Optimal Coherent Phantom Track Design using Virtual Motion Camouflage

Yunjun Xu and Gareth Basset

Abstract—The virtual motion camouflage based trajectory planning methodology, augmented with the derived early termination conditions, is proposed to design the optimal collaborative trajectories for electronic combat air vehicles in constructing a coherent phantom track. In this problem, a realistic 6DOF dynamics model and severe state, control, control rate, and geometric constraints are all considered. An innovative virtual motion camouflage based method is applied to solve the inner-loop optimization problem in a very quick fashion. For the outer-loop, the early termination conditions are used to reduce the computational time by not executing the inner-loop optimization if these conditions are violated by the guessed phantom track. The early termination conditions include the necessary conditions derived based on the motion camouflage steering law and feasibility conditions derived according to the state, control, and control rate constraints.

I. INTRODUCTION

As an example of collaborative control systems, phantom tracking (PT) generation through cooperative electronic air vehicles (ECAVs) has attracted much attention recently [1-4]. This type of technique deception belongs to a broad range of electronic attack methodologies and has been regarded as one of the top ten military goals for unmanned aviation [5].

However, due to the high dimensionality of the problem and the involvement of severe equality and inequality constraints, it has always been a challenge to design the optimal trajectory for this mission. Most of the current work is focused on finding feasibility conditions for the case where only kinematic or very simple dynamic relations are considered [1, 3, 4].

In [6], inspired by a phenomenon of insect motion, the virtual motion camouflage (VMC) based method has been proposed to design a real-time (1) feasible constant speed coherent mission, (2) maximum-duration constant speed coherent mission, and (3) optimal ECAV trajectory mission for a given PT. In these designs, a complicated dynamics model, constrained control and state variables, and tight geometry constraints have all been considered. As an extension of the work in [6], in this paper, a fast (close to real-time) trajectory planning method based on the derived early termination mechanism and the VMC method is proposed to solve for the optimal coherent PT with the minimized total energy consumption of all the ECAVs. The advantages of the proposed approach are as follows: (1) the problem dimension of inner-loop optimization is dramatically reduced; and (2) the derived early termination strategies are used to reduce the computational cost by not wasting the CPU time in the inner-loop optimization if the PT guessed from the outer-loop doesn’t satisfy these conditions.

In the next section, the problem definition of the coherent optimal PT mission and the overall two-loop algorithm structure are given. This is followed by the concepts of the PT, ECAV, and motion camouflage (MC) in Section III. Then, for the inner-loop, the VMC based ECAVs’ trajectories optimization is developed in Section IV. The outer-loop optimization for the PT is described in Section V. Numerical examples are shown and conclusions are drawn in the end.

II. OPTIMAL COHERENT PT MISSION AND SOLUTION ARCHITECTURE

A. Problem Definition

A 6DOF dynamics model [7, 8] will be used in the optimal coherent PT mission design to govern the motions of all ECAVs and the phantom aircraft as

\[
\dot{\begin{bmatrix}
    x_e \\
    x_n \\
    \dot{v} \\
    \dot{\gamma} \\
    \dot{\mu}
\end{bmatrix}} =
\begin{bmatrix}
    V \cos \chi \cos \gamma \\
    V \sin \chi \cos \gamma \\
    V \sin \gamma \\
    (g / V)(k_n \cos \mu - \cos \gamma) \\
    (g / V)(k_n \sin \mu) / (V \cos \gamma)
\end{bmatrix}
\]

(1)

where \([x_e, x_n, \gamma, \mu, \chi, \gamma]^{T}\) is the east-north-up coordinate of the aircraft (ECAVs or phantom), \(V\) is the air speed (0 \(\leq V \leq 200 \text{ m/s}\)), \(\chi\) is the heading angle \((-50^\circ \leq \chi \leq 50^\circ\)), and \(\gamma\) is the flight path angle \((-25^\circ \leq \gamma \leq 25^\circ\)). The control variables are the applied thrust \(T\) (0 \(\leq T \leq 229,124 \text{ N}\)), load factor \(n\) \((-1.5 \leq n \leq 3\)), and bank angle \(\mu\) \((-25^\circ \leq \mu \leq 25^\circ\)).

