Hybrid System’s Model and Algorithm for Highway Traffic Monitoring

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Abstract—We propose a method for the early detection and localization of highway traffic congestion onset and its propagation using a stochastic linear hybrid system model (SLHS) and a state-dependent-transition hybrid estimation (SDTHE) algorithm. The SLHS model is used to model the congested and non-congested scenarios of the highway. Using the SDTHE algorithm, we estimate the states (continuous and discrete states) of the highway that will provide us with the traffic congestion information. The performance of the algorithm is analyzed using the correct detection and identification (CDID) indices, false alarm rate (FA) indices, time-to-detection (TTD) delays as well as the run time. We use a set of constructed data that represent the various congestion onset and propagation scenarios. The validation of the algorithm is done using real traffic data obtained from highway I-405 S in California using the Freeway Performance Measurement System (PEMS).

I. INTRODUCTION

In dynamic systems, there are processes that incur malfunctions due to a fault in the system or the system components. The effects of the fault can lead to a failure of the system. It is important to have a method that will be able to detect the fault as early as possible, to identify or diagnose the fault and to implement a method to counteract the fault, known as reconfiguration, in order to avoid the system’s failure. In our world today, with the advancement of technology, so is the complexity of our systems, hence the complexity of the mathematical models that represent the system. An example of a complex system is the highway traffic system.

There has been a rigorous analysis of traffic flow using mathematical models based on macroscopic or microscopic models [1]. The macroscopic models depict the traffic flow as if it were a fluid flow while the microscopic models follow the movement of an individual vehicle in time and space. The microscopic model may not be a good model for online applications compared to the macroscopic model, because it has a complex structure and is computationally expensive [1], [2]. Furthermore, it may not be effective in providing representative and efficient information for analysis. The macroscopic model used for traffic flow representation is based on the hydrodynamic or kinematic wave theory [1], [2], [3], [4], [5]. Even though the traffic itself is not like a fluid flow, it can be modeled to resemble one, due to sufficient similarities of the traffic dynamics to a fluid flow [2].

There has been a lot of work done in the macroscopic modeling of traffic flow. Examples include the simple continuum model (SCM) in 1955 [6], [2], [7], [5] and the high order continuum model in 1971 [8], [2], [3], [9]. The cell transmission model (CTM) developed by Daganzo in 1994 [3], [4], [5] has a modified version known as the switching mode model (SMM) [10]. The SMM is a hybrid system that represents the status of a highway segment, by switching among a discrete set of modes that represent the traffic conditions of the highway segment. This model used to estimate unmeasured densities on the highway but may not provide us with propagation information that might be needed for reconfiguration and further congestion alleviation.

In this paper motivated by the SMM [10], we propose a stochastic linear hybrid system (SLHS) model for highway traffic monitoring which can describe a congestion onset and its localization. This model also tackles an important aspect of traffic monitoring by describing the progression of the congestion in the highway segment, whether the traffic density is worsening or recuperating by modeling its dynamics. This is very important in traffic control because in order to provide the correct reconfiguration, one needs the correct information of the state of traffic and its progression. This will be helpful in daily traffic commuting [12] and in the event of disaster evacuation plans [3], where traffic density data and its progression provide accurate information required for developing tools that will help alleviate the duration of a congestion and the chances of accidents that occur due to congestion. We propose the State-Dependent Transition Hybrid Estimation (SDTHE) algorithm for highway traffic monitoring.

The rest of this paper is organized as follows: Section II elaborates on the highway traffic modeling problem formulation. Section III describes the SLHS model. The SDTHE algorithm is presented in Section IV. Simulations and results are presented in Section V and Section VI finalizes the paper with conclusions. The Appendix is provided in Section VII.

