Abstract—This paper considers the asymptotic stability problem for switched Hopfield neural networks with time-varying delay under hysteretic switching rule. The parameter uncertainties are considered and assumed to be norm bounded. Single Lyapunov function method is used to analyze the stability property and design the hysteretic switching rule, which is designed according to current state and the previous value of switched signal. Sufficient conditions are given in terms of linear matrix inequalities (LMIs) to guarantee the stability of the system. An example illustrates the effectiveness of the proposed theory.

I. INTRODUCTION

Recently, the Hopfield neural networks have been studied widely and applied to signal and image processing, quadratic optimization, and fixed point computation [1,2]. When neural networks are used to solve optimization problems, it is essential to determine the existence of a unique equilibrium point and its stability properties, which are fundamental for neural networks designs. Various sufficient conditions for the global asymptotic stability of the equilibrium point of neural networks have been proposed.

As is well known, time delays are likely to be present due to the finite switching speed of amplifiers that are the core elements for implementing artificial neurons in models of neural networks. Time delays that occurred in the interaction between neurons will affect the stability of a network by creating instability, oscillation and chaos phenomena [3]. Therefore, the study of neural networks with time delays is more important in practice, and a large number of results have been available in [4-7]. On the other hand, uncertainty is also the important cause of instability of neural networks. Uncertainty can be commonly encountered due to the modeling inaccuracies and/or changes in the environment. Similarly, impractical implementation of neural networks, some vital data such as the neuron firing rates and the weight coefficients are usually acquired and processed by means of the statistical estimates [8]. Therefore, the robust stability analysis of neural networks has been widely studied and a great number of results on this topic have been reported [9, 10]. Furthermore, the connections between different nodes are not always stable, and link failure and new link creation may happen at times. Hence, the abrupt changes in the network structure may occur, and switches between some different topologies are inevitable for many real world dynamical networks [11].

Switched systems, as an important special class of hybrid control systems, have received great attention of researchers in recent years [12-18]. Switched systems have numerous applications in control of mechanical systems, the automotive industry, aircraft and air traffic control, switching power converters, and many other fields. Switched systems consist of a family of continuous-time or discrete-time subsystems and a rule specifying which subsystem will be activated at each instant of time. Usually the switching rule is a piecewise constant function dependent on the state or time. Recently, many results on the stability of switched system with time-varying delay and parametric uncertainties have been reported [19-23]. Based on the existing results, the switched Hopfield neural networks are able to realize.

Stability of switched neural networks has been considered in [8,11,24,25,26]. In [24], Huang et al. derived some conditions for global exponential stability of switched Hopfield neural networks with parameter uncertainties and time-varying delay. This was the first time to introduce and study the switched Hopfield neural networks. Yuan et al. [8] combined Cohen–Grossberg neural networks with an arbitrary switching rule, and analyzed the robust stability of switched Cohen–Grossberg neural networks with mixed time-varying delay. Li et al. [25] and Lou et al. [26] studied the switched recurrent neural networks with time-varying delay. Some new sufficient conditions were obtained to ensure global asymptotical stability and global robust stability. In 2009, Lu et al. [11] formulated the switched stochastic dynamical network model. By applying the average dwell time approach and multiple Lyapunov functions, the conditions were derived to guarantee the exponential stabilization of switched stochastic dynamical networks. In these papers mentioned above, each of the sub-neural networks is assumed to be stable.

In this paper, we study the stability of a class of switched Hopfield neural networks with time-varying delay and parameter uncertainties. On the premise of Hurwitz convex combination, single Lyapunov function method is used to design the hysteretic switching rule according to current state and the previous value of switched signal to guarantee the switched system stable. Sufficient conditions are given in terms of linear matrix inequalities (LMIs). An example illustrates the effectiveness of the proposed theory.

Notations: The notations used throughout this paper are standard. The superscript T and -1 denote the transpose and the inverse of any square matrix; \( \mathbb{R} \) denotes the set of real numbers.
numbers; \( \mathbb{R}^n \) denotes the \( n \)-dimensional Euclidean space; \( \mathbb{R}^{n \times m} \) denotes the set of all \( n \times m \) real matrices; \( \text{diag}(\star) \) denotes a block-diagonal matrix; \( A > 0 \) \( (A < 0, A \leq 0) \) denotes a positive (negative, negative semi-definite) matrix; \( I \) denotes the identity matrix. The symbol ‘ \( \star \) ’ within the matrix represents the symmetric term of the matrix.

