Observer-based Nonlinear Control Allocation

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Abstract—In this paper, a Lyapunov-based approach is proposed for control allocation problem based on output feedback and model reference control. A nonlinear observer design approach for a general class of nonlinear systems is used to estimate the unmeasurable system states. The proposed approach ensures that the estimated states exponentially converge to their true values and the designed closed-loop system converges to a given stable reference model as \( t \to \infty \).

Index Terms: nonlinear system, control allocation, nonlinear observer.

1. INTRODUCTION

Control allocation is the process of mapping virtual control inputs (such as torque and force) into actual actuator deflections in the design of control systems [6], [8], [3], [12]. Essentially, it is considered as a constrained optimization problem as one usually wants to fully utilize all actuators in order to minimize power consumption, drag and other costs related to the use of control, subject to constraints such as actuator position and rate limits. In the design of control allocation, full state information is required. However, in practice, states may not be measurable. Hence, estimation of these unmeasurable states becomes inevitable.

The unmeasurable states are generally estimated based on available measurements and the knowledge of the physical system. For linear systems, the property of observability guarantees the existence of an observer. Luenberger or Kalman observers are known to give a systematic solution [13]. In the case of nonlinear systems, observability in general depends on the input of the system. In other words, observability of a nonlinear system does not exclude the existence of inputs for which two distinct initial states generate identical measured outputs. Hence, in general, observer gains can be expected to depend on the applied input [15]. This makes the design of a nonlinear observer for a general nonlinear system a challenging problem. Although various results have been proposed over the past decades [15], [4], [5], [2], [14], [20], [10], [11], [7], [9], [16], [17], [18], [1], none of them can claim to provide a general solution with the same convergence properties as in the linear case. The existing observers generally depend closely on some specific structures of the nonlinear systems under consideration.

In this work, we propose a Lyapunov-based approach for control allocation design via output feedback and model reference control. The corresponding results extend those developed in [12], [3] from state feedback to output feedback. The Lyapunov-type state observer for a general class of nonlinear systems in [17], [18] is adopted in the design. The proposed approach ensures that the state estimation error converges exponentially to zero and the closed-loop system converges to a given stable reference model as \( t \to \infty \).

Throughout this paper, given a real map \( f(\mathbf{v}, \mathbf{w}) , (\mathbf{v}, \mathbf{w}) \in \mathbb{R}^n \times \mathbb{R}^m \), \( D_{\mathbf{v}}f(\mathbf{v}_0, \mathbf{w}_0) \) denotes its derivative with respect to \( \mathbf{v} \) at the point \((\mathbf{v}_0, \mathbf{w}_0)\). For given real map \( h(\mathbf{v}) \) with \( \mathbf{v} \in \mathbb{R}^n \), \( Dh(\mathbf{v}_0) \) denotes its derivative with respect to \( \mathbf{v} \) at the point \( \mathbf{v}_0 \). In addition, \( \| \cdot \| \) represent Euclidean norm.
2. Problem Formulation

Consider the following nonlinear system:

\[
\begin{align*}
\dot{x} &= f(x, u) \\
y &= h(x)
\end{align*}
\]  

(1)

where \( x \in \mathcal{X} \subset \mathbb{R}^{n_x} \) is the state vector with \( \mathcal{X} \) an open subset of \( \mathbb{R}^{n_x} \) and \( 0 \in \mathcal{X} \), \( y \in \mathbb{R}^l \) is the measurement output vector, and \( u \in \mathbb{R}^m \) is the control input vector satisfying

\[
u \in \Omega \triangleq \left\{ u=[u_1 u_2 \cdots u_m]^T | u_i \leq \bar{u}_i, i=1,2,\ldots,m \right\}
\]

(2)

with \( \bar{u} = [\bar{u}_1 \bar{u}_2 \cdots \bar{u}_m]^T \) and \( \bar{u} = [\bar{u}_1 \bar{u}_2 \cdots \bar{u}_m]^T \) being vectors of lower and upper control limits, respectively. In this paper, the system (1) is assumed to satisfy the following assumption:

Assumption 1: The function \( f(x, u) \) is smooth and the output function \( h(x) \) is continuously differentiable.

As control allocation need full state information, the state estimation for the system (1) is required.

