obtain the analysis estimates of the state, the EnKF performs an ensemble of parallel data assimilation cycles, where for $i = 1, \ldots, q$

$$x_i^a = x_i^f + \tilde{K}_i (z_i^f - h(x_i^f)^T)$$

The perturbed observations $z_i^f$ are given by

$$z_i^f = z_i + \upsilon_i$$

where $\upsilon_i$ is a zero-mean random variable with a normal distribution and covariance $R_i$. The sample error covariance matrix computed from the $\upsilon_i$ converges to $R_i \text{ as } q \to \infty$. We approximate the analysis error covariance $P_i^a$ by $\tilde{P}_i^a$, where

$$\tilde{P}_i^a = \frac{1}{q-1} E_i^a (E_i^a)^T$$

and $E_i^a$ is defined by (18) with $x_i^f$ replaced by $x_i^a$ and $\pi_i^f$ replaced by the mean of the analysis estimate ensemble members. We use the classical Kalman filter gain expression and the approximations of the error covariance to determine the filter gain $\tilde{K}_i$ by

$$\tilde{K}_i = \tilde{P}_i^e (\tilde{P}_i^a)^{-1}$$

The last step is the prediction of error statistics in the forecast step:

$$x_i^f = f(x_i^a, u_i) + \upsilon_i$$

where $\upsilon_i$ is the measurement noise and obeys the Gaussian distribution $N(0, R_i)$, with $R_i = \text{diag}(R_{i1}, R_{i2}, \ldots, R_{iq})$.

III. NONLINEAR FILTER FOR EXTENDED TARGET TRACKING

The common EKF approach is no longer competent for our problem since the high nonlinearity. In our problem, the measurement equation (9) is severely nonlinear so that the EKF approach can just work satisfactorily when the shape parameters are partly known [1], but can hardly make joint estimation of length and aspect ratio simultaneously. As a superior alternative, some new Kalman filters are adopted for nonlinear systems. Then the unscented Kalman filter (UKF) and the ensemble Kalman filter (EnKF) are employed. Since the dynamic model is linear and Gaussian, we conceptually use the UKF and EnKF for the nonlinear part of the measurement equation and leave the remaining part to the conventional KF algorithm.

A. The UKF Formulation

In the dynamic and measurement equations of the system, only the $h(*)$ function is nonlinear.

$$x_{k+1} = F_k x_k + B_k w_k$$

$$z_k = h(x_k) + v_k$$

So the predictive mean and covariance can be calculated exactly using KF equations

$$\hat{x}_{k+1} = F_k \hat{x}_k + B_k \hat{P}_k$$

$$P_{k+1} = F_k P_k F_k^T + B_k Q_k B_k^T$$

The state vector $x_k$ with mean $\hat{x}_k$ and covariance $P_k$ can be approximated by sigma points $\chi_{k+1}$ selected from the columns of $\hat{x}_k \pm (a_k \sqrt{P_k})$, $i = 0, \ldots, 2L$. The opposite weight $\omega_i$ is $\omega_0 = 1/(L+1)$, $\omega_i = 1/(2L)$ ( $i = 1, 2, \ldots, 2L$ ). Then the updated mean and covariance can be calculated through the transformed sigma points

$$\hat{z}_k = h(\chi_k), \hat{z}_k = \sum_{i=0}^{2L} \omega_i z_{i,k}$$

$$P_a = \sum_{i=0}^{2L} \omega_i (z_{i,k} - \hat{z}_k)(z_{i,k} - \hat{z}_k)^T + R_k$$

and the KF will continue as the arrival of new measurement $z_k$

$$x_k = \hat{x}_{k+1} + W_k(z_k - \hat{z}_k), P_k = P_{k+1} - W_k P_a W_k^T$$

where $W_k = P_{k+1} P_a^{-1}$.
where the values \( w_i \) are sampled from a normal distribution with average zero and covariance \( Q \). The sample error covariance matrix computed from the \( w_i \) converges to \( Q \) as \( q \to \infty \). Finally, we summarize the analysis and forecast steps. Analysis Step:

\[
\hat{\textbf{x}}_i = \hat{P}_{ii} \left( \hat{P}_{ii} \right)^{-1}, \]

\[
x_i = \textbf{x}_i + \hat{\textbf{x}}_i \left( z_i - \textbf{h}(\textbf{x}_i) \right)^T, \quad \tilde{\textbf{x}}_i = \frac{1}{q} \sum_{i=1}^{q} x_i. \tag{26}
\]

Forecast Step:

\[
x_{i+1} = \textbf{F}_i \textbf{x}_i + \textbf{B}_i w_i, \quad \tilde{\textbf{x}}'_i = \frac{1}{q} \sum_{i=1}^{q} x'_i,
\]

\[
E'_i = [x'_i - \tilde{x}'_i] \cdots [x'_i - \tilde{x}'_i], \quad E'_i = [z'_i - \tilde{z}'_i] \cdots [z'_i - \tilde{z}'_i],
\]

\[
\hat{P}'_{i+1} = \frac{1}{q-1} E'_i (E'_i)^T. \tag{27}
\]

Unlike the extended Kalman filter, the evaluation of the filter gain \( \hat{K}_i \) in the EnKF does not involve an approximation of the nonlinearity \( f(x, u) \) and \( h(x) \). Hence, the computational burden of evaluating the Jacobians of \( f(x, u) \) and \( h(x) \) is absent in the EnKF. Furthermore, note that equations in the UKF involves evaluation of \( P'_i \in \mathbb{R}^{n \times n} \), which is an \( O(n^3) \) operation. However, in (26)-(27) of the EnKF, only \( P'_{i+1} \in \mathbb{R}^{n \times p} \) and \( \hat{P}'_{i+1} \in \mathbb{R}^{n \times p} \) are evaluated, which is an \( O(pn) \) operation. Hence, if \( q \ll n \), then the computational burden of evaluating the approximate covariance in the EnKF is less than the computational burden of determining the approximate covariance in the UKF. However, (26) implies that \( q \) parallel copies of the model have to be simulated, and, when \( q \) is large, the computational burden of the forecast step in the EnKF is large. Alternatively, in the UKF, only one copy of the model is simulated to obtain the state estimates. Hence, if \( n \) is very large and \( q \ll n \), then the EnKF is computationally less intensive than the UKF.