II. PROBLEM FORMULATION

Consider the following Hopfield neural networks with time-varying delay and parameter uncertainties:

\[
\dot{z}(t) = -(A + \Delta A(t))z(t) + (B + \Delta B(t))y(z(t - \tau(t))) + u,
\]

where \( z(t) = [z_1(t), z_2(t), \ldots, z_n(t)]^T \in \mathbb{R}^n \) is the neural state vector, \( u = [u_1, u_2, \ldots, u_n]^T \) is a constant external input vector, \( A = \text{diag}(a_1, a_2, \ldots, a_n) > 0 \) \( (\in \mathbb{R}^{n \times n}) \) is a positive diagonal matrix, \( B \in \mathbb{R}^{n \times m} \) is the interconnection matrix, \( \Delta A(t) \) and \( \Delta B(t) \) are parameter uncertainties in the matrix \( A \) and \( B \), 

\[
y(z(t - \tau(t))) = [y_1(z_1(t - \tau(t))), y_2(z_2(t - \tau(t))), \ldots, y_n(z_n(t - \tau(t)))]^T
\]

is the neuron activation function vector.

We suppose that the switched Hopfield neural networks have only one equilibrium point, denoted by \( \dot{z}^* = [z_1^*, z_2^*, \ldots, z_n^*]^T \). For the purpose of simplicity, we can shift the equilibrium \( z^* \) to the origin by the transformation \( x(t) = z(t) - z^* \), and the system (1) can be rewritten as the following form:

\[
\dot{x}(t) = - (A + \Delta A(t))x(t) + (B + \Delta B(t))g(x(t - \tau(t))),
\]

where \( x(t) = [x_1(t), x_2(t), \ldots, x_n(t)]^T \in \mathbb{R}^n \) is the state vector of the transformed system, \( g(x(\bullet)) = [g_1(x_1(\bullet)), \ldots, g_n(x_n(\bullet))]^T \) and \( g_i(x_i(\bullet)) = y_i(x_i(\bullet) + z_i^*) - y_i(x_i^*) \).

The switched Hopfield neural networks consist of a set of Hopfield neural networks. Each of the Hopfield neural networks is regarded as an individual subsystem. The operation mode of the switched neural networks is determined by a switching rule. The model of switched Hopfield neural networks is described as follows:

\[
\dot{x}(t) = -(A_{i\sigma} + \Delta A_{i\sigma}(t))x(t) + (B_{i\sigma} + \Delta B_{i\sigma}(t))g_i(x(t - \tau(t))),
\]

where \( \sigma \in \Gamma = \{1, 2, \ldots, m\} \) is piecewise constant switching signal, \( \sigma = i \) means that the \( i \)-th subsystem is activated, \( \{A_{i\sigma}, \Delta A_{i\sigma}(t), B_{i\sigma}, \Delta B_{i\sigma}(t)\}, (i \in \Gamma) \) is a family of matrices. The uncertainties \( \Delta A(t), \Delta B(t) \) can be represented as

\[
[\Delta A(t) \quad \Delta B(t)] = M[F_{ii}(t)N_{ii} \quad F_{ii}(t)N_{ii}],
\]

where \( M, N_{ii}, N_{ii} \) are constant matrices with appropriate dimensions, \( F_{ii}(t) \) and \( F_{ii}(t) \) are unknown, real, and possibly time-varying matrices satisfying \( F_{ii}^\tau(t)F_{ii}(t) \leq I, \ F_{ii}^\tau(t)F_{ii}(t) \leq I \).

To give our main results in the next section, we need to present the following assumptions.

Assumption 1. There exists a positive diagonal matrix \( K = \text{diag}(k_1, k_2, \ldots, k_n) > 0 \), such that the activation function \( g_j(x) \) satisfy

\[
|g_j(x)| \leq k_j |x|
\]

for \( j = 1, 2, \ldots, n \).

Assumption 2. \( \tau(t) \) denotes the time-varying delay satisfying the following two cases,

(a) \( \tau(t) \) is differentiable and bounded with a constant delay-derivative bound:

\[
0 \leq \tau(t) \leq \tau, \quad \dot{\tau}(t) \leq \mu;
\]

(b) \( \tau(t) \) is continuous and bounded:

\[
0 \leq \tau(t) \leq \tau.
\]

Assumption 3. There exists a Hurwitz linear convex combination of \( S \), i.e.,

\[
\gamma_{\alpha_1, \alpha_2, \ldots, \alpha_n}(S_1, S_2, \ldots, S_m) = \sum_{j=1}^{m} \alpha_j S_j : \alpha_1, \alpha_2, \ldots, \alpha_n \in [0,1], \sum_{j=1}^{m} \alpha_j = 1 \}
\]

The following lemmas will be used in the proof of our main results.