Consider a dynamic observer for (1) of the following form

\[
\dot{\hat{x}} = f(\hat{x}, u) - \Phi(\hat{x}, u)[y - h(\hat{x})]
\]

(3)

Define the error \( e \) as

\[
e = x - \hat{x}
\]

(4)

We wish to design the mapping \( \Phi(\hat{x}, u) \) such that the trajectory of \( e \) with the dynamics

\[
\dot{e} = f(x, u) - f(\hat{x}, u) + \Phi(\hat{x}, u)[y - h(\hat{x})]
\]

(5)

converge to zero as \( t \to +\infty \), uniformly on \( u \in \Omega \), for every \( x(0) \in \mathcal{X} \) subject to \( e(0) = x(0) - \hat{x}(0) \) near zero.

The aim of this paper is to design a control allocation law via the state observer (3) such that the closed-loop system matches the following reference model which represents the desired dynamics.

\[
\dot{x} = A_d x + B_d r
\]

(6)

where \( A_d \in \mathbb{R}^{n_x \times n_x} \), \( B_d \in \mathbb{R}^{n_x \times n_r} \) and the reference \( r \in \mathbb{R}^{n_r} \) satisfy the following assumption:

Assumption 2: \( A_d \) is Hurwitz, and \( r \in \Sigma \subset \mathbb{R}^{n_r} \) is continuously differentiable where \( \Sigma \) is an open subset defined by: for each \( r \in \Sigma \), there exist \( x \in \mathcal{X} \) and \( u \in \Omega \) such that the system (1) matches the reference system (6).

With the state estimate \( \hat{x} \), we introduce the cost function

\[
J_1(\hat{x}, r, u) = \frac{1}{2} \tau^T(\hat{x}, r, u)H_1 \tau(\hat{x}, r, u)
\]

(7)

to minimize the matching error \( \tau(\hat{x}, r, u) \) where

\[
\tau(\hat{x}, r, u) \triangleq f(\hat{x}, u) - A_d \hat{x} - B_d r
\]

(8)

and \( 0 < H_1 \in \mathbb{R}^{n_x \times n_x} \) is a known weighting matrix.

Further, we introduce a cost function to minimize power consumption

\[
J_2(u) = \frac{1}{2} u^T H_2 u
\]

(9)

where \( 0 < H_2 \in \mathbb{R}^{m \times m} \) is a known weighting matrix.

Now the control allocation problem is formulated in terms of solving the following nonlinear static minimization problem:

\[
\min_{u} J(\hat{x}, r, u) \quad \text{subject to} \quad u \in \Omega
\]

(10)

where \( \hat{x} \) converges to \( x \) exponentially

with

\[
J(\hat{x}, r, u) = J_1(\hat{x}, r, u) + J_2(u).
\]

Define

\[
\Delta(u) = [S(u_1) \ S(u_2) \cdots S(u_m)]
\]

(11)

where

\[
S(u_i)=\min((u_i-\bar{u}_i)^3, (\bar{u}_i-u_i)^3, 0), i=1,2,\ldots,m
\]

(12)

Then the constraint condition \( u \in \Omega \) is equivalent to

\[
\Delta(u) = 0
\]

(13)

Here we reformulate the first line of the optimization problem (10) by minimizing the Lagrangian

\[
L(\dot{x}, r, u, \lambda) = J(\hat{x}, r, u) + \epsilon \Delta(u) \lambda
\]

(14)

with \( \epsilon > 0 \) a constant and \( \lambda \in \mathbb{R}^m \) a Lagrange multiplier.

Assume that

Assumption 3: \( \frac{\partial^2 L}{\partial u^2} > 0 \).

We have the following lemma immediately.

Lemma 1: ([19], p. 42) If Assumptions 1 and 3 hold, the problem (14) achieve local minima if and only if \( \frac{\partial L}{\partial \lambda} = 0 \) and \( \frac{\partial L}{\partial u} = 0 \).
To solve the control allocation problem with the state estimate \( \hat{x} \) from the observer (3), we consider the following control Lyapunov-like function

\[
V(\hat{x}, e, r, u, \lambda) = V_m(\hat{x}, r, u, \lambda) + \frac{1}{2} e^T P e
\]

with \( P > 0 \) and

\[
V_m(\hat{x}, r, u, \lambda) = \frac{1}{2} \left[ \left( \frac{\partial L}{\partial u} \right)^T \frac{\partial L}{\partial u} + \left( \frac{\partial L}{\partial \lambda} \right)^T \frac{\partial L}{\partial \lambda} \right]
\]

The function \( V_m \) is designed to attract \((u, \lambda)\) so as to minimize \( L \). The second term of (15) is used to achieve the exponential convergence of the observer error \( e \).