II. HIGHWAY TRAFFIC MODELING PROBLEM FORMULATION

Throughout this paper, we consider a highway traffic scenario shown in Figure 1, in which a segment of the highway is divided into four cells. The traffic flow \( q \) (veh/min), or density \( \rho \) (veh/mile), is measured using the loop detectors that are located at the upstream \( q_U \) and downstream \( \rho_D \) locations as well as the on- and off-ramp locations \( r_1 \) and \( r_2 \) respectively. Using the triangular fundamental diagram, we develop the necessary parameters required for the highway traffic model, as we will discuss below. In our case, each
of the cells have the same length for simplicity but they can have variable lengths. A congestion is defined when the number of vehicles within a cell is greater than what is required for optimum travel at the required speed limit and the required spacing between vehicles. The critical density $\rho_c$, is the density that characterizes a congestion onset. If a cell is congested, the cell density is greater than the critical density and we denote it with a C. If a cell is not congested, then the cell density is less than or equal to the critical density (i.e. it is in free-flow) and we denote it with an F, as seen in Figure 1(Top). Each cell has the same critical density since they have the same lengths. Given the fact that each cell has the same critical density, the speed of the vehicles $v$ and the congestion wave speed $w$ (if applicable) will vary depending on the location and degree of the congestion in the highway segment (Note: $v \geq w$), which in turn will affect $q_{M}$. $q_{M}$ is the maximum flow allowed within a cell and $q_{J}$ is the jam density beyond which there is no flow possible.

So we have expanded our model based on the CTM [10]. The CTM is based on the conservation of flow, in which the density of each cell $i$ evolves as:

$$\rho_{i}(k+1) = \rho_{i}(k) + \frac{\Delta t}{l}(q_{i}(k) - q_{i+1}(k))$$  \hspace{1cm} (1)

where $\rho_{i}$ is the density of the cell $i$, $q$ is the flow, $l$ is the length of the cell and $\Delta t$ is the sampling time.

Let $x(k) = [\rho_{1} \rho_{2} \rho_{3} \rho_{4}]^T$, $u(k) = [q_{1}(k) r_{1}(k) r_{2}(k) \rho_{4}]^T$ and $z(k) = [q_{1}(k) q_{4}(k)]^T$. The measurements $z(k)$ are obtained from the upstream and downstream locations i.e. cells 1 and 4. Then we have:

$$x(k) = A_{m}(k)x(k-1) + B_{m}(k)u(k) + B_{J,m}(k)\rho_{J} + B_{q,m}(k)q_{M}$$ \hspace{1cm} (2)

$$z(k) = C_{m}(k)x(k)$$ \hspace{1cm} (3)

where $A_{m}$, $B_{m}$, $B_{J,m}$, $B_{q,m}$ and $C_{m}$ are the system matrices.

For each cell the system parameters are dependent on the traffic conditions in the highway segment: if the traffic is in free flow, the system parameters are computed from the F section in Figure 1 (Top); if the traffic is congested, the parameters are derived from the C section in Figure 1 (Top). Thus, for each cell, equations (2) and (3) will have two different sets of parameters for the two traffic conditions. Therefore, a hybrid system with the interacting continuous and discrete dynamics, can accurately model highway traffic conditions. In this case, a traffic condition change such as a free-flow to congestion, takes place when the traffic density in a cell reaches the critical traffic density.

In addition, the highway traffic has various uncertainties such as the uncertainties in the loop detector measurements and the uneven behavior of individual drivers with respect to speeds and uneven spacing. Thus we model the highway traffic as a Stochastic Linear Hybrid System (SLHS) model. This is a hybrid system with uncertainties whose mode transitions are dependent on the continuous state. We develop a stochastic guard condition for each mode which describes when the hybrid system makes a transition from one mode to the next. Hence our model is agile and accurate in detecting and localizing the congestion onset by providing the identity of the discrete mode representing the traffic condition in operation.

So, for each congestion onset mode (where the congestion onset takes place in a cell) we model a viable set of modes for congestion propagation. Instead of having $g^h$ discrete modes (where $g=2$, (for the conditions F and C), and $h=4$, for the number of cells in our example), we only need 3 modes at each step. This makes the algorithm viable for online applications, since we work with only three discrete modes at each time step instead of $2^4$ discrete modes (see Figure 2 for congestion onset of mode FFFC). It is also important to note that as the number of discrete modes increases in a hybrid estimation algorithm, the accuracy in estimating the correct mode deteriorates.