Lemma 1 [27]. (Schur Complement) The LMI

\[
\begin{bmatrix}
G & K
\end{bmatrix} > 0,
\]

where \( G \) and \( L \) are symmetric matrix, is equivalent to

\[
L > 0, \quad G - KL^1K^T > 0.
\]

Lemma 2 [21, 28]. For any real matrices \( Q > 0, E, M, N \) and \( F \) with appropriate dimensions, and any scalar \( \varepsilon > 0 \), if \( F^T F \leq I \), then

(a) \( M F N + N^T F^T M^T \leq \varepsilon M M^T + \varepsilon^{-1} N^T N \)

(b) if \( \varepsilon I - M^T Q M > 0 \), then

\[
(E + M F N)^T (E + M F N) \leq \varepsilon (E + M F N)^T (E + M F N)^T,
\]

\( \times M^T (E + \varepsilon N^T N) \)

Lemma 3 [25]. For any positive definite constant matrix \( W \in \mathbb{R}^{n \times n} \), scalar \( d > 0 \), \( t \in [0, +\infty) \), vector function \( v(t) : [t - d, t] \rightarrow \mathbb{R}^n \),

\[
\int_{t-d}^{t} v(s) ds \leq d \int_{t-d}^{t} v^T(s) W v(s) ds.
\]

III. MAIN RESULTS

In this section, the sufficient conditions are derived by single Lyapunov Function method, and stability criteria are expressed by LMIs. The hysteretic switching rule will be designed to guarantee the stability of the switched system (3), in which the subsystems are not necessarily stable.

Theorem 1. Under the Assumption 2 (a), if there exist \( \bar{A} \in \gamma_{\alpha_1, \alpha_2, \ldots, \alpha_n}(A_1, A_2, \ldots, A_m), \bar{N}_i \in \gamma_{\alpha_1, \alpha_2, \ldots, \alpha_n}(N_1, N_2, \ldots, N_m) \),
symmetric matrices $P \succ 0$, $Q \succ 0$, $R \succ 0$, and any matrices $Y$, $Z$, a diagonal matrix $K$, positive scalars $\rho$, $\psi$, $\xi$, $\varepsilon_1$, $\varepsilon_2$, satisfying $\varepsilon_1I - M^T RM \succ 0$, and $\varepsilon_1I - M^T (R + \xi^{-1} I)M \succ 0$ such that the following LMIs are feasible
\[
\begin{bmatrix}
-\overline{A}P - PA & PM & N_R^T & \overline{A}RM \\
* & -\rho^{-1}I & 0 & 0 \\
* & * & -(\rho^{-1} + \varepsilon_1)^{-1}I & 0 \\
* & * & * & -(\varepsilon_1I - M^T RM)
\end{bmatrix} < 0,
\]
(7)

where
\[
\begin{align*}
\hat{H}_i &= \begin{bmatrix}
\varphi_i & -Y + Z^T - Y \\
* & \varphi_i & -Z \\
* & * & -\varepsilon^2 R
\end{bmatrix}, \\
\hat{R}_{ii} &= \begin{bmatrix}
(RA_i - P)^T & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix}, \\
\hat{G}_i &= \begin{bmatrix}
KB_i^T(R + \xi^{-1} I)B_iK - Z - Z^T
\end{bmatrix}, \\
\hat{L}_i &= \begin{bmatrix}
-\xi^{-1}I \\
0 \\
0
\end{bmatrix}, \\
\hat{M} &= \begin{bmatrix}
RM \\
0
\end{bmatrix}, \\
L_i &= -\psi I, \quad i = 1, \ldots, m.
\end{align*}
\]

Then, there exist switching rules $\sigma$ that make system (3) asymptotically stable.

