Nonlinear Controller design for Permanent Magnet Synchronous Motor Using Adaptive Weighted PSO

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Abstract—A nonlinear speed controller design for permanent magnet synchronous motor (PMSM) based on adaptive backstepping technique is presented in this paper. In order to improve the ability of torque estimation, the differential action is applied in the torque observe equation. To achieve good dynamic performance and robust, the controller parameters are optimized by adaptive weighted particle swarm optimization (PSO), which is an efficient and simple tool for multi-objective and multi-dimensional problem. The proposed controller is verified by simulation, and the results show that the controller has robust and good dynamic response.

I. INTRODUCTION

Due to high torque to current ratio, large power to weight ratio, high efficiency, high power factor, and robustness, permanent magnet synchronous motors (PMSM) are used in robots, electrically powered air compressor drives in vehicles, and other industrial applications [1]. To achieve the high performance, the vector control of PMSM has been applied [2]. The controllers based on advanced control theories and techniques such as, robust control, fuzzy control, neural net control, etc have further improved the robust and dynamic of PMSM drives.

Because of its simple and reliable, proportional integral derivate (PID) controller is widely used in industrial applications. PID controller is based on classic control theory, and the linear system can obtain the good performance with PID controller. But for a nonlinear system, PID controller is difficult to make the system gain the fast dynamic response and robust. PMSM drive is a nonlinear system with the variable parameters and complicated application environment, so the traditional PID controller and classic control theory are unfit for designing the controller for PMSM.

Advance control theory developing, the design of the nonlinear system controller becomes feasible. The adaptive backstepping is an effective design tool for nonlinear system [3]. The adaptive backstepping is a systemic and recursive design method for nonlinear feedback control, which is able to keep robust and stability for uncertain system. PMSM with speed controller using adaptive backstepping technique obtains fast and accuracy response, quick recovery from disturbances, and insensitivity to parameter variations [4, 5].

II. MODEL OF PMSM DRIVE

The mathematical model of PMSM in a synchronous rotating rotor d-q reference frame can be described as follows [8, 9]

\[
\begin{align*}
\dot{v}_d &= \frac{R + pL_d}{L_d} i_d - \frac{pL_d}{L_d} \omega i_q + \frac{pL_d}{L_d} \Phi_m \\
\dot{v}_q &= \frac{pL_d}{L_d} i_d - \frac{R + pL_d}{L_d} \omega i_q + \frac{pL_d}{L_d} \Phi_m \Phi_m
\end{align*}
\]

(1)

\[
T_e = \frac{3P}{2} \left( \Phi_m i_q + (L_d - L_q) i_d i_q \right)
\]

(2)

\[
T_e = T_i + B\omega_r + J \frac{d\omega_r}{dt}
\]

(3)

where \(v_d\) and \(v_q\) are the d- and q-axes stator voltages, respectively; \(i_d\) and \(i_q\) are the d- and q-axes currents, respectively; \(R\) is the state resistance per phase; \(L_d\) and \(L_q\) are the d- and q-axes stator inductances, respectively; \(T_i\) and \(T_L\) are the electromagnetic and load torques, respectively; \(J\) is the moment of inertia of the motor; \(B\) is the friction coefficient of the motor; \(P\) is the number of pole pairs; \(\omega_r\) is the rotor mechanical speed; \(p\) is the differential operator \((d/dt)\); and \(\Phi_m\) is the permanent magnet flux linkage. The mathematical model of PMSM can be rewritten as follows.
\[
\frac{di_d}{dt} = \frac{v_d - R}{L_d} i_d + P\omega_i \frac{L_m}{L_d} i_d
\]
\[
\frac{di_q}{dt} = \frac{v_q - R}{L_q} i_q - P\omega_i \frac{L_m}{L_q} i_q - P\omega_i \phi_m \frac{L_m}{L_q}
\]
\[
\frac{d\omega}{dt} = \frac{3P\phi_m}{2J} i_q + \frac{3P}{2J} (L_d - L_q) i_d i_q - \frac{B}{J} \omega - \frac{1}{J} T_L
\]

III. ADAPTIVE BACKSTEEPING CONTROL DESIGN FOR PMSM

The basic idea of adaptive backstepping is identification of control variable, and the control variable is forced to a stabilizing function. The corresponding error variable can be stabilized by proper selection via Lyapunov’s stability theory. The parameters which are not easy measured directly can be estimated effectively by adaptive backstepping technique. This method is fit for speed control for PMSM. In this paper, the inertia and the load torque are uncertain, and the equation of torque estimation with differential action can estimate the load torque quickly.