Since we now have a SLHS model whose continuous states represent traffic densities in each cell and the discrete states (or modes) describe the traffic conditions (e.g. congestion or free-flow) of each cell, we can now formulate highway traffic monitoring as a hybrid state estimation problem. For hybrid estimation, we use the State-Dependent-Transition Hybrid Estimation (SDTHE) algorithm, that uses the SLHS model and the stochastic guard conditions for mode transitions [13].
III. STOCHASTIC LINEAR HYBRID SYSTEM (SLHS) MODEL

In this section, we discuss the SLHS model in detail. The SLHS model is represented as follows [13]:

\[
x(k) = A_{m(k)}x(k - 1) + B_{m(k)}u(k) + B_{J,m(k)}\rho_J + B_{q,m(k)}\eta_M + \eta_{m(k)}(k - 1)
\]

\[
z(k) = C_m(k)x(k) + \zeta_{m(k)}(k)
\]

where \( k \) is the discrete time; \( x(k) \in \mathbb{R}^d \) is the continuous state, where each state represents the density of each cell; \( u(k) \) is the control input; \( m(k) \in M = \{1, 2, ..., m\} \) is the discrete state or mode at a given time \( k \); \( \eta_m(k) \) and \( \zeta_m(k) \) are the white Gaussian noise with zero mean and covariances \( Q_m \) and \( R_m \), respectively; \( A_m, B_m, B_{q,m}, B_{J,m} \) and \( C_m \) are the system matrices corresponding to mode \( m \). \( z(k) \) are the upstream and downstream flow measurements. The mode evolution is governed by:

\[
\Pi(x(k)) = [\pi_{ij}(x(k))]_{i,j}
\]

where \( \Sigma_{i=1}^m \pi_{ij} = 1 \) (5)

\[
\pi_{ij}(x(k)) \text{ is the conditional mode transition probability from mode } i \text{ to mode } j (i,j = 1,2,...,m), \text{ conditioned on the continuous state } x(k):
\]

\[
\pi_{ij}(x(k)) := p[m(k + 1) = j|m(k) = i, x(k)] \tag{6}
\]

In developing the SLHS model for a highway traffic system, we model the discrete states so that each mode represents a specific congestion location and the continuous states represent the traffic density for each cell in the highway segment. At each time \( k \), a single discrete state (or mode) is activated to represent the actual traffic situation. The mode will shift to another mode based on a set of guard conditions that trigger the activation of the next mode. The continuous state will have an additive randomness due to the errors in the loop detector measurements and the uneven behavior of individual drivers with respect to speed and spacing. Hence, the mode transitions are conditioned on the stochastically evolving continuous states based on the stochastic guard conditions. Now the mode propagation is based on a Markov chain, which means that the mode evolution at time \( k \) to the next mode at time \( k + 1 \) is dependent on only the mode at time \( k \). The continuous-state-dependent transition matrix provides us with the probability values for the discrete mode transitions and the discrete mode with the highest probability is activated.

Let us now take a closer look at the development of the guard conditions and the discrete modes.

A. Guard conditions

In the SLHS, for the system to make a transition from one mode to another, a stochastic guard condition must be met. Each guard condition is expressed as a system of linear inequalities. For instance, for mode \( i \) to transition to mode \( j \), the following condition must be met:

\[
L_{ij}x(k) - \Theta_{ij} \geq 0 \tag{7}
\]

where \( L_{ij} \) is a \( l \times n \) constant matrix and \( \Theta_{ij} \) is a \( l \)-dimensional random vector. Let us consider an example: for mode 1 (FFFF) to transition to mode 2 (FFFC), the guard condition is given as follows:

\[
\rho_4 \geq \rho_c + \sigma \tag{8}
\]

where \( \sigma \) is the standard deviation from the mean. (for simplicity in our scenarios \( \sigma = 4 \text{veh/mile} \))

An illustration for the guard conditions in our application is shown in Figure 3: given the density measurements in cells 1 and 4, if \( L_{ij}x(k) \geq \Theta_{ij} \), where \( \Theta_{ij} \) is a random vector with values of the continuous state to trigger a transition, then the mode will transition from mode \( i \): free-flow, to mode \( j \): congestion.

The conditional mode transition probability is computed as:

\[
\pi_{ij}(x(k)) = p[L_{ij}x(k) - \Theta_{ij} \geq 0|m(k) = i, x(k)]
\]

\[
= p[\Theta_{ij} \leq L_{ij}x(k)|m(k) = i, x(k)] \tag{9}
\]

\( \Theta_{ij} \) is modeled as a \( l \)-dimensional Gaussian pdf given by

\[
p[\Theta_{ij}] = N_l(\Theta_{ij}; \mu_{ij}, \Sigma_{ij}) \tag{10}
\]

where \( \mu_{ij} \) is the constant mean and \( \Sigma_{ij} \) is the covariance.