**Proof.** Since $\overline{A} \in \gamma_{\alpha_1,\alpha_2,\ldots,\alpha_m}(A_1, A_2, \ldots, A_m)$, there exist $\alpha_i \in [0,1]$, $i = 1, 2, \ldots, m$, satisfying $\sum_{i=1}^m \alpha_i = 1$ and $\overline{A} = \sum_{i=1}^m \alpha_i A_i$.

and $\overline{N}_R = \sum_{i=1}^m \alpha_i N_{Ri}$. From (7), we have

\[
\begin{bmatrix}
-\overline{A}P - PA & PM & \overline{N}_R^T & \overline{A}RM \\
* & -\rho^{-1}I & 0 & 0 \\
* & * & -(\rho^{-1} + \varepsilon_1)^{-1}I & 0 \\
* & * & * & -(\varepsilon_1I - M^T RM)
\end{bmatrix} < 0
\]

Using Lemma 1 we can easily calculate that
\[
\sum_{i=1}^m \alpha_i x^T \left[-A_i P - P A_i + \rho P M M^T P + \left(\rho^{-1} + \varepsilon_i\right) N_{Ri}^T N_{Ri}ight]
\]
\[+ A_i R M \left(\varepsilon_i I - M^T R M\right)^{-1} M^T R A_i \right] x < 0.
\]
(10)

Let
\[
\Omega_1 = \left\{ x \mid x^T \left[-A_i P - P A_i + \rho P M M^T P + \left(\rho^{-1} + \varepsilon_i\right) N_{Ri}^T N_{Ri}\right]
\]
\[+ A_i R M \left(\varepsilon_i I - M^T R M\right)^{-1} M^T R A_i \right] x < 0 \}
\]
(11)

From (10), we obtain
\[
\bigcup_{i=1}^m \Omega_i = \mathbb{R}^n \setminus \{0\}.
\]

The hysteresis switching rule $\sigma$ for system (3) is designed as follows
\[
\sigma(0) = \min \{ \Omega_i \mid x(0) \in \Omega_i \}
\]
for $t > 0$
\[
\sigma(t) = \begin{cases}
\bar{i}, & \text{if } x(t) \in \Omega_i, \sigma(t) = \bar{i} \\
\min \{ \Omega_k \mid x(t) \in \Omega_k \}, & \text{if } x(t) \notin \Omega_i, \sigma(t) = \bar{i}
\end{cases}
\]
(12)

We choose the following Lyapunov function
\[
V(x, t) = V_1 + V_2 + V_3
\]
(13)

where $V_1 = x^T(t) P x(t)$, $V_2 = \int_{\tau - t}^{\tau} g^T(s) Q g(s) ds$, and $V_3 = \tau^{-1} \int_{\tau - t}^{\tau} \dot{x}(s) R \dot{x}(s) ds d\theta$.

Calculating the time derivative of $V$ along the trajectory of (3), we can get
\[
\dot{V}_1 = x^T(t) (\overline{A}P - PA_i)x(t)
\]
\[+ x^T(t) \left(-A_i P - P A_i\right)x(t)
\]
\[+ x^T(t) \left(-\Delta A_i^T(t) P - P \Delta A_i(t)\right)x(t)
\]
\[+ g^T(x(t - \tau(t))) B_i \Delta B_i(t) P x(t)
\]
\[+ g^T(x(t - \tau(t))) B_i \Delta B_i(t) g(x(t - \tau(t)))
\]
\[\dot{V}_2 \leq g^T(x(t)) Q g(x(t)) - (1 - \mu) g^T(x(t - \tau(t))) Q g(x(t - \tau(t)))
\]
\[\dot{V}_3 \leq \dot{x}^T(t) R \dot{x}(t) - \tau^{-1} \int_{\tau - t}^{\tau} \dot{x}^T(s) R \dot{x}(s) ds
\]

By using Lemma 2 (a), there exists $\rho > 0$ such that the following inequality holds
\[
x^T(t) \left(-\Delta A_i^T(t) P - P \Delta A_i(t)\right)x(t)
\]
\[\leq x^T(t) \left(\rho P M M^T P + \rho^{-1} N_{Ri}^T N_{Ri}\right) x(t).
\]
(14)

From Lemma 2 (a), (b) we obtain
\[
\dot{x}^T(t) R \dot{x}(t) + g^T(x(t - \tau(t))) (B_i + \Delta B_i(t))^T P x(t)
\]
\[+ x^T(t) P (B_i + \Delta B_i(t)) g(x(t - \tau(t)))
\]
\[\leq x^T(t) A_i \left[R + R M \left(\varepsilon_i I - M^T R M\right)^{-1} M^T R\right] A_i x(t)
\]
\[+ \varepsilon_2 x^T(t) N_{Ri}^T N_{Ri} x(t) + g^T(x(t - \tau(t))) B_i^T \left[(R + \xi^{-1} I) + (R + \xi^{-1} I)\right]
\]

\[ x^T \{ \varepsilon, I - M^T (R + \varepsilon^{-1} I) M \}^{-1} M^T (R + \varepsilon^{-1} I) B g(x(t) - x(t - \tau(t))) + \varepsilon g^T (x(t) - x(t - \tau(t))) N_{\text{m}} N_{\text{b}} g(x(t) - x(t - \tau(t))) + \varepsilon x^T (t) (RA_1 - P + R\Delta A_1 (t))^T (RA_1 - P + R\Delta A_1 (t)) x(t) \].