Following the observer design in [18], we define a neighborhood \( Q \) of zero with \( Q \subset \mathcal{X} \), a neighborhood \( W \) of \( \mathcal{X} \) with \( \{x - e : x \in \mathcal{X}, e \in Q\} \subset W \), and a closed ball \( S \) of radius \( r > 0 \), centered at zero, such that \( S \subset Q \). Then define the boundary of \( S \) as \( \partial S \). Figure 1 illustrates the geometrical relationship of these defined sets.

![Figure 1. Geometrical representation of sets](image)

Let \( \mathcal{H} \) denote the set of the continuously differentiable output mappings \( h(x) : \mathcal{X} \to \mathbb{R}^l \) such that for every \( m_0 \in Q \) and \( \hat{x} \in W \), we have

\[
R(\hat{x}, m_0) \geq 0
\]

(17)

\[
\ker R(\hat{x}, m_0) \subset \ker D_h(\hat{x})
\]

(18)

where

\[
R(\hat{x}, m_0) = [D_h(\hat{x})]^{T} D_h(\hat{x} + m_0) + [D_h(\hat{x} + m_0)]^{T} D_h(\hat{x})
\]

(19)

We assume that

\[
\lambda(\hat{x}, m_0) = \hat{x}^T D_h(\hat{x}) (\hat{x} + m_0) + (\hat{x} + m_0)^T D_h(\hat{x}) \hat{x}
\]

Assumption 4: \( h(x) \) in the system (1) belongs to the set \( \mathcal{H} \), namely, \( h(x) \in \mathcal{H} \).

Further, we define

\[
N_{\Delta} \triangleq \left\{ e \in \mathbb{R}^{n_x} : e^T P D_x f(\hat{x} + m_1, u)e \leq -k_0 \|e\|^2 \right\}
\]

(20)

and assume that

\[
\text{Assumption 5: There exist a positive definite matrix } P \in \mathbb{R}^{n_x \times n_x} \text{ and a positive constant } k_0 \text{ such that } \ker D_h(\hat{x}) \subset N \text{ holds for any } (\hat{x}, m_1, u) \in W \times Q \times \Omega.
\]

Remark 1: Assumption 5 ensures that the estimation error system (5) is stable in the case of \( h(x) = h(\hat{x}) \) and \( x \neq \hat{x} \). In particular, for linear systems, the condition in Assumption 5 is used to ensure detectability.

3. MAIN RESULTS

Denote

\[
\begin{bmatrix}
\alpha \\
\beta
\end{bmatrix} = \begin{bmatrix}
\frac{\partial^2 L}{\partial u^2} & \frac{\partial^2 L}{\partial \lambda \partial u} \\
\frac{\partial^2 L}{\partial u \partial \lambda} & 0_{m \times m}
\end{bmatrix}
\]

(21)

and define

\[
M \triangleq \left\{ \nu \in \mathbb{R}^{n_x} : \nu = r \|e\|^{-1} e, \ e \in N \cap S \right\}
\]

(22)

Let

\[
\gamma_1(\hat{x}, u) = \max \left\{ \nu^T (\nu^T D_x f(\hat{x} + m_1, u)) + k_0 \right\}, \ m_1 \in S, (\hat{x}, u) \in W \times \Omega
\]

(23)

\[
\gamma_2(\hat{x}) = \min \left\{ \frac{1}{2} \nu^T R(\hat{x}, m_0) \nu, m_0 \in S, \nu \in \partial S - M, \hat{x} \in W \right\}
\]

(24)

Notice that, since \( m_0, m_1 \in S \) and \( (\hat{x}, u) \in W \times \Omega \), according to Assumption 1, \( D_x f(\hat{x} + m_1, u) \) is bounded. From \( k_0 > 0 \), we have \( 0 < \gamma_1(\hat{x}, u) < +\infty \). According to Assumption 4, we have \( \ker R(\hat{x}, m_0) \subset \ker D_h(\hat{x}) \) and \( R(\hat{x}, m_0) \geq 0 \) which ensures that \( 0 < \nu^T R(\hat{x}, m_0) \nu < +\infty \) for every \( \nu \in \partial S - M, m_0 \in S \) and \( \hat{x} \in W \). Thus, we have \( 0 < \gamma_2(\hat{x}) < +\infty \).

Theorem 1: Consider the system (1) with \( x \in \mathcal{X} \) and \( u \in \Omega \). Suppose that Assumptions 1-5 are satisfied. For a given asymptotically stable matrix \( A_d \) and a matrix \( B_d \), given symmetric positive-definite matrices \( \Gamma_1 \) and \( \Gamma_2 \), and a given
positive constant $\omega$, and for $e(0)$ near zero, \( \left( \frac{\partial L}{\partial \lambda} \frac{\partial L}{\partial u} \right) e \) exponentially converges to zero as $t \to +\infty$, and the dynamics of the nonlinear system (1) converges to that of the stable reference model $\dot{x} = A_d x + B_d r$ if the following dynamic update law
\[
\begin{cases}
\dot{u} = -\Gamma_1 \alpha + \xi_1 \\
\dot{\lambda} = -\Gamma_2 \beta + \xi_2
\end{cases}
\tag{25}
\]
and the observer system
\[
\dot{x} = f(\tilde{x}, u) - \Phi(\tilde{x}, u) [y - h(\tilde{x})]
\tag{26}
\]
are adopted. Here $\alpha \in \mathbb{R}^m$, $\beta \in \mathbb{R}^m$ are as in (21), and $\xi_1 \in \mathbb{R}^m$ and $\xi_2 \in \mathbb{R}^m$ satisfy
\[
\alpha^T \xi_1 + \beta^T \xi_2 + \delta + \omega V_m = 0
\tag{27}
\]
with $V_m$ as in (16) and
\[
\delta = \frac{\partial L}{\partial u} + \frac{\partial^2 L}{\partial u \partial \lambda} \dot{u} + \frac{\partial L}{\partial u} \frac{\partial^2 L}{\partial \lambda^2} \dot{\lambda}
\tag{28}
\]
and the mapping
\[
\Phi(\tilde{x}, u) = -\theta(\tilde{x}, u) P^{-1} [Dh(\tilde{x})]^T
\tag{29}
\]
where
\[
\theta(\tilde{x}, u) \geq \frac{\gamma_1(\tilde{x}, u)}{\gamma_2(\tilde{x})}
\tag{30}
\]
with $\gamma_1(\tilde{x}, u) > 0$ and $\gamma_2(\tilde{x}) > 0$ defined as in (23) and (24).

**Proof:** From the Lyapunov-like function (15), we obtain its time derivative as
\[
\dot{V} = \left( \frac{\partial L}{\partial u} \right)^T \frac{\partial^2 L}{\partial u \partial \lambda} \dot{u} + \frac{\partial L}{\partial u} \frac{\partial^2 L}{\partial \lambda^2} \dot{\lambda}
\tag{31}
\]
Substituting $\dot{e}$ in (5), $\alpha$, $\beta$ and $\phi$ as in (21) and $\delta$ as in (28) into (31), we have
\[
\dot{V} = \alpha^T \dot{u} + \beta^T \dot{\lambda} + \delta
+ e^T \left[ f(x, u) - f(\tilde{x}, u) + \Phi(\tilde{x}, u) [y - h(\tilde{x})] \right]
\tag{32}
\]
Consider $e \in S$. Since $S$ is convex, according to Mean Value Theorem, there exists $m_0, m_1 \in S$ such that
\[
f(x, u) - f(\tilde{x}, u) = D_x f(\tilde{x} + m_1, u) e
\tag{33}
\]
\[
y - h(\tilde{x}) = Dh(\tilde{x} + m_0) e
\tag{34}
\]
Then substituting (25), (27), (33) and (34) into (32), we obtain
\[
\dot{V} = -\alpha^T \Gamma_1 \alpha - \beta^T \Gamma_2 \beta - \omega V_m
+ e^T \left[ D_x f(\tilde{x} + m_1, u) + \Phi(\tilde{x}, u) Dh(\tilde{x} + m_0) \right] e
\tag{35}
\]
After substituting $\Phi(\tilde{x}, u)$ as in (29) and $R(\tilde{x}, m_0)$ as in (19), (35) can be rewritten as
\[
\dot{V} = -\alpha^T \Gamma_1 \alpha - \beta^T \Gamma_2 \beta - \omega V_m
+ e^T P D_x f(\tilde{x} + m_1, u) e - \frac{\theta(\tilde{x}, u)}{2} e^T R(\tilde{x}, m_0) e
\tag{36}
\]
Since the matrices $\Gamma_1 > 0$ and $\Gamma_2 > 0$, we have
\[
\dot{V} \leq -\omega V_m + e^T P D_x f(\tilde{x} + m_1, u) e - \frac{\theta(\tilde{x}, u)}{2} e^T R(\tilde{x}, m_0) e
\tag{37}
\]
For $e = 0$, we have
\[
\dot{V} \leq -\omega V_m = -\omega V
\tag{38}
\]
Since $\omega > 0$, $V$ exponentially converges to zero as $t \to +\infty$. For any nonzero $e \in S$, let $\nu = r ||e||^{-1} e$. Then we have
\[
\dot{V} \leq -\omega V_m + \frac{1}{r^2} ||e||^2 \nu^T P D_x f(\tilde{x} + m_1, u) \nu - \frac{\theta(\tilde{x}, u)}{2} ||e||^2 \nu^T R(\tilde{x}, m_0) \nu
\tag{39}
\]
First let us consider nonzero $e \in N \cap S$. From $\nu = r ||e||^{-1} e$, we have $\nu \in M$. Since $m_0, m_1 \in S \subseteq Q$, $\tilde{x} \in W$ and $u \in \Omega$, according to Assumptions 1-5, it follows that
\[
\nu^T R(\tilde{x}, m_0) \nu \geq 0
\tag{40}
\]
and
\[
\nu^T P D_x f(\tilde{x} + m_1, u) \nu \leq -k_0 ||\nu||^2
\tag{41}
\]
with the constant $k_0 > 0$.
From (39), we have
\[
\dot{V} \leq -\omega V_m - k_0 ||e||^2 \leq -\sigma V
\tag{42}
\]
for some constant $\sigma > 0$.
Then we consider nonzero $e \in S - N \cap S$, namely, $\nu \in \partial S - M$. From (39), taking into account (23)-(24), we obtain
\[
\dot{V} \leq -\omega V_m + \frac{1}{r^2} ||e||^2 \left[ \gamma_1(\tilde{x}, u) - k_0 r^2 - \theta(\tilde{x}, u) \gamma_2(\tilde{x}) \right]
\tag{43}
\]
Since $\theta(\hat{x}, u)$ satisfy the condition (30), we obtain (42) again. Thus, for all $e \in S$, \( \left( \frac{\partial L}{\partial \lambda}, \frac{\partial L}{\partial u}, e \right) \) exponentially converges to zero as $t \to +\infty$. By Assumption 2, we have $\tau(x, r, u)$ converges to zero as $t \to +\infty$, namely, the system (1) converges to $\dot{x} = A_d x + B_d r$. This completes the proof.

To solve $\xi_1$ and $\xi_2$ from (27), the method proposed in [12] is used.

4. Example

Consider the pendulum system
\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2 \\
y
\end{bmatrix} = \begin{bmatrix}
x_2 \\
-\sin x_1 + u_1 \cos x_1 + u_2 \sin x_1 \\
x_1 + x_2
\end{bmatrix}
\]  
(44)

with $x = [x_1 \ x_2]^T \in \mathbb{R}^2$, $u = [u_1 \ u_2]^T \in \Omega$ and
\[
\Omega = \left\{ u = [u_1 u_2]^T : |u_1| \leq 1, -0.5 \leq u_2 \leq 0.5 \right\}
\]  
(45)

Choose
\[
P = \begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix}
\]

For $e \neq 0$ and $e \in \ker[1 \ 1]$, we have $e_1 = -e_2$ and
\[
e^T P D_x f(x, u) e|_{e_1 = -e_2}
= \begin{bmatrix} e_1 & -e_1 \\ - \cos x_1 - u_1 \sin x_1 + u_2 \cos x_1 & 0 \\ \cos x_1 + u_1 \sin x_1 - u_2 \cos x_1 - 3e_1^2 & 0 \\ -0.5986\|e\|^2 |_{e_1 = -e_2} \leq -k_0 \|e\|^2 |_{e_1 = -e_2}
\]

with $0 < k_0 < 0.5986$. Hence, Assumption 5 is satisfied. Let $S$ be the ball of radius $r = 1$, centered at zero and $\partial S$ is the boundary of $S$. Define $M \subset \partial S$ and
\[
M = \left\{ \nu = [\nu_1 \nu_2]^T \in \mathbb{R}^2 : \|\nu\| = 1, 3\nu_1 \nu_2 + 1.8028|\nu_1 \nu_2| < -k_0 \right\}
\]
Obviously,
\[
\partial S - M = \left\{ \nu = [\nu_1 \nu_2]^T \in \mathbb{R}^2 : \|\nu\| = 1, 3\nu_1 \nu_2 + 1.8028|\nu_1 \nu_2| \geq -k_0 \right\}
\]
As $\gamma_1(\hat{x}, u) = 3 \times 1.8028 + k_0$ and
\[
\gamma_2(\hat{x}) = \min \left\{ (\nu_1 + \nu_2)^2, \nu \in \partial S - M \right\} = 1 - \frac{2k_0}{3-1.8028}
\]
choosing $k_0 = 0.5$, we have $\gamma_1(\hat{x}, u) = 35.8699$. Let $\theta(\hat{x}, u) = 36 > 35.8699$ and we have $\Phi(\hat{x}, u) = -[12 \ 36]^T$.

Now the nonlinear observer becomes
\[
\begin{bmatrix}
\dot{\hat{x}}_1 \\
\dot{\hat{x}}_2
\end{bmatrix} = \begin{bmatrix}
x_2 \\
-\sin \hat{x}_1 + u_1 \cos \hat{x}_1 + \frac{12}{36}(y - \hat{x}_1 - \hat{x}_2)
\end{bmatrix}
\]

Choose the reference model (6) where
\[
A_d = \begin{bmatrix} 0 & 1 \\ -25 & -10 \end{bmatrix}, \quad B_d = \begin{bmatrix} 0 \\ 25 \end{bmatrix}
\]

Set $H_1 = I_2$, $H_2 = 10^{-6} I_2$, $\epsilon = 10^6$, $\omega = 1$, $\Gamma_1 = 2I_2$, and $\Gamma_2 = 2I_2$. Set $x_1(0) = 0.3$ and $x_2(0) = 0.5$. Using the proposed approach, we have the simulation result of the pendulum system (44)-(46) shown in Figure 2 where the reference input is given by
\[
\begin{align*}
\alpha &= 6(t_{f_1} - t)^5 - 15(t_{f_1} - t)^4 + 10(t_{f_1} - t)^3, \quad 0 \leq t < t_1 \\
r_f &= -r_f \begin{bmatrix} -t_{f_2} & t_{f_2} \end{bmatrix}^{5} - 15(t_{f_2} - t)^4 + 10(t_{f_2} - t)^3 + r_f, \\
& \text{if} \quad t_2 \leq t < t_f
\end{align*}
\]
with $t_1 = 10s$, $t_2 = 20s$, $t_f = 30s$ and $r_f = 0.5$, and the control $u_2$ is stuck at $-0.5$ from $t = 12s$ onward.

From Figure 2, we note that the states $x_1$ and $x_2$ match the reference $r$ and $\hat{r}$ well even when $u_2$ is stuck at $-0.5$. It is also noted that the estimation errors $e_1(= x_1 - \hat{x}_1)$ and $e_2(= x_2 - \hat{x}_2)$ exponentially converge to zero. Moreover, Figure 2 shows that the controls $u_1$ and $u_2$ respectively remain within $[-1, 1]$ and $[-0.5, 0.5]$ even though the control $u_1$ reaches saturation during the first 1.2 second which activates the Lagrange multiplier $\lambda_1$. In addition, some responses, such as the allocation errors $\tau_1$ and $\tau_2$, the Lyapunov-like function $V_m$ and the output $y$ are also shown in Figure 2.

5. Concluding Remarks

Sufficient Lyapunov-like conditions have been proposed for the control allocation design via output feedback. The proposed approach is applicable to a wide class of nonlinear systems. As the initial estimation error $e(0)$ need be near zero and the predefined dynamics of the closed-loop is
Fig. 2. Simulation results of the nonlinear observer-based pendulum control described by a linear stable reference model, the proposed approach will present a local nature.

REFERENCES