The drag used here is calculated by

\[
D = 0.5 \rho V^2 SC_d + 2k_n^2 \rho^2 W^2 / (\rho V^2 S)
\]

(2)

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The constants used in the model [7, 8] are: wing area \( S = 37.16 \text{m}^2 \), zero lift drag coefficient \( C_{d0} = 0.02 \), load factor effectiveness \( k_s = 1 \), induced drag coefficient \( k = 0.1 \), gravitational coefficient \( g = 9.81 \text{kg} / \text{m}^2 \), atmospheric density \( \rho = 1.2251 \text{kg} / \text{m}^3 \), and the weight \( W = 14515 \text{g} \).

The problem in this paper is to design the optimal collaborative trajectories for ECAVs to achieve a coherent PT with the minimum total energy consumption. The geometric characteristic of the mission profile is that each ECAV must be on the line connecting its corresponding radar and the PT during its flight. The performance index is the total energy consumption used in the coherent mission, defined as

\[
J = \sum_{i=1}^{N_{EC}} \int_{t_0}^{t_f} T_{EC} \, dt
\]

where \( N_{EC} \) is the number of the ECAVs involved in the mission, \( T_{EC} \) is the thrust used by the \( i^{th} \) ECAV, and \( t_0 \) and \( t_f \) are the initial and final time of the mission, respectively.

In this constrained nonlinear optimal trajectory design problem, in addition to the dynamic constraint (equality constraint) as shown in Eq. (1), there are state and control inequality constraints, and geometric equality constraints involved. Also, to be more realistic, the rate of the control variables need to be constrained and the ECAV should not be too close to the PT or its corresponding radar. The rates of the control variables are assumed to be \(-3 \times 10^4 \leq \dot{\mathbf{x}} \leq 3 \times 10^4 \text{ N/s} \), \(-0.5 \leq \dot{n} \leq 0.5 \text{ /s} \), and \(-10^4 \text{ /s} \leq \dot{\mu} \leq 10^4 \text{ /s} \), respectively. Here the relative distance from an ECAV to PT and its corresponding radar is described by the path control parameter (PCP), \( \dot{\mathbf{v}} \), to be described in Section III. The proximity of the ECAV to the PT and to its corresponding radar location is constrained by \( v_{\min} \leq v \leq v_{\max} \).

B. Solution Architecture

The overall structure of the algorithm proposed in the paper is shown in Fig. 1. The inner-loop finds the minimum energy trajectory for each ECAV based on the given PT from the outer-loop iterations, while the outer-loop selects the optimal PT such that the total energy of the ECAVs can be minimized.

It is obvious that the computational cost will be high for this high dimensional, nonlinear, and constrained optimization problem. In this paper, in order to have a fast solution, three specific steps are taken here. First, the virtual motion camouflage based method is used in the inner-loop to obtain a very quick optimal solution for the selected PT based on the outer-loop iterations. Second, the differential inclusion technique combined with a pseudo-spectral method is used in the outer-loop to find the optimal PT. Third, just like what has been done in the sequential quadratic approach (SQP) [9], early termination mechanisms are used based on the derived feasibility and necessary conditions to dramatically reduce the computational cost. For this challenging coherent PT mission design problem, early termination mechanisms will avoid wasting time in the inner-loop if a poor or infeasible PT is given in the outer-loop. Also, since the system is decentralized, i.e. the optimal trajectory design is conducted at the level of each ECAV instead of the PT, the computational cost will be maintained roughly the same regardless of the number of radars present within the network.

III. PHANTOM TRACK AND MOTION CAMOUFLAGE FOR THE INNER-LOOP

Two moving objects, a prey and an aggressor, are involved in the motion camouflage. The path (e.g. the position vector) of the aggressor \( \mathbf{x}_a(t) \) is confined by the motion of the prey \( \mathbf{x}_p(t) \), the selected reference point \( \mathbf{x}_r(t) \), and the path control parameter (PCP) \( \mathbf{v}(t) \) (defined originally in [8]) as

\[
\mathbf{x}_a = \mathbf{x}_r + \mathbf{v}_p \quad (4)
\]

in which the relative position between the prey and the reference point is defined as \( \mathbf{x}_{p-r} = \mathbf{x}_p - \mathbf{x}_r \).