From Lemma 3 we can easily obtain
\[ -\tau^{-2} \int_{t-\tau(t)}^{t} \dot{x}^T (s) R \dot{x}(s) ds \leq -\tau^{-2} \int_{t-\tau(t)}^{t} \dot{x}(s) ds \int_{t-\tau(t)}^{t} \dot{x}(s) ds. \]

From the Leibniz-Newton formula, it is clear that
\[ 2 \left[ x^T (t) Y + x^T (t - \tau(t)) Y \right] \times \left[ R(t) - R(t - \tau(t)) \right] \dot{x}(s) ds = 0. \]

By combining inequalities (14)-(17), we obtain
\[ \dot{V}(x, t) = x^T (t) \left(-A P - PA + \rho P M M^T P + \left(\rho^2 + \varepsilon_1 \right) N_{\text{m}}^T N_{\text{di}}^T + A R M \right) x(t) + \varepsilon_1 \left[ -Y + z^T \right] \left[ -Y + z^T \right] x(t) + \varepsilon_1 \left[ -Y + z^T \right] \left[ -Y + z^T \right] x(t - \tau(t)) \geq 0. \]

By Lemma 1, the matrix
\[ \begin{bmatrix} \Xi & -Y + z^T & -Y \\ \ast & \Phi & -Z \\ \ast & \ast & -\tau^2 R \end{bmatrix} \]

is equivalent to
\[ U_i = \begin{bmatrix} H_i & \tilde{R}_i & \tilde{G}_i \\ \ast & L_1 & 0 \\ \ast & \ast & L_2 \end{bmatrix}, \]
\[ H_i = \begin{bmatrix} A_{\text{RA}} + KQK + Y + Y^T & -Y + z^T & -Y \\ \ast & \varphi_{2_1} & -Z \\ \ast & \ast & -\tau^2 R \end{bmatrix}, \]
\[ \varphi_{2_1} = -(1 - \mu) KQK + \varepsilon_2 K N_{\text{m}}^T N_{\text{b}} K + KB_1^T \left( R + \varepsilon^{-1} I \right) B K - Z - Z^T. \]

From (8) and (11) we can obtain \( \dot{V}(x, t) < 0 \).

According to the single Lyapunov function method, we can apply the Lemma 3 to obtain
can conclude that the switched neural networks (3) is globally asymptotically stable under the hysteretic switching rule. The proof is completed.

**Remark 1.** The hysteretic switching rule, which is determined by the current state and the previous value of switched signal, is different from the general switching rules. If the current state belongs to the state sub-region which is directed by the previous value of switched signal, the switching signal keeps invariant. Contrarily, if the current state dose not belongs to the state sub-region, the switching signal changes into the minimum number of the subsystem according to the current state sub-region.

Under Assumption 2 (b), we can easily obtain the following corollary.

**Corollary 1.** Under Assumption 2 (b), if there exist $\bar{A} \in \gamma_{\alpha_1,\alpha_2,\cdots,\alpha_m} (A_1, A_2, \cdots, A_m)$ , $\bar{N}_i \in \gamma_{\alpha_1,\alpha_2,\cdots,\alpha_m} (N_1, N_2, \cdots, N_m)$, symmetric matrices $P > 0$, $R > 0$, and any matrices $Y$, $Z$, a diagonal matrix $K$, positive scalars $\rho, \psi, \xi, \varepsilon_1, \varepsilon_2$, satisfying

$$
\varepsilon_1 I - M^T R M > 0,
$$

and

$$
\varepsilon_2 I - M^T (R + \xi^T I) M > 0
$$

such that the following LMIs (7) and (8) with $0$ symmetric matrices $0, 0$ are feasible. Then, there exists switching rule (12) that makes system (3) asymptotically stable.

**Proof:** Choose the following Lyapunov function $V(x(t)) = V_1 + V_2$, where $V_1$ and $V_2$ are described in (13). The proof process is similar to the Theorem 1’s.