B. Discrete States (Modes)

For the initial detection and localization of a congestion, the SLHS model has 6 modes: FFFF (mode 1), FFFC (mode 2), FFCF (mode 3), FCFF (mode 4), CFFF (mode 5) and CCCC (mode 6), where each letter represents the state of the cell in the 4-cell highway segment in Figure 1 (Below) (F is free-flow and C is congested). The information of the localization of the congestion onset and propagation is provided from the identity of the discrete mode.

The congestion propagation scenario shows us how the initially congested cell affects the adjacent cells and eventually adjacent segments. Since a congested shock wave always travels upstream, the propagation will also be in the upstream direction as well. This is under the assumption that the location of the congestion is stationary in the sense that it restricts traffic flow in the downstream direction. With the congestion onset detection given by either of the modes FFFC, FFCF, FCFF and CFFF, we show how the congestion will propagate for each of the given modes by designing the intermediate modes that will depict the behavior of the propagated congested cells. For example, we consider the
highway segment to be in free-flow mode (FFFF), then a congestion takes place in cell 2 (FCFF), the propagation will take place with only three possible modes. If the congestion alleviates, it will transition back to mode FFFF. If it stays the same it will remain in mode FCFF. If it worsens, then it will propagate to mode CCFF. This is computationally cost effective for the hybrid estimation algorithm, so that we do not need to switch among \(2^4 = 16\) modes, as was discussed in Section II.

IV. STATE-DEPENDENT TRANSITION HYBRID ESTIMATION ALGORITHM (SDTHE)

In this section, we review the SDTHE algorithm in detail. The structure of the SDTHE algorithm is based on the Interacting Multiple Model but differs from it based on the application of the guard conditions. The details of each step is elaborated below.

1) Mode transition probability.
For \(i,j = 1,2,...,m\), we compute the mode-transition probabilities denoted by \(\pi_{ij}(k-1)\):

\[
\pi_{ij}(k-1) = p[m(k) = j|m(k-1) = i, Z^{k-1}] = \Phi_i(L_{ij}\hat{x}_i(k-1|k-1) - \mu_{ij}, \Sigma_{ij} + L_{ij}P_i(k-1|k-1)L_{ij}^T) \quad (11)
\]

where \(\hat{x}_i(k-1|k-1)\) and \(P_i(k-1|k-1)\) are the continuous state estimates and covariances respectively for mode \(i\) at time \(k-1\). \(\mu_{ij}\) and \(\Sigma_{ij}\) are the mean and covariance and \(\Phi_i(y, \Sigma)\) is the \(d\)-dimensional Gaussian cdf. The mode transition probability is evaluated analytically as a multivariate Gaussian cdf, which is computationally cost effective versus using Gaussian quadratures or Monte Carlo integration.

2) Mixing and initialization.
The initial conditions \(\hat{x}_{j0}(k-1)\) and \(P_{j0}(k-1)\) for the Kalman filter matched to each mode are computed as follows:

\[
\hat{x}_{j0}(k-1) = \Sigma_{i=1}^{m} \pi_{ij}(k-1)\hat{x}_i(k-1|k-1) \quad (12)
\]

\[
P_{j0}(k-1) = \Sigma_{i=1}^{m} P_i(k-1|k-1) + [\hat{x}_i(k-1) - \hat{x}_{j0}(k-1)]^T \pi_{ij}(k-1) \quad (13)
\]

where

\[
\alpha_{ji}(k-1) = \frac{1}{c_j} \pi_{ij}(k-1) \alpha_i(k-1|k-1) \quad (14)
\]

and \(\alpha_{ji}(k-1)\) and \(c_j\) are the prior mode probability and the normalizing constant respectively.