The pair of radar, ECAV, and PT can be naturally modeled using the MC framework and thus the results obtained in the previous studies [10-11] can be applied here. The east-north-up position of the \( i^{th} \) radar \( \mathbf{x}_i = [x_{i,E}, x_{i,N}, x_{i,U}]^T \) can be regarded as the reference point, and the phantom track \( \mathbf{x}_{p} = [x_{p,E}, x_{p,N}, x_{p,U}]^T \) is a virtual prey trajectory, while the \( i^{th} \) ECAV can be regarded as the aggressor and its motion \( \mathbf{x}_{EC_i} = [x_{EC_{i,E}}, x_{EC_{i,N}}, x_{EC_{i,U}}]^T \) is constrained by the motion camouflage rule as \( \mathbf{x}_{EC_i} = \mathbf{x}_i + \mathbf{v}_i (\mathbf{x}_p - \mathbf{x}_i) \).
IV. INNER-LOOP - OPTIMAL TRAJECTORY DESIGN FOR ECAVS THROUGH VMC

To handle the complicated state, control, and geometric constraints, the VMC approach will be applied in the inner-loop to solve for the optimal trajectory of the ECAV quickly once given the PT.

A. VMC Formulation

In the VMC framework [6], the state vector \( \mathbf{x} \) is separated into two parts: the “position” state \( \mathbf{x}_a \) (i.e. \( \mathbf{x}_a = [x_L, x_N, x_T]^T \)) and the corresponding state rate \( \mathbf{x}_w \) (i.e. \( \mathbf{x}_w = [V, \psi, \chi]^T \)). The subscript \( EC_i \) is omitted here for brevity. The state and state derivatives, which will be used to find \( \mathbf{x}_a \) and \( \mathbf{u}_a \), are governed by

\[
\dot{\mathbf{x}}_a = \dot{x}_a + i\mathbf{V}_{p-r} + \nu\mathbf{p}_{p-r} \tag{5}
\]

and

\[
\ddot{\mathbf{x}}_a = \ddot{x}_a + 2i\mathbf{V}_{p-r} + \nu\mathbf{p}_{p-r} + i\mathbf{p}_{p-r} \tag{6}
\]

As we can see, the position states and their corresponding derivatives are functions of the path control parameter (PCP), guessed virtual prey motion (provided by the outer-loop), the reference point (radar location), and their corresponding derivatives. For the inner-loop, the selected PCP \( \nu(t) \) determines both the speed and curvature of the trajectory in the constructed space.

Correspondingly, the dynamics model \( \dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{u}, t) \) (Eq. 1) will be rewritten as \( \dot{\mathbf{x}}_a(t) = \mathbf{f}_a(\mathbf{x}, t) \) and \( \ddot{\mathbf{x}}_a(t) = \mathbf{f}_a(\mathbf{x}, t) \). For the particular dynamics model used in this paper, all state and control variables can be represented by the PCPs and their derivatives based on Eqs. (1), (4), (5), and (6) through differential inclusion approaches as

\[
V = \left( \dot{x}_N^2 + \dot{x}_E^2 \right)^{1/2} \tag{7}
\]

\[
\dot{V} = \left( \dot{x}_N^2 + \dot{x}_E^2 \right) / V \tag{8}
\]

\[
\gamma = \sin^{-1}(\dot{x}_N / V) \tag{9}
\]

\[
\chi = \tan^{-1}(\dot{x}_N / \dot{x}_E) \tag{10}
\]

\[
\dot{\gamma} = \left[ 1 / \sqrt{1-(\dot{x}_N / V)^2} \right] (\ddot{x}_N V - \dot{x}_E \dot{V}) / V^2 \tag{11}
\]

\[
\dot{\chi} = \left[ 1 / \left[ 1 + (\dot{x}_N / \dot{x}_E)^2 \right] \right] (\ddot{x}_N \dot{x}_E - \dot{x}_N \ddot{x}_E / \dot{x}_E^2) \tag{12}
\]

\[
\tan \mu = \left[ \dot{\gamma} \cos \gamma + \dot{\chi}/(\dot{\gamma} \sin \gamma) \right] / \left[ \dot{\gamma} \sin \gamma + \dot{\chi} \cos \gamma \right] \tag{13}
\]

\[
n = \left\{ \begin{array}{ll}
\dot{V} \cos \gamma / \left( g k_{\text{c}}, \sin \mu \right) & \sin \mu \neq 0 \\
(\dot{V} \cos \gamma / g) + \cos \gamma \cos \mu & \cos \mu \neq 0
\end{array} \right.
\]

and

\[
T = \dot{W}(\sin \gamma + \dot{V} / g) + D \tag{15}
\]

For the purpose of brevity, the rate of the control variables are not shown here.