**Remark 2.** Some similar models of other switched neural networks can be designed in the similar way, such as recurrent neural networks, cellular neural networks, bidirectional associative memory (BAM) neural networks, and so on.

### IV. ILLUSTRATIVE FORMULATION

Consider the switched Hopfield neural networks system

$$
\dot{x}(i) = -(A_i + \Delta A_i(i))x(i) + (B_i + \Delta B_i(i))g(x(t-\tau(t))), \quad (i=1,2)
$$

(22)

where $g(x) = \frac{1 - e^{-x}}{1 + e^{-x}}$, and the parameters are

$$
A_i = \begin{bmatrix} 0.6 & 0 \\ 0 & 0.5 \end{bmatrix}, \quad N_1 = \begin{bmatrix} 0.7 & -0.9 \\ 0.6 & -0.7 \end{bmatrix}, \quad B_i = \begin{bmatrix} -0.7 & -3.6 \\ -5 & -0.2 \end{bmatrix},
$$

$$
A_{11} = \begin{bmatrix} -0.2 & 2 \\ 1.2 & -0.1 \end{bmatrix}, \quad A_{12} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.6 \end{bmatrix}, \quad N_{12} = \begin{bmatrix} -0.6 & 1 \\ -0.3 & 0.6 \end{bmatrix},
$$

$$
B_2 = \begin{bmatrix} 2 & -4 \\ 5 & 2 \end{bmatrix}, \quad N_{21} = \begin{bmatrix} 0 & -2 \\ -0.1 & -0.3 \end{bmatrix}, \quad M = \begin{bmatrix} 0.1 & 0.5 \\ 0.1 & 0.3 \end{bmatrix},
$$

from Assumptions 1, 2 (a), we can obtain $\tau = 1$, $\mu = 0.5$, $K = \text{diag}(1.2, 1.2)$, and $F(t)$ is any matrix satisfying (5).

Choosing the initial values as $x_0 = \begin{bmatrix} 1 & -1 \end{bmatrix}$, the simulation results for the state responses of two systems are shown in Fig. 1 and Fig. 2, respectively. We can see the subsystems are unstable.

Let $\rho = 2, \varepsilon_1 = 1, \varepsilon_2 = 0.1, \psi = 0.2, \xi = 15$. From Theorem 1, we choose the combination coefficients $\alpha_1 = \alpha_2 = 0.5$. By solving LMIs, we get

$$
P = \begin{bmatrix} 0.6211 & -0.0259 \\ -0.0259 & 0.6789 \end{bmatrix}, \quad Q = \begin{bmatrix} 0.6650 & 0.2646 \\ 0.2646 & 0.3173 \end{bmatrix},
$$

$$
R = \begin{bmatrix} 0.0876 & 0.0093 \\ 0.0093 & 0.0954 \end{bmatrix}.
$$

The hysteretic switching rule is

$$
\sigma(t) = \begin{cases} 1, & \text{if } (x(0) \in \Omega_1) \text{ or } (x(t) \in \Omega_2) \text{ and } \sigma(t^-) = 1 \\
2, & \text{if } (x(0) \in \Omega_2) \text{ or } (x(t) \in \Omega_1) \text{ and } \sigma(t^-) = 2 \\
\text{or } (x(t) \notin \Omega_1 \text{ and } \sigma(t^-) = 1) \\
\text{or } (x(t) \notin \Omega_2 \text{ and } \sigma(t^-) = 2) \\
\end{cases}
$$

where

$$
\Omega_1 = \{ x | 0.721x_1^2 - 2.8456x_1x_2 + 1.3527x_2^2 < 0 \},
$$

$$
\Omega_2 = \{ x | 0.7413x_1^2 - 2.057x_1x_2 + 1.3071x_2^2 < 0 \}.
$$

The simulation results are shown by Fig. 3 and Fig. 4, which shows the switched Hopfield neural networks system (22) asymptotically converges to the unique equilibrium.
In this paper, we have studied the stability of the uncertain switched delay Hopfield neural networks. The switched Hopfield neural networks composed of a set of Hopfield neural networks. Each of the Hopfield neural networks is regarded as an individual subsystem, and does not need to be stable. By single Lyapunov method, the hysteretic switching rule has been designed to guarantee the stability of the switched neural networks. The sufficient condition for the switched neural networks asymptotically stable has been derived. The results are formulated in terms of LMIs, and it is easy to be solved. The simulation example shows the effectiveness of the present result.

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