\[ \text{IV. SIMULATION RESULTS AND PERFORMANCE ANALYSIS} \]

Non-maneuvering and maneuvering Target Tracking Performances are considered on the final approach. The simulation results are organized to facilitate a comparison of the UKF, the EnKF and the EKF, as well as to illustrate the estimation performance.

A. Non-maneuvering Target Tracking Performance

The non-maneuvering extended target tracking performance is evaluated by simulations over noisy trajectories of constant velocity targets. The target follows a constant speed path, initially moving in a straight line and then turning at a constant rate per time step. The purpose of the non-maneuvering tracking performance analysis is to demonstrate that both the dynamic states and the shape parameters can be estimated more accurately under the help of target extent measurements. The initial target state is \( x_0 = (-10000, 15, -5000, 20, 50, 0.2) \). The observer is static, located at the origin of the \( x-y \) plane, and obtains measurements of range, bearing, along-range extent and cross-range extent at a time interval of 2 seconds. The measurements are corrupted by a zero-mean, white Gaussian noise \( R_i = \text{diag}[\sigma_r^2, \sigma_b^2, \sigma_{x'}^2, \sigma_{y'}^2] \), with the standard deviations \( \sigma_r = 3 \) units, \( \sigma_b = 0.1^\circ \), and \( \sigma_{x'} = \sigma_{y'} = 1 \) units. Fig. 2 and Fig. 3 show the target position trajectory and the velocity estimation errors, along with the \( 2\sigma \) error bounds, that are reformed by the precisely measured target extents.

While estimating the shape parameters, sensors that measure down-range extent and cross-range extent simultaneously can obtain more accurate estimation than that
only measure one dimension, as shown in Fig. 4.

The computational time required to calculate the estimates is also given. The time taken to simulate 200 s in the model is also shown in Figure 5 for comparison. The time required to obtain the state estimates using the UKF and EnKF is shown.

The results show that both the dynamic states and the shape parameters can be estimated more accurately under the help of target extent measurements. The UKF and EnKF all provide more accurate and reliable estimation performance, without increasing any computational complexity.

**B. Maneuvering Target Tracking and Model Adaptation**

When the tracking model is perfectly matched with the practical situation, the effect of nonlinear filters might perform ideally. However, maneuvers and miss associations happen frequently in real environment of surveillance systems. Flexibility on model is the key point for the real filter. Unfortunately, nonlinear filters might be intensively unstable and unpredictable under mismatched model. An effective solution to improve the model flexibility is to introduce the Interacting Multiple Model (IMM) technique [16].

The IMM estimator is a suboptimal hybrid filter that is able to estimate the state of a dynamic system with several behavior modes which can switch from one to another. It is assumed that the total number of target maneuver models \( r \) is three as the typical case. To implement the Markov model, a probability \( P_{ij} \) that the target makes transition from model state \( i \) to state \( j \) is known a priori and expressed in a probability transition matrix \( P = [P_{ij}]_{3 \times 3} \), which satisfies \( \sum_{i=1}^{3} P_{ij} = 1 \). Therefore, the overall state estimation and covariance matrix is calculated as a weighted sum of each single one[17,18].

Nonlinear filters will be challenged under target maneuvers if the tracking algorithm is designed on single model. When the model can not follow with the actual movement, the nonlinear filter will be strongly unstable as shown in Fig. 9, while the KF will only be affected on accuracy. Model mismatch is a fatal problem preventing the applications of nonlinear filtering.

The testing scenario is a compound of uniform motions and maneuvers, as shown in Fig. 6. The initial state of the motion is \( x_0 = (-10000, 45, -5100, 60, 50, 0.2) \), the maneuvers are coordinate turns with angular velocity \( \omega = \pm 0.1 \text{rad/s} \), and the noise terms are the same as above. The experimental models are constant velocity movement and constant rate coordinated turn maneuvers with the angular velocity \( \omega \).

To illustrate the tolerance of the imprecise prior information of the motion, the value of \( \sigma_\omega \) adopted in the tracking
algorithm is set $1.5\,\sigma_0$. The result of model probability estimation is shown in Fig. 7 and the estimation of the shape parameters of the extended target is shown in Fig. 8.

The simulation results indicate that it is possible to effectively maneuvering target tracking with nonlinear Kalman filter by combined the IMM technique to the filter equations. Obviously, Fig. 8 shows that the computational burden associated with the UKF is greater than the EnKF, and the UKF have more accurate and reliable estimation performance than EnKF in this application.

V. CONCLUSION

The UKF and the EnKF algorithms for tracking an ellipse modelled extended target is developed and studied in this paper. As the shape information is observed by high resolution sensors and exploited by the nonlinear filters, the tracking accuracy of the dynamic state is also significantly improved. Compared with each other, the nonlinear filters make a superior alternative. For the computational cost, the EnKF represents great superiority than UKF. The motion model adaption is solved by introducing the IMM technique so that the algorithm is able to work under most of the motion modes. Simulation results for a target with three motion types are given to illustrate the ability of the UKF and the EnKF algorithms to track an extended target and estimate the shape parameters under highly nonlinear model and switching maneuvers.

REFERENCES