B. Collocation through a Pseudospectral Approach

After the state and control variables are represented by the PCPs through the VMC formulation and differential inclusion, the PCP history \( \nu(t) \) is discretized into \( i = 0, ..., N \) nodes, with \( \nu_0 = \nu(t_0) \) and \( \nu_N = \nu(t_N) \). There will be \( N+1 \) parameters to be optimized in the inner-loop. The vector form of the discretized PCP \( \nu(t) \), \( i = 0, ..., N \) is denoted as \( \mathbf{v} \). In this paper, the PCP time history is approximated using the Lagrange interpolation polynomial [12] as

\[
\nu(T) = \sum_{i=0}^{N} \nu_i \Phi(T) \tag{16}
\]

where the scaled time \( T = (2t - t_0 - t_f) / (t_f - t_0) \in [-1,1] \) is the zeros of \( \hat{L}_N \), the derivative of the Legendre polynomial \( L_N \). The base functions \( \Phi(T) \), \( i = 0, ..., N \) are the Lagrange interpolating polynomials of order \( N \)

\[
\Phi(T) = \frac{1}{N(N+1)L_N(T)} (T^2 - 1)L_N(T), i = 0, ..., N \tag{17}
\]

Through this collocation, in the original time scale \( t \), the \( n^\text{th} \) order derivatives of the PCP vector is [12]

\[
d^n v / dt^n = \left[ 2 / (t_f - t_0)^2 \right] D^nu \tag{18}
\]

where the differentiation matrix \( D \) is defined in [16]. The discretized form of the performance index can be written as

\[
J = \rho [\nu_k] + \left[ (t_f - t_0) / 2 \right] \sum_{k=0}^{N} L(v, \dot{v}, ..., \nu) \omega_k \tag{19}
\]

where \( \omega_k \) is the weight for the \( k^\text{th} \) LGL node. The state and control equality and inequality constraints can be discretized as

\[
h(v, \dot{v}, ...) = 0 \tag{20}
\]

and

\[
g_i(v, \dot{v}, ..., \nu) \geq 0, k = 0, ..., N \tag{21}
\]

A nonlinear program package can be used to solve the constrained optimal trajectory control problem for Eqs. (19) through (21).

C. Dimension Reduction for the Inner-Loop Optimization

In this section the dimensions of the inner-loop optimization problem formulated in the VMC approach and a typical collocation method will be compared in Table 1.

In a typical direct collocation method without using the VMC mechanism, the number of parameters to be optimized is \( 9(N+1) \), including six state and three control variables discretized at \( N+1 \) nodes. In the proposed VMC approach, there will be only \( N+1 \) PCP parameters in the optimization. Notice that the number can be further reduced with the feasibility condition proposed later. The complexity of the problem is reduced roughly by eight times from the collocation method. The smaller problem is then in principle solved more quickly [13].

In addition to the dimension reduction, the problem formulated in VMC reduces the difficulties associated with the equality constraints in solving the achieved NLP. Specifically, there is no equality constraint involved in the
VMC approach, while in the direct collocation method, there are roughly \(8(N+1)\) equality constraints. In these \(8(N+1)\) equality constraints, \(6(N+1)\) come from the dynamics equation constraints, while the remaining \(2(N+1)\) come from the line of sight (LOS) constraints of the coherent PT mission.

**Table 1** Dimension comparison for each of the ECAV trajectory design

<table>
<thead>
<tr>
<th></th>
<th>Collocation</th>
<th>VMC</th>
</tr>
</thead>
<tbody>
<tr>
<td># of parameters</td>
<td>(9(N+1))</td>
<td>(N-2)</td>
</tr>
<tr>
<td># of E.C.</td>
<td>~ (8(N+1))</td>
<td>0</td>
</tr>
</tbody>
</table>

**Remark 1:** The dimension reduction comparison here is for the case when only one pair of PT, ECAV, and radar is involved. The reduction will be even more significant if there are multiple pairs involved in the coherent PT mission design.

**Remark 2:** For the coherent PT mission design, the results obtained from the VMC approach will be the same as the one achieved in the full space search, similar to what can be done in a typical collocation. Therefore there is no need to check the optimality of the solution in the original full space [10-11].

**D. Necessary Condition based on Boundary Values**

Based on the accessibility condition (Eqs. 7-15), once we know the PCP variables, all the state and control variables of each ECAV can be calculated. Therefore there will be a total of \(N+1\) PCP nodes to be optimized for the trajectory of each ECAV in the inner-loop. Here, Lemma 1 will be used to check the necessary condition for the initial velocity of each ECAV, while Lemma 2 (or Lemma 3) will be used to calculate two (or one) extra PCPs given initial and final conditions (or just the initial condition) of each ECAV. Also Lemmas 1 and 2 will give the guideline for the minimum number of nodes needed in discretization. Notice that these three lemmas have been proven in [6].

**Lemma 1:** The three components in the initial velocity of the ECAV, \(\dot{x}_{ECAV,0}\), \(\dot{x}_{ECAV,0}',\) and \(\dot{x}_{ECAV,0}''\), have to satisfy the following two equations:

\[
a_2 \dot{x}_{ECAV,0} - a_{11} \dot{x}_{ECAV,0}' = a_2 v_{0} \left[ \dot{x}_{p,E,0} + \frac{2D_{00}}{t_f - t_0} a_1 \right] + a_2 v_N \left[ \frac{2D_{0N}}{t_f - t_0} a_1 \right] \tag{22}
\]

and

\[
a_3 \dot{x}_{ECAV,0} - a_{11} \dot{x}_{ECAV,0}'' = a_3 v_{0} \left[ \dot{x}_{p,E,0} + \frac{2D_{00}}{t_f - t_0} a_1 \right] + a_3 v_N \left[ \frac{2D_{0N}}{t_f - t_0} a_1 \right] \tag{23}
\]

while the three components in the final velocity of the ECAV are related by the following two equations:

\[
b_2 \dot{x}_{ECAV,N} - b_3 \dot{x}_{ECAV,N}'' = b_2 v_{0} \left[ \frac{2D_{00}}{t_f - t_0} b_1 + \frac{2D_{0N}}{t_f - t_0} b_1 \right] + b_2 v_N \left[ \dot{x}_{p,N} + \frac{2D_{NN}}{t_f - t_0} b_2 \right] \tag{24}
\]

and

\[
b_3 \dot{x}_{ECAV,N} - b_3 \dot{x}_{ECAV,N}'' = b_3 v_{0} \left[ \frac{2D_{00}}{t_f - t_0} b_1 + \frac{2D_{NN}}{t_f - t_0} b_1 \right] + b_3 v_N \left[ \dot{x}_{p,N} + \frac{2D_{NN}}{t_f - t_0} b_2 \right] \tag{25}
\]

in which \(D_{ij}\) is the entry in the \(i^{th}\) row and \(j^{th}\) column of the differentiation matrix \(D\),

\[
(x_{p,0} - x_{r,0}) \leq a_i = [a_1, a_2, a_3]^T,
\]

and

\[
(x_{p,N} - x_{r,N}) \leq b_i = [b_1, b_2, b_3]^T.
\]

**Lemma 2:** There are only \(N-3\) independent PCP variables that need to be optimized. In addition to the initial and final PCP variables that are defined by the initial and final PT, ECAV, and radar locations, there are two PCPs that can be calculated without optimization by

\[
\begin{bmatrix}
2D_{00} v_{0} \\
2D_{0N} v_{0} \\
2D_{NN} v_{0}
\end{bmatrix} = \begin{bmatrix}
2D_{00} b_1 \\
2D_{0N} b_1 \\
2D_{NN} b_1
\end{bmatrix}
\]

**Lemma 3:** There are only \(N-2\) independent PCP variables that need to be optimized if the final state rate is free. In addition to the initial and final PCP variables defined by the initial and final PT, ECAV, and radar locations, there is another PCP that can be calculated without optimization by

\[
\begin{bmatrix}
2D_{00} a_{11} v_{0} \\
2D_{0N} a_{11} v_{0} \\
2D_{NN} a_{11} v_{0}
\end{bmatrix} = \begin{bmatrix}
2D_{00} b_1 \\
2D_{0N} b_1 \\
2D_{NN} b_1
\end{bmatrix} \frac{N-2}{j=0} \sum_{j=0}^{N-2} 2D_{0j} a_{11} v_{j}
\]

**Remark 3:** The initial guess of the PCPs \((v_{0}, i = 2,\ldots,N-2)\) or \((v_{N}, i = 2,\ldots,N-1)\) should be close to the initial and final values of the PCPs \((v_{0} v_{N})\). Because the motion of the ECAV is limited by the speed, thrust, g-load, and bank angle, a big difference between the initial and final PCPs means that the solution might be outside of the feasible region of the ECAV and that convergence may be difficult. Also, the ECAV should not be too close to the radar or the phantom to avoid being detected by the enemy. Therefore, the PCP bounds should be set properly.
Remark 4: As in [6], the inner-loop optimization only takes about 1.8 seconds for a 50-second simulation to converge, even if the algorithm is coded in MATLAB. Note that the computations were performed on a desktop with Dual CPUs (E6550) at the frequency of 2.33 GHz, with a 2.95-GB random-access memory.

Remark 5: Since the system is decentralized, i.e. the optimal trajectory design is conducted at the level of each ECAV, the computational cost will be maintained roughly the same regardless of the number of radars in the network.

V. OUTER-LOOP - OPTIMAL COHERENT PHANTOM TRACK

In this section, the outer-loop to generate the optimal coherent PT for all ECAVs will be discussed. Early termination strategies will be described based on the motion camouflage steering law and its necessary condition.

A. Necessary Conditions based on PCP Propagation

The necessary condition of the coherent PT is derived based on the steering law of the MC as shown in the next two lemmas. The proofs for them can be found in Refs. [6, 10].

Lemma 4: The propagation of the PCP for each ECAV is governed by

\[
\dot{v} = -\frac{\left(x_p - x_{pr}\right) \cdot \dot{x}_p}{\|x_p\|^2} \pm \sqrt{\left(\frac{1}{2}\|x_p\|^2 + \frac{\|v_p\|^2}{\|x_p\|^2} + \frac{V_{EC}^2}{\|x_p\|^2}\right)}
\]

(28)

Remark 6: In most cases, the location of the reference point (e.g. the radar network) is fixed. Therefore, the PCP governing equation for the ECAV can be simplified as

\[
\dot{v} = -\frac{x_p^T \cdot \dot{x}_p}{\|x_p\|^2} \pm \sqrt{\frac{\left(x_p^T \cdot \dot{x}_p\right) \cdot \frac{\|v_p\|^2}{\|x_p\|^2} + \frac{V_{EC}^2}{\|x_p\|^2}}
\]

(29)

Lemma 5: For a MC to be valid, the speed of the coherent PT must satisfy

\[
V_p^2 - \left(\frac{x_p^T \cdot \dot{x}_p}{\|x_p\|^2}\right)^2 \leq V_{EC}^2 / v^2
\]

(30)

for all the ECAV and radar pairs.

B. Early Termination

For the two-loop structure, just like the sequential quadratic programming [9], it seems wasteful to expend much effort on any inner-loop optimization using a PT that violates the necessary and feasibility conditions.

Lemma 6: The position, speed, and velocity of the PT at each discretized node generated in the outer-loop must satisfy the following equation:

\[
V_p^2 - \left(\frac{x_p^T \cdot \dot{x}_p}{\|x_p\|^2}\right)^2 \leq V_{EC, max}^2 / v^2
\]

(31)

where \(v_b\) is selected according to the initial and final PCP values. Note that since the control variables of the ECAV or PT are limited, a big difference between \(v_b\) and \(v_f\) will cause the ECAV to violate the control constraints. The value of \(v_b\) should satisfy \(v_{min} < v_b < \min(v_b, v_f)\).

In addition to Lemma 6 (necessary condition), the PT guessed in the outer-loop needs to satisfy the following feasibility conditions described in Section II.A, i.e. all the state, control, and control rate constraints. If the generated PT doesn’t satisfy these necessary and feasibility conditions, the inner-loop will be not executed and a large number will be used as the performance index for the inner-loop.

C. Outer-Loop Optimization

The parameters to be optimized in the outer-loop are the east-north-up coordinates of the PT, \([x_{PT,E}, x_{PT,N}, x_{PT,U}]^T\), Through Eqs. 7-15, the speed, flight path angle, heading angle, thrust, g-load, and bank angle can be calculated using the guessed PT in each iteration. The position history \([x_{PT,E}, x_{PT,N}, x_{PT,U}]^T\) is discretized into \(i = 0, \ldots, N\) LGL nodes, with known initial and final positions. Similar to the approach used in Section IV.B, the LGL based pseudospectral method and a NLP algorithm (e.g. the fmincon) will be used here to solve the constrained optimization problem in the outer-loop.

VI. NUMERICAL SIMULATION

A. Simulation Scenario

The advantage of the proposed early termination and VMC strategies for the coherent optimal phantom track design will be shown in the simulation here. The initial and final positions of the PT are set to be \([-6709.4, -4357.1, 3600]\) m and \([-1270.2, -1435, 3260.6]\) m. Four ECAVs and four radars are involved, and these radars are located at \([1000, -4000, 10]\) m, \([0, 4000, 60]\) m, \([-10000, -7000, -30]\) m, and \([-5000, -9000, 50]\) m. The initial and final PCPs are listed in Table 2-A, in which one ECAV’s PCP increases while the remaining ones decrease.

<table>
<thead>
<tr>
<th>ECAVs</th>
<th>Initial/final PCPs</th>
</tr>
</thead>
<tbody>
<tr>
<td>ECAV1</td>
<td>0.7/0.65</td>
</tr>
<tr>
<td>ECAV2</td>
<td>0.5/0.44</td>
</tr>
<tr>
<td>ECAV3</td>
<td>0.5/6</td>
</tr>
<tr>
<td>ECAV4</td>
<td>0.6/55</td>
</tr>
</tbody>
</table>

B. Simulation Results

Cases with different number of nodes and different number of ECAVs are all tested. To be brief, only a couple cases are shown here. As shown in Table 2-B, the performance indices achieved in the cases with different number of ECAVs involved are much smaller than the values achieved in [6], where the PT is assumed to be given (not optimized). Also, the overall runtime is not significantly affected as the number of ECAVs increases, which shows the advantages coming from the fully decentralized structure. As can be seen, with as many as five vehicles involved (4 ECAVs plus one phantom), the computational cost in optimization is only 9.29 seconds when coded in MATLAB for a 50-second flight simulation.

<table>
<thead>
<tr>
<th>ECAVs involved</th>
<th>CPU time (s)</th>
<th>Indices</th>
</tr>
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<tr>
<td>1</td>
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<td>1.26x10^9</td>
</tr>
<tr>
<td>1, 2</td>
<td>9.78</td>
<td>5.59x10^9</td>
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<tr>
<td>1, 2, 3</td>
<td>11.02</td>
<td>3.48x10^10</td>
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</table>
Only the simulation results for the case with 8-node and 3-ECAV are shown in Fig. 2 – Fig. 8. In Fig. 2, the optimal phantom and ECAVs have a similar shape for the 3D trajectory. As shown in Fig. 3, the speeds of all ECAVs and PT are within the speed limit. The flight path angle (Fig. 4) and the heading angle (Fig. 5) of the ECAVs are following the trend of the PT. To achieve the minimum energy consumption, the thrust commands of the ECAVs used are smaller than that of the PT (Fig. 6). Also as shown in Fig. 7 and Fig. 8, the g-load and the bank angle are within the constraints and the values of the ECAVs have similar shapes as those of the PT.

VII. CONCLUSION

In this paper, the virtual motion camouflage based subspace optimization method together with the early termination strategy are used to design the coherent minimum energy phantom track mission considering the 6DOF dynamics model, state and control equality and inequality constraints, and geometry constraints. The advantages of the approach are: through this bio-inspired approach, the dimension of the model described by dynamic relations can be represented by a single-degree-of-freedom vector, and thus the inner-loop optimization can be dramatically decreased. In the meantime, the necessary and feasibility conditions derived based on the VMC steering law can be used as the early termination mechanisms for the outer-loop optimization. The simulation has been used to show these advantages.

Fig. 8. Bank angle for the PT and ECAVs

ACKNOWLEDGMENT

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REFERENCES