3) Filtering.
The posterior mean \(\hat{x}_j(k)\) and covariance \(P_j(k)\) are then computed for each Kalman filter \(j\) using the initial conditions and the measurements.

\[
\hat{x}_j(k) = A_j\hat{x}_{j0}(k-1) + K_j(r_j(k)) \quad (15)
\]

\[
P_j(k) = [I - K_jC_j]P_j(k|k-1) \quad (16)
\]

where

\[
r_j(k) = z(k) - C_j\hat{x}_j(k|k-1) \quad (17)
\]

\[
P_j(k|k-1) = A_jP_{j0}(k-1)A_j^T + Q_j \quad (18)
\]

\[
K_j = P_j(k|k-1)C_j^T S_j(k)^{-1} \quad (19)
\]

\[
S_j(k) = C_jP_j(k|k-1)C_j^T + R_j \quad (20)
\]

4) Mode Probability Update.
For \(j = 1,2,...,m\), the prior probability of mode \(j\) defined by:

\[
\alpha_j(k|k-1) = p[m(k) = j|Z^{k-1}] = \Sigma_{i=1}^{m} \pi_{ij}(k-1|k-1) \quad (17)
\]

The posterior probability of mode \(j\) is computed as:

\[
\alpha_j(k) = \frac{1}{\delta} p[z(k)|m(k) = j, Z^{k-1}] \times \alpha_j(k|k-1) \quad (18)
\]

with \(\delta\) as the normalizing constant and \(\Lambda_j\) defined as the likelihood function for each Kalman filter \(j\) as:

\[
\Lambda_j(k) = p[z(k)|m(k) = j, Z^{k-1}]p[m(k) = j|Z^{k-1}] = \mathcal{N}_p(r_j(k); 0, S_j(k)) \quad (19)
\]

5) Output.
The output of the SDTHE are the continuous and the discrete state estimates:

\[
\hat{x}(k) = \Sigma_{j=1}^{m} \hat{x}_j(k)\alpha_j(k|k) \quad (20)
\]

\[
\hat{m}(k) = \arg \max_{j} \alpha_j(k|k) \quad (21)
\]

So, at each time \(k\), using the information up to time \(k\), both the continuous and discrete state estimates are computed. The discrete states are distinguished to represent the congestion in each cell. The computation of the discrete state estimate will detect and localize the congestion, by matching the continuous dynamics of the highway segment to the mode that represents the congestion.

V. SIMULATIONS AND RESULTS

In this section, we consider the 4-cell highway segment example in Figure 1 and present the scenarios of congestion onset using constructed data as well as real data. By applying the SDTHE algorithm, our goal is to see whether we can detect the congestion onset and propagation and identify its location accurately. The location of the congestion will be identified as within the area of a specified cell. In our scenarios, the parameters used in the highway system matrices are computed using the triangular fundamental diagram. The system parameters can be found in the Appendix. The simulations are performed for 100 Monte Carlo runs. The
performance of the algorithm is measured with the correct detection and identification (CDID), false alarm (FA), run time and the time-to-detection (TTD) delay. The CDID tells us how accurate the detected mode is. The FA shows us how many times the algorithm detects a mode that is not the correct mode in operation. The time it takes for the algorithm to detect a mode transition from when it happened is presented as the TTD delay.

A. Congestion onset detection, propagation and localization

We present a scenario for a congestion onset and propagation. When a cell is congested, after a certain amount of time, the congestion will propagate upstream into the adjacent cell and eventually into the adjacent highway segment and so on. This is a realistic scenario of what takes place on the highways. A congested highway segment does not have to be fully jammed or have zero velocity for it to be congested. The critical density for a highway means that there will be a decrease in traffic flow and that the highway system will be operating below its par capability. In this scenario, we will take a look at the detection, propagation and localization for a congestion taking place in cell 2. We address the congestion propagation scenario after a congestion onset. Another case may be the one in which the congestion onset is alleviated where the highway segment returns to its initial fully free-flow mode. Since congestion propagates upstream, the scenario starts with an initially fully free-flow mode (FFFF), to the congestion onset in cell 2 (FCFF) and the congestion propagation into cell 1 as mode CCFF. Mode FCFF occurs at 20 minutes and mode CCFF occurs at 30 minutes. We assume that the entire scenario of interest takes place for 40 minutes. In the continuous dynamics of mode FCFF, cells 1, 3 and 4 are in free flow mode with velocities $v_1$, $v_3$ and $v_4$ equal to 40mph, 63mph and 70mph respectively. While the congested wave in cell 2 has a speed of $w_2 = 14.26$ mph between cells 1 and 2 and a speed of $w_3 = 16$ mph between cells 2 and 3. As seen from Figure 4, at the vicinity of 20 min, we see the density of cell 2 increasing. At 30 min, we see the density of cell 1 increasing as well. The congestion in cell 2 propagates upstream into cell 1, while the densities in cells 3 and 4 decreased. We also expect a TTD delay since it takes time for the information to propagate to cell 1, where the measurements are taken. The maximum TTD delay for this scenario is 2.1 minutes, while the run time for this scenario is 1.3777 sec using MATLAB. The density and mode probability plots are shown in Figure 4 as well as their performance indices in Table I.

B. Traffic Data from I-405 S in Los Angeles, CA

For validation of the proposed highway model and the traffic monitoring algorithm, we use real traffic data representing a highway segment in Los Angeles, California on November the 26th at 2400 to November the 27th at 0100, where there is usually a lot of traffic since it represents the day before Thanksgiving. The data was obtained from the Freeway Performance Measurement System (PEMS) in California, which is a collaboration of UC Berkeley, PATH and Caltrans [14]. The data is obtained for a duration of 25 hours. Due to the huge amount of information, we will use a segment of the data for better visual presentation and analysis between 4:10am and 8:20am shown as between 50-100 with 5-minute time-steps in Figure 5. We show the cell densities for the entire 25 hours, but the mode plots are zoomed in to the times described above. The sampling time for the measurements is 5 minutes. Since this is real traffic data, the two key points here are that there is an up-flow of the congested wave from the adjacent highway segment from the downstream location as well as un-metered on-ramp traffic. This explains the sudden transitions from the FFFF mode to the FFCC modes seen in Figure 5 (Top). Now as 5:25 am approaches around the 65th minute time-step, you can see the traffic congestion beginning to build up as the modes interchange between FFCC to FCCC then CCC and then an interchange between FCCC and CCC. We compare the estimated modes and the real modes in Figure 5 (Top). This provides us with a way to determine the effectiveness of our algorithm. For this case the overall CDID and the FA are 88% and 12% respectively for the 4:10am through 8:20am duration. This

<table>
<thead>
<tr>
<th>Mode</th>
<th>CDID (%)</th>
<th>FA (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mode 1: FFFF</td>
<td>97.82</td>
<td>2.18</td>
</tr>
<tr>
<td>Mode 4: FCFF</td>
<td>89.45</td>
<td>10.55</td>
</tr>
<tr>
<td>Mode 7: CCFF</td>
<td>100</td>
<td>0</td>
</tr>
<tr>
<td>Overall</td>
<td>98.275</td>
<td>3.725</td>
</tr>
</tbody>
</table>

Fig. 4. (Top) Estimated and Actual modes; (Below) Cell densities for 100 Monte Carlo runs for congestion propagation in cell 2.
takes into account for delays greater than 5 minutes. The length of the highway segment is about 2 miles long and each cell has approximately 0.5 miles. The critical density here was 200 vehicles/mile. The number is high because we have 4 lanes on the highway, and also it was noticed that beyond this value the speed was less than the speed limit of 70mph. The run time for this scenario is 12.22sec for the entire 25 hour duration with a mean TTD delay of less than 5min because the sampling time is done for every 5 minutes. This is reasonable for the performance in the detection and localization of modes. Other algorithms used for incident detection had a 17.4% FA rate and a mean time to detection of 5.5 minutes [15]. In the PEMS system, the data fidelity for a 5-minute time-step observation period is between 70% to 80% (Note that these are based on different scenarios). Therefore, we have demonstrated that the proposed model and algorithm accurately detect a traffic congestion and propagation as well as identifying its location.

VI. CONCLUSIONS

We have designed a stochastic linear hybrid system that accurately represents the states of the highway traffic condition. Using the SDTHE algorithm, we are able to pin-point the congestion location and its propagation. The performance of the algorithm is analyzed using performance indices such as the correct detection and identification, false alarm rate, time-to-detection delay and simulation run time, rendering our algorithm with a good performance capability for online applications. We have tested various traffic scenarios and presented an illustrative example for the congestion onset detection and congestion propagation. The algorithm was also validated with real traffic data from highway I-405 S in California.

VII. APPENDIX

\[
\Delta t (\text{Sampling time}) = 5 \text{ sec} \\
l = 0.4 \text{ miles} \\
\rho_J = [688, 688, 688, 688, 688]^T [\text{veh/miles}] \\
q_m = [133, 133, 133, 133]^T [\text{veh/hr}]
\]

VIII. ACKNOWLEDGEMENTS

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