Robust Anti-windup Control of SISO Systems

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Abstract—This paper provides some perspectives on the robust anti-windup (AW) control problem by focusing on the simplest case of AW compensation for uncertain linear single-input-single-output (SISO) systems. Using Quantitative Feedback Theory (QFT)-style frequency domain analysis, and through numerical examples, it is shown that even for simple systems, certain popular AW techniques may fail to provide robust stability when saturation is encountered. A QFT-based AW method, in which uncertainty is explicitly incorporated into the design procedure, is revised to employ less conservative stability multipliers and employed to overcome these robustness deficiencies. Through a robust AW example the paper highlights: the importance of explicitly designing the AW controller to be robust to system uncertainty; the potential failure in designing for robustness based only on the nominal plant, with IMC AW shown to unstable in one case; and the significant reduction in the problem difficulty that can arise from the use of less conservative saturation multipliers.

I. INTRODUCTION

The theory supporting anti-windup (AW) controller synthesis is quite mature, with recent theoretical developments offering AW controller synthesis approaches based on the solution of LMIs (see for example [11], [2], [15] and references therein). These synthesis approaches provide the ability to construct AW controllers that ensure the system subject to saturation constraints is stable, satisfies some upper bound on a chosen performance measure, and if system uncertainty is considered, that these properties are robust to norm bounded uncertainty in the plant [13]. It is this robustness characteristic that we consider in more detail in this paper. It is well-known from linear robust control theory that consideration of plant uncertainty is vital for the design of controllers which function well in practice and, indeed, the more knowledge of the uncertainty characteristics, the greater the possibility for reducing conservatism in controller design. It thus stands to reason that uncertainty should also be carefully considered in AW design and, although a few papers have tackled this subject ([13], [1], [9], [17]) most papers are content to assume that if the controller is robust, in an appropriate sense, then the controller with AW compensation will continue to be robust. This view is fallacious, as shown in [13], [9].

One of the issues in considering uncertainty in AW design is that dealing with saturation and uncertainty at the same time is quite hard [13], particularly with results based on sector-bounds and quadratic Lyapunov functions. However, using results from the theory of integral quadratic constraints (IQCs) [8], it is possible to characterise the saturation nonlinearity less conservatively using more general stability multipliers. For analysis, LMI algorithms offer an efficient way to perform this IQC based robust stability test, although some restrictions on the multiplier structure and/or class apply [7], [14], [6]. However, for synthesis, the problem appears to be non-convex, both due to the IQCs for the (structured) uncertainty, which occur in linear robust control, and to the more complex IQCs for the saturation nonlinearity. Therefore, the AW designer is forced to resort to the most basic norm-bounded uncertainty and sector-based saturation models in order to obtain convex synthesis routines [13], [1], [9].

The objective of this paper is to highlight the conservatism that arises from the restriction of the synthesis approach to be convex, and to reaffirm the importance of explicit uncertainty modelling in robust AW design. A robust AW design example that illustrates the importance of explicit uncertainty consideration is given, and various AW compensators are designed and their robustness properties examined. A QFT-style design procedure [4] which can take advantage of the more general multipliers available for saturation nonlinearities in a convenient style is proposed. Due to its lower conservatism, this procedure can more comfortably accommodate uncertainty and saturation simultaneously, and it is shown how low order AW compensators can be designed to explicitly handle uncertainty using less conservative stability multipliers calculated using the results of [14].

The paper is structured as follows. First the robust problem typical in AW control is summarised. Then the classical robust AW design procedure proposed in [4] is reviewed. A robust AW example is then presented to highlight the aforementioned robustness and conservatism issues.

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II. ROBUST AW PROBLEM

The AW control problem considered herein is that of designing a linear SISO AW controller for an existing linear control system, comprising an uncertain linear plant $P \in P^\Delta$ and a given linear controller $G$, such that the system subject to input constraints is robustly stable and the effects of saturation on the performance objectives are reduced. This system is generically depicted in Fig. 1, with $P \in P^\Delta$ the uncertain plant, $G = [G_1 \ G_2]$ the linear controller, and $\Lambda$ the AW compensator. $N(.)$ represents the standard saturation nonlinearity defined as

$$\text{sat}(u) = \text{sign}(u) \min \{|u|, \bar{u}\}, \quad \bar{u} > 0$$

(1)

To aid the understanding of the AW problem, the system in Fig. 1 can be represented in the equivalent form in Fig. 2, typically referred to as the mismatch system [16], [3]. The nonlinearity in this loop is the deadzone nonlinearity which satisfies the identity $\text{sat}(u) + \text{Dzn}(u) = u$ where

$$\text{Dzn}(u) = \text{sign}(u) \max \{|u| - \bar{u}\}, \quad \bar{u} > 0$$

(2)

The elements $T_n$ and $S_n$ of the mismatch system are defined in Eqs. (3) to (5), where $\Lambda = [\Lambda_1^T \ \Lambda_2^T]^T$ is assumed to be in an external AW configuration and hence $v_2$ feeds to the input of the controller and $L_n$ is the loop-transmission around the saturation nonlinearity (in negative feedback).

$$T_n(\Lambda) = (I + L_n)^{-1} L_n,$$

(3)

$$S_n(\Lambda) = (I + L_n)^{-1},$$

(4)

$$L_n(\Lambda) = (I + (\Lambda_1 - G_2 \Lambda_2)^{-1})^{-1}(-G_2 P - (\Lambda_1 - G_2 \Lambda_2)).$$

(5)

The following assumptions are made throughout

- $P \in P^\Delta$ and $G$ are linear time invariant (LTI) systems
- Every $P \in P^\Delta$ is asymptotically stable
- When $N(.) = I$, the linear closed-loop is well-posed and assumed to possess desirable performance properties for all $P \in P^\Delta$
- The performance map of interest is the nonlinear map $\mathcal{J}_{y_n,u_{lin}}$ from $u_{lin}$ to $y_d$ [15], [3].

The above assumptions essentially ensure that the nominal closed-loop linear system is robustly stable and yields desirable behaviour, in the sense that we would like the effect of saturation on the ideal linear response, captured by the “disturbance” $y_d$, to be as small as possible in order to ensure the real output $y$ is as close as possible to the ideal linear output $y_{lin}$ during periods of saturation. The asymptotic stability assumption on $P \in P^\Delta$ allows us to consider global stability results; otherwise we are forced to consider results which are local in some respects. Note that the nonlinear map $\mathcal{J}_{y_n,u_{lin}}$, which basically dictates how much effect saturation has on the performance output, can be described by the following equations:

$$\mathcal{J}_{y_n,u_{lin}} \sim \begin{cases} y_n = -T_n(\Lambda)\bar{u} \\ y_d = P S_n(\Lambda)\bar{u} \\ \bar{u} = \text{Dzn}(u_{lin} - u_d) \end{cases}$$

(6)

Figure 2 then allows us to define the following intuitive AW problem (see [12], [15] for similar problem definitions).

**Definition 1: Robust AW Control Problem.** The anti-windup compensator $\Lambda$ is said to solve the robust $\mathcal{L}_2$ AW problem for the uncertain LTI plant family $P^\Delta$ with LTI controller $G$, if $\forall P \in P^\Delta$ the following conditions are satisfied

1. The operator $\mathcal{J}_{y_n,u_{lin}}$ is well-posed and internally stable.
2. Assuming zero initial conditions for $\mathcal{J}_{y_n,u_{lin}}$, if $\text{dist}(u_{lin}, \mathcal{U}) = 0$ for the compact set $\mathcal{U} \subset \{u \in \mathbb{R} : \text{Sat}(u) = u\}$, then $y_d = 0$, $\forall t \geq 0$.
3. If $\text{dist}(u_{lin}, \mathcal{U}) \in \mathcal{L}_2$, then $y_d \in \mathcal{L}_2$.

The AW compensator $\Lambda$ is said to solve the robust AW problem with performance level $\gamma$ if, in addition to items 1, 2 and 3, the following is satisfied:

4. The operator $\mathcal{J}_{y_n,u_{lin}}$ is $\mathcal{L}_2$ stable, with finite $\mathcal{L}_2$ gain $\gamma$.

This robust AW problem basically assumes no deviation from linear performance if the control signal is not large enough to cause saturation and recovery of the linear performance level if saturation is encountered for a finite period of time. Normally the above properties are only required to hold for a given plant $P$; here we require them to be satisfied for every $P \in P^\Delta$. Under the assumption that $\text{sat}(.)$ is simply sector bounded, this robust AW problem can be solved using LMIs [13]. However, the uncertainty considered is limited to being unstructured (and perhaps additive) and the AW compensator synthesis procedure is based, in essence, on the Circle Criterion which assumes a static stability multiplier.

III. STABILITY RESULTS

Perusal of Fig. 2 and Eq. (6) reveals that the basic stability problem in AW synthesis is that of ensuring stability of the nonlinear loop. This is of the standard Lur’e form where $\mathcal{L}(s)$ represents the linear part of the system ($-T_n$ in this case) and $\phi(.)$ represents the nonlinear part of the system (the deadzone in this case). As $P \in P^\Delta$ and the closed loop are assumed to be asymptotically stable, and $\Lambda$ will be designed asymptotically stable, it follows that $P S_n \in \mathcal{RH}_\infty$ for all $P \in P^\Delta$ and hence the stability problem in Fig. 2 is purely that of ensuring stability of the nonlinear loop. In this paper we use three stability methods for studying the stability of the nonlinear loop, as described below.

A. Describing function

In the describing function method, the nonlinearity $\phi$ is replaced by its describing function, $N(j\omega, A)$ which in the case of the saturation or deadzone is simply the locus $N \in [0,1]$ [5]. Stability is then verified using linear techniques on the Nyquist or Nichols chart and, if there is an intersection between the frequency response of $\mathcal{L}(s)$ and the locus $-1/N \in [-\infty, -1]$, the system is predicted to be unstable. Note that no intersection between the frequency response of $\mathcal{L}(s)$ and the $-1/N$ locus does not necessarily imply stability; however intersection is an approximate necessary condition for instability. Hence, in this paper we take the intersection of the linear frequency response of $L_n$ (resp. $-T_n$) and the describing function for the saturation (resp. deadzone) nonlinearity as an indication of likely instability.

B. Circle Criterion

Perhaps the most common way of treating the saturation and deadzone nonlinearities in the AW literature is through sector bounds. In this case, with either $\phi = \text{sat}(.)$ or $\phi = \text{Dzn}(.)$, it follows that

$$\phi(.) \in \text{Sector}[0,1]$$

(7)
It is then well known (see [8], [5] for example) that the system will be asymptotically stable if $L(s) \in \mathcal{R}_\infty$ and the following passivity condition is satisfied

$$(L(j\omega) + 1)^* + (L'(j\omega) + 1) > 0 \quad (8)$$

which is equivalent to $L(s) + 1$ being strictly positive real (SPR). Although the Circle Criterion provides only a sufficient condition for stability, it is attractive because it normally leads to convex algorithms. It is however highly conservative for the case of saturation nonlinearities.

**C. Stability using Zames-Falb multipliers**

A similar, but significantly less conservative, method of proving stability is when $\phi(.)$ also satisfies a slope-restriction, namely

$$\phi(.) \in \text{Slope}[0,1] \quad (9)$$

In this case, it is well known [18], [8] that the system will be asymptotically stable if there exists a multiplier $M(s) = H_0 - H(s)$ where $\|H\|_1 \leq H_0$ such that the following passivity condition is satisfied

$$(L(j\omega) + 1)^* M(j\omega)^* + M(j\omega)(L'(j\omega) + 1) > 0 \quad (10)$$

or equivalently that $M(j\omega)(L'(j\omega) + 1)$ be SPR. This is one of the least conservative methods for checking stability, but it is somewhat harder to apply than the Circle Criterion due to the difficulty of searching for the stability multiplier $M(s)$. However, a tractable method, based on LMI, for finding such multipliers has recently been developed [14] and it is those results to which we will appeal here.

**IV. CLASSICAL ROBUST AW DESIGN APPROACH**

This section introduces a classical approach to designing AW compensators which approximately satisfy the AW problems defined earlier. There are two aspects to the problem: the basic (abstract) stability requirement and the performance requirement. The abstract stability requirements, for both circle multipliers and Zames-Falb multipliers, as mentioned above, can be expressed as a passivity condition. In turn this can be transformed into a simple graphical representation in the Nichols chart. This can be done in a robust manner by specifying feasibility regions for the various plants $P \in \mathcal{P}_\Delta$.

The performance requirement present in Definition 1 is more problematic for classical approaches, as it is difficult to transpose the $\mathcal{L}_2$ gain condition directly to the Nichols Chart. Rather than using the upper-bound on the gain of $\mathcal{F}_{\Delta, \text{lin}}$ as the performance measure, as is standard, the approach adopted here follows [4] in which the performance measure is taken as the linear lower bound of this map. The bound is obtained by treating the saturation nonlinearity as a linear element, with linear gain in the set $[0,1]$. Based on this linear measure of performance, a relaxed version of Definition 1 can be proposed.

**Definition 2: Relaxed Robust AW Control Problem.**

The anti-windup compensator $\Lambda$ is said to solve the relaxed robust $\mathcal{L}_2$ AW problem for the uncertain LTI plant family $\mathcal{P}_\Delta$ with LTI controller $G$, if $\forall P \in \mathcal{P}_\Delta$ conditions 1 to 3 of Definition 1 are satisfied and, in addition the following condition is satisfied:

1. The operator $\mathcal{F}_{\Delta, \text{lin}}$, defining the map from signals $u_{\text{lin}}$ to $y_{\Delta}$ in the mismatch system with $Dzn(\cdot) \in \omega$, is well-defined and stable, with $\|\mathcal{F}_{\Delta, \text{lin}}\|_{\omega} < \gamma$.

**A. Classical QFT-style Design Method**

Here the QFT-style classical design method proposed in [4] to satisfy the Relaxed Robust AW Control Problem is reviewed and expanded. In particular, it is shown how the Zames-Falb multipliers can be incorporated in order to add more flexibility to the design. The approach is based on the mismatch system depicted in Fig. 2 and ensures absolute stability of the nonlinear loop when the deadzone $Dzn(\cdot)$ is modelled using either sector bounds ($Dzn(\cdot) \in \text{Sector}[0,1]$) or slope restrictions ($Dzn(\cdot) \in \text{Slope}[0,1]$). Performance is enforced using Definition 2 where the nonlinear operator $\mathcal{F}_{\Delta, \text{lin}}$ is replaced by its linear equivalent $\mathcal{F}_{\Delta, \text{lin}}^{\text{lin}}$. Both performance and stability will be enforced in a robust manner by replacing $P \in \mathcal{P}_\Delta$ with a finite number of plants in the family at a finite number of frequencies, as is typical in the QFT design method [10].

For simplicity of presentation in the following local subsections, it is assumed that $\Lambda = 0$ and therefore that $\Lambda = [\Lambda_1^T 0]$. This limits, to some extent, the freedom in the external AW configuration (see [4] for further discussion) but ensures that the AW compensator is simply a scalar system.

**B. Stability**

As mentioned above, the stability problem in Fig. 2 reduces to ensuring stability of the nonlinear loop. As $Dzn(\cdot) \in \text{Slope}[0,1]$, it thus follows that the system is asymptotically stable if there exists a multiplier $M(s) = H_0 - H(s)$ where $\|H\|_1 \leq H_0$, such that the following transfer function is SPR:

$$(-T_n + 1)M = S_nM \quad (11)$$

Thus we have the following stability requirement

$$S_nM = \frac{M(1+\Lambda_1)}{(1-C_2P)} \quad \text{SPR} \quad \forall P \in \mathcal{P}_\Delta$$

This SPR condition amounts to a constraint on the phase of the transfer function in Eq. (12). This can be expressed as the following polynomial in the free AW controller $\Lambda_1$ and hence handled in the standard QFT approach [10]:

$$c_1 \hat{c}_3 + c_1 \hat{c}_2 + r_{\Lambda_1} e^{j\theta_{\Lambda_1}} (c_3 \hat{c}_2) + e^{-j\theta_{\Lambda_1}} (c_2 \hat{c}_3) \geq 0, \quad (13)$$

where the bar denotes complex transpose, $\Lambda = r_{\Lambda_1} e^{j\theta_{\Lambda_1}}, c_1 = c_3 = M$ and $c_2 = (1-G_2P)$. By satisfying the bounds on the frequency response of $\Lambda_1$ that result from Eq. (13), at all the design frequencies, and for all plants in the plant family $\mathcal{P}_\Delta$, the system can be rendered SPR and hence absolutely stable.

In the original approach of [4], the multiplier $M(s)$ was chosen either as a static multiplier or as a Popov multiplier, as the deadzone was simply assumed to satisfy standard sector constraints $Dzn(\cdot) \in \text{Sector}[0,1]$. In this paper, the less conservative Zames-Falb multipliers are used in order to greatly reduce conservatism and to allow simpler, lower order compensators to be used to guarantee stability. The downside of using Zames-Falb multipliers is that it is difficult to choose appropriate multipliers while simultaneously synthesising the AW compensator. For the designs presented herein, an iterative design procedure is used in which the QFT methodology of [4] is combined with the quasi-convex algorithm reported in [14].

6747
C. Performance

In [4] multiple design objectives were considered. Here it suffices to review only the objective on the map from \( u_{\text{lim}} \) to \( y_{\text{d}} \) in Fig. 2, related to the recovery of the system’s ideal linear response. As the employed lower bound on the maps is linear, the design problem amounts to a constraint on a linear closed-loop transfer function which can be enforced via standard QFT methods [10].

1) Recovery of Linear Response: Here we enforce a constraint on the map \( \mathcal{F}_{l}^{(\text{in})} \), which corresponds to \( \mathcal{F}_{l}^{(\text{in})} \) in Fig. 2, but with Dzn(\( )\) \( \in [0,1] \). This can be posed as follows:

\[
P_{1} \cdot |\mathcal{F}_{l}^{(\text{in})}(n)_{i}| < |W_{P_{1}}|, \ \forall \omega, \ \forall P \in P^A, \ \forall n_{i} \in [0,1],
\]

where

\[
\mathcal{F}_{l}^{(\text{in})}(n) = \frac{P_{n_{i}} + \Lambda_{1}P_{n_{i}}}{(1 - G_{2}P(1 - n_{i})) + \Lambda_{1}n_{i}}.
\]

D. Design Procedure

Based on the stability (S1) and performance (P1) objectives, the Relaxed Robust AW Control Problem is solved by representing the feasible region for the design of \( \Lambda_{1} \) at each frequency in the Nichols chart and loop-shaping \( \Lambda_{1} \). As all the specifications give rise to first or second order polynomials in \( \Lambda_{1} \), the generation of these bounds and the intersection of the feasible sets at each frequency can be performed using standard QFT software.

V. ROBUST ANTI-WINDUP DESIGN EXAMPLES

Here we consider a modified version of the the SISO AW problem considered in [13] as a robust AW controller design problem. The plant \( P(s) \sim (A_{P}, B_{P}, C_{P}, D_{P}) \) is described by the state-space matrices:

\[
\begin{bmatrix}
A_{P} & B_{P} \\
C_{P} & D_{P}
\end{bmatrix} =
\begin{bmatrix}
-70 & 8 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & a_{32} & a_{33} & 10
\end{bmatrix}.
\]

(15)

The plant family \( P_{A} = \{P_{1}, P_{2}, P_{3}\} \) comprises three plants, the parameters of which are given in Table I. Plants \( P_{1} \) and \( P_{2} \) are those given in [13] but with a lower damping ratio and the addition of a high frequency pole. Plant \( P_{3} \) is \( P_{2} \) with an increase in the natural frequency of the lightly damped mode. Notably, the parametric uncertainty present in this plant family is not easily represented using additive uncertainty. The feedback controller \( G(s) \sim (A_{g}, B_{gr}, B_{g}, C_{gr}, D_{gr}, D_{g}) \) is also slightly modified from that in [13] to be described by the state-space matrices:

\[
\begin{bmatrix}
A_{g} & B_{gr} & B_{g} \\
C_{gr} & D_{gr} & D_{g}
\end{bmatrix} =
\begin{bmatrix}
-0.001 & -7000 & 2500 & 250 & 0 & -100 \\
0 & -80 & 0 & 2.5 & 0 & -1 \\
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -2.5 & 1 & 0 \\
0.998 & -1 \times 10^{-4} & 5000 & 500 & 0 & -200
\end{bmatrix}.
\]

(16)

This controller is the same as in [13], except for an increase in gain and the addition of a low frequency lag element. The controller robustly stabilises all three plant cases, with reasonable gain reduction and gain and phase margins, as seen in Fig. 3 and Table I.

The interest in this example here is the robustness problem that arises from having to design an AW controller that simultaneously provides for stability and performance during periods of input magnitude saturation, with the control input saturated at \( \pm 1 \). In the proceeding sections, several AW control solutions will be considered.

1) Unconstrained Linear System: Fig. 4 shows the responses to a pulse reference of magnitude 1 and no input saturation limits enforced. The closed-loop unconstrained system of the plant \( (15) \) and controller \( (16) \) shows good response properties for all three plant cases, with the output accurately tracking the reference in all cases.

2) Constrained Uncompensated System: Figure 5 shows that the response of the same systems with the control input saturated at \( \pm 1 \); for all systems, the good reference tracking is lost. In particular, the system containing \( P_{1} \) has an oscillatory response and the system containing \( P_{2} \) becomes unstable. This stability and performance degradation is not surprising: Fig. 4 shows the unconstrained control signal often violates the \( \pm 1 \) saturation limit and the system with \( P_{2} \) is seen to satisfy the describing function condition in Fig. 3.

3) IMC AW Controller: In [13] it was shown that the IMC controller is optimally robust for unstructured additive uncertainty. For highly structured plant descriptions, it is natural to expect that IMC AW may not be appropriate and indeed, this was implied in [9]. Here we show that,
not only is the IMC AW compensator not optimally robust for structured plant uncertainty, but in fact, it can drive the system unstable when the system without AW compensation is stable. Fig.6 shows $L_n$ for all three plant cases (zoomed around the stability point), with the IMC AW compensator designed based on the nominal plant $P_2$ (ie $A_1 = 0$ and $A_2 = -P_2$). Note that $L_n = 0$ for plant case $P_2$ and is therefore not seen in Fig.6. Considering the loop transmissions in Fig.6, it is clear that this IMC AW compensator applied on plant case $P_1$ or $P_3$ is likely to perform poorly. In particular, for $P_3$ the loop transmission $L_n$ is stable but crosses twice the ray $\{k, \pm 180^\circ\}$, $k \in [1, \infty]$ and therefore satisfies the describing function condition for the saturation nonlinearity. Fig.7 shows the response of the systems. While the IMC AW compensator performs poorly for all systems, for the system with $P_3$, IMC AW leads to an unstable system while the saturated system without AW is stable. The systems with $P_1$ and $P_2$ are stable. Evidently, IMC AW is not a good choice of AW compensator here and, as could be expected given that it is only designed based on the plant $P_2$, offers no assurance of robust stability. In fact the use this IMC AW compensator for the plant $P_3$ leads to worse performance than no AW at all.

4) Robust Full Order LMI AW Controller: The robust AW problem can also be addressed using the full order robust dynamic AW compensator synthesis approach in [13]. Again $P_2$ was chosen as the nominal plant which was used to design the AW compensator, and the performance and robustness weights were chosen as $W_p = 1$ and $W_r = 100$. Fig.8 shows $L_n$ for all three plant cases with this AW compensator. The features are more favourable than the IMC case, with importantly no satisfaction of the describing function condition. Note however that the properties of the systems with $P_1$ and $P_3$ are not guaranteed by the design, and hence if they are not acceptable, only tuning of $W_p$ and $W_r$ is available as a design freedom. Fig.9 shows the response of the systems with this AW compensator, where it is seen that the systems are stable and the saturated response is much improved, with the linear response almost recovered, although the response of the system with $P_3$ is quite poor.

5) Robust QFT AW Controller: To provide AW controllers that are explicitly robust for all three plant cases, an AW controller which meets the requirements in Definition 2 for all $P \in \{P_1, P_2, P_3\}$ is designed. Note that the design procedure...
VI. CONCLUSIONS

This paper has revisited the robust AW control problem and various techniques suitable for its solution. It has been shown, through a simple numerical example, that consideration of uncertainty is vital in obtaining a good AW design. In particular the IMC AW compensator is shown to have no guaranteed robustness properties when the uncertainty is highly structured. Although an existing robust AW synthesis method [13] was found to perform adequately on this example, this was more out of luck than design, as the uncertainty considered is difficult to account for explicitly in that framework. A QFT-style robust AW design procedure was shown to be able to design explicitly for robustness to the structured plant uncertainty and highlights the subsequent advantage of guaranteed robustness of the AW controllers. The QFT-style procedure also allowed for the design to less conservative models of the saturation nonlinearity and hence less conservative multipliers than the standard circle multipliers. Zames-Falb multipliers were optimally synthesised and included in the design procedure and this was shown to reduce greatly the design conservatism, allowing in this instance robust absolute stability to be achieved when it is infeasible to design for using circle multipliers. While such classical AW design methods are limited mainly to SISO systems, they do highlight the “loss of information” present in many LMI design methods and they show what can be gained when detailed uncertainty and saturation nonlinearity models are used in the design procedure.

REFERENCES