Delay-range dependent stability analysis for T–S fuzzy systems with time-varying delay

Min Kook Song, Jin Bae Park, and Young Hoon Joo

Abstract—This paper is concerned with the stability analysis of Takagi–Sugeno (T–S) fuzzy systems with time varying delays by using novel quadratic Lyapunov-Krasovskii functionals. The novel Lyapunov-Krasovskii functionals comes from some existing ones employed in the previous results and add another part which is constructed by dividing the delay range into several segments and choosing proper functionals with free weighted matrices corresponding to different segments. Then using these delay partitioning idea, some new delay-range-dependent stability criteria are derived for T–S fuzzy systems. All the sufficient conditions are established in terms of linear matrix inequalities (LMIs), which can be solved efficiently by using the LMI algorithm. It is shown that these criteria for T–S fuzzy systems with time varying delays are always less conservative than the previous results. Two numerical examples are given to illustrate the less conservatism of the proposed method.

I. INTRODUCTION

It has been shown that the existence of time delays is often one of the main causes of instability and poor performance of a control system [3], [4]. Therefore, the problem of the stability analysis of nonlinear time-delay systems has attracted considerable attention during the last decade. There has been an increasing interest in the study of Takagi–Sugeno(T–S) fuzzy systems since that this system is a powerful tool for modeling complex nonlinear systems. It is known that, by using the T–S fuzzy model, a nonlinear system can be described as a weighted sum of some simple linear subsystems and then can be stabilized by a model-based fuzzy control. Recently, the T–S fuzzy systems with time delays have been studied [7], [8], [13]–[15], [18]–[20]. In [14], [15], the stability analysis for such T–S fuzzy systems with time delays were considered, and state feedback fuzzy controllers and fuzzy observers were designed via the so-called parallel distributed compensation (PDC) scheme. Among these references, great efforts have been focused on effective reduction of the conservation of the T–S fuzzy systems with time-delays. Many effective methods, such as new bounding technique for cross terms [11,12], augmented Lyapunov functional method [8] and free-weighing matrix method [13,14] have been proposed. In [13], Tian investigated the robust stability and stabilization of the T–S fuzzy systems by introducing slack matrices, delay-dependent conditions were obtained for the asymptotic stability of the fuzzy systems. However, some useful information was ignored when enlarging \(d(t)\) to \(d_2\), as described in [9], which can bring considerable conservatism. The time-varying delay in an interval has strong application background, which commonly exists in many practical systems. For example, it has been described in [13], [19], [20] that the lower bound of the delay in the networked control systems is often larger than zero. The investigation for the systems with interval time-varying delay has been caused considerable attention, see [18–20] and the references there in.

In this paper, we consider the problem of stability analysis for T–S fuzzy system with interval time-varying delay. By using the novel Lyapunov-Krasovskii functionals, some new delay-range-dependent stability