The objective of speed control design is to track rotor speed \(\omega_r\). The error of speed is given by
\[
e_{\omega} = \omega_r - \dot{\omega}
\]
and the speed error dynamic from (6)
\[
\frac{de_{\omega}}{dt} = \frac{1}{J} \left( T_L + B\omega - \frac{3P}{2J} \phi_m \dot{i}_q + \left( L_d - L_q \right) i_d \dot{i}_q \right)
\]
Speed error must be reduced to zero. The d-q axis current \(i_d\) and \(i_q\) can be identified by a-b-c axis stator currents which are measured by sensor. \(i_d\) and \(i_q\) are the control elements.

To determine the stabilizing function, the first Lyapunov function is chosen as
\[
V = \frac{e_{\omega}^2}{2}
\]
The derivates of the Lyapunov function (9) from (8) is computed as
\[
\dot{V} = e_{\omega} \frac{de_{\omega}}{dt} = \frac{e_{\omega}}{J} \left( B\omega + T_L \right) - \frac{3P}{2J} \phi_m \left( \dot{i}_q + \left( L_d - L_q \right) i_d \dot{i}_q \right)
\]
to be sure the system is stability, we imposed
\[
\dot{V} = -k_e e_{\omega}^2 \leq 0
\]
where \(k_e\) is a positive constant feedback gain, and (10) can be written as
\[
\dot{V} = -k_e e_{\omega}^2 + \frac{e_{\omega}}{J} \left( T_L + B\omega + k_m J e_{\omega} - \frac{3P}{2J} \phi_m \dot{i}_q \right)
\]
The load torque \(T_L\) and the inertia \(J\) cannot be measure directly. Therefore from the above function, the stabilizing functions can be defined as
\[
i_d' = 0
\]
where \(\dot{T}_L\) is the estimated load torque, and \(\dot{J}\) is the estimated inertia of the PMSM. From (14), the speed error can be written as
\[
\frac{de_{\omega}}{dt} = \frac{1}{J} \left( -\dot{T}_L + \frac{3P}{2} \phi_m e_{\omega} + \frac{3P}{2} \left( L_d - L_q \right) e_d i_q - k_m \dot{J} e_{\omega} \right)
\]
(15)

The new Lyapunov function can be defined for the whole system with the estimation errors as
\[
V = \frac{1}{2} e_{\omega}^2 + \frac{1}{2} e_{i_d}^2 + \frac{1}{2} e_{i_q}^2 + \frac{1}{2\theta_1} \dot{J}^2 + \frac{1}{2\theta_2} \left( \dot{T}_L + k_m J e_{\omega} \right)^2
\]
(20)
where \(\theta_1, \theta_2\) are the adaptive gain, and \(\theta_1 \geq 0, \theta_2 \geq 0\). The above error can be differentiatied
\[
\dot{V} = e_{\omega} \dot{e}_{\omega} + e_{i_d} \dot{e}_{i_d} + e_{i_q} \dot{e}_{i_q} + \frac{1}{\theta_1} \dot{J} \left( \ddot{T}_L + k_m J e_{\omega} \right)
\]
\[
= -k_e \dot{e}_{\omega}^2 - k_d \dot{e}_{i_d}^2 - k_d \dot{e}_{i_q}^2
\]
\[
+ e_{\omega} \left( \frac{R i_d - P\omega_i L_d i_d - v_d}{L_d} + \frac{3P}{2J} \left( L_d - L_q \right) e_d i_q + k_m e_{\omega} \right)
\]
\[
+ e_{i_d} \left( \frac{R e_{i_d} - P\omega_i L_d e_{i_d} - v_d}{L_d} + \frac{3P}{2J} \left( L_d - L_q \right) \dot{i}_q i_d + k_m e_{\omega} \right)
\]
\[
+ e_{i_q} \left( \frac{R e_{i_q} - P\omega_i L_d e_{i_q} - v_d}{L_d} + \frac{3P}{2J} \left( L_d - L_q \right) \dot{i}_q i_q + k_m e_{\omega} \right)
\]
\[
+ \dot{J} \left( \frac{1}{\theta_1} \dot{J} \dot{e}_{\omega} + \frac{2k_m k_m J}{3P\phi_m} e_{\omega}^2 + \frac{2k_m k_m J}{3P\phi_m} e_{\omega} \right)
\]
\[
+ \left( \ddot{T}_L + k_m J e_{\omega} \right) \left( \frac{1}{\theta_2} \ddot{T}_L + k_m J e_{\omega} \right) - \frac{e_{\omega}}{J} \left( \frac{2}{3P\phi_m} e_{\omega} \right)
\]
where \( \alpha = \frac{3P\phi_m}{2} e_\alpha^2 + \frac{3P}{2} (L_d - L_q) e_i \); \( k_d \) and \( k_q \) are positive constant. To be sure the global asymptotic stability, the d-q axis control voltages are defined as

\[
\begin{align*}
  v_d &= R_i q_d + P e_o L_q i_q + \frac{3P}{2} (L_d - L_q) L_d e_o i_q + k_d L_d e_d \\
  v_q &= R_i q_q + P e_o L_q i_q + P e_o \phi_m + \frac{2L_q}{3P\phi_m} \left( k_o J - B \right) \left( \frac{3P\phi_m}{2} e_\alpha + \frac{3P}{2} (L_d - L_q) e_i \right) \\
  &\quad - \frac{k_m L_q e_q}{3P\phi_m} J e_o + \frac{2k_m}{3P\phi_m} \left( k_o J - B \right) L_q e_q \\
  &\quad + k_q L_q e_q + L_k a e_a + \frac{2P L_q}{2J} \phi_m e_o.
\end{align*}
\]

From the above functions, (21) can be simplified as

\[
\begin{align*}
  \dot{\rho}_i &= -k_a e_o^2 - k_d e_d^2 - k_q e_q^2 \\
  &\quad + \left[ \frac{1}{\theta_1} \frac{J - k_d}{J} e_\alpha^2 + \frac{2k_m k_a}{3P\phi_m} e_a^2 \right] e_i. \\
  \dot{\theta}_1 &= \frac{1}{\theta_2} \left( \tilde{T}_L + k_m J e_q \right) - \frac{e_a}{J} \\
  \dot{\tilde{T}}_L &= \theta_2 \left( \frac{e_a}{J} + \frac{2k_o J - B}{3P\phi_m} e_q \right) - k_m J e_q.
\end{align*}
\]

From (24), the update laws for the parameters can be estimated as

\[
\dot{T}_L = \theta_2 \left( \frac{e_a}{J} + \frac{2k_o J - B}{3P\phi_m} e_q \right) - k_m J e_q.
\]

The particles tend to fly
better and better positions. The procedure can be described as

\[ v_i(t) = wv_i(t-1) + \alpha r_1 [x_{pbest} - x_i(t-1)] + \alpha r_2 [x_{gbest} - x_i(t-1)] \]

(28)

\[ x_i(t) = x_i(t-1) + v_i(t) \]

(29)

where \( w \) is the inertia weight factor; \( \alpha \) is positive constant, defined as acceleration coefficient; \( r_1 \) and \( r_2 \) are two random functions in the range of \([0, 1]\); \( x_i \) represents the position of \( i \)th particle and \( x_{pbest} \), the best previous position of \( i \)th particle; \( x_{gbest} \) is the position of best particle among the entire population; \( v_i \) is velocity for the \( i \)th particle. The current position of every particle is evaluated by (28), (29).

To improve the reaching capability of PSO algorithm, adaptive weighted PSO has been proposed. The acceleration factor \( \alpha \) in (28) is represented as follow

\[ \alpha = \alpha_0 + \frac{t}{N_i} \]

(30)

where \( N_i \) denotes the number of iterations, \( t \) represents the current generation, and suggested range for \( \alpha_0 \) is \([0.5, 1]\). The inertia weight is changed at every generation via the following function

\[ w = w_0 + r_1 (1 - w_0) \]

(31)

where \( w_0 \) is positive constant in \([0, 1]\), and the suggested range for \( w_0 \) is \([0.5, 1]\); \( r_1 \) is a random function in the range of \([0, 1]\).

B. Optimization Procedure

According section III, the controller can be design, which makes the system stabilizing. The controller parameters, as \( k_o, k_d, k_q, m, \theta_1, \) and \( \theta_2 \), are determined the performance of the system. The selection of parameters is a difficult task. Therefore the optimization of the parameters is a necessary part of design controller, and the optimal controller can guarantee that the system obtain the fast dynamic response, and robust. The adaptive weighted PSO is applied in optimization the controller. The procedure is described as

- Initialize the particles. The swarm has 6-dimension particles which guarantee \( k_o, k_d, k_q, m, \theta_1, \) and \( \theta_2 \) positive.
- Generate the controller according the section III, and the controller is applied in the model of PMSM drive
- Evaluate the fitness for each particle. The fitness function is derived from the integral time multiplied by absolute error (ITAE), and the fitness function can be defined as

\[ f_i = \int_0^t \delta |e_o| + k_1 |e_o| \frac{d}{dt} + k_2 e_o^2 \]

(32)

where \( \delta = \begin{cases} k_1 & e_o \geq 0 \\ k_2 & e_o < 0 \end{cases} \), and \( k_1, k_2, k_3, k_4 \) are the adaptive constant.
- Update \( x_{pbest} \) and \( x_{gbest} \)
  \[ x_{pbest} = x_i \] if \( f_i < f_{pbest} \)
  \[ x_{gbest} = x_i \] if \( f_i < f_{gbest} \)
- Generate \( \alpha \) and \( w \), using (30) and (31).
- Update velocity \( v_i \) and position \( x_i \), using (28) and (29).

- If attain the maximum iteration, exit, and otherwise go to iterate.

V. SIMULATION RESULTS

In order to verify the effectiveness of the proposed controller, the model of PMSM drive is built, which is described as fig.1. The nominal parameters of the simulated PMSM are listed in Table I.

Under the same optimality condition and same fitness function, the parameters of controller with the differential action and without proposed method are optimized by adaptive weighted PSO, respectively. In fig 2, the controller using the adaptive weighted PSO has better dynamic and steady-state performance than the controller’s without optimization. The speed response time of system with optimal controller is 0.13 s, and its overshoot is 0.6%. In fig 3 (a), the proposed controller has better dynamic performance than the normal controller’s using the backstepping. The step torque is applied at \( t = 0.5 \)s, and the proposed controller can achieve stable operation and acceptable performance. In fig 3 (b), the stator current \( i_s \) is shown. In fig 3(c), the proposed controller can estimate the load faster than the normal backstepping controller. The electromagnetic torque is shown in fig 3(d).

![Fig. 2. Speed response of the optimal controller and controller without optimization.](image-url)
VI. CONCLUSION

The backstepping based speed controller with differential action has been proposed, which can improve the ability of torque estimation. The proposed controller has used adaptive weighted PSO to obtain the optimal parameters of the controller. The simulation results show that the proposed controller fast estimate the load torque for PMSM drive, and make the PMSM drive own good performance.

REFERENCES


Fig. 3. (a) Speed response (b) Motor stator current $i_a$ (c) Estimated values of load torque (d) Electromagnetic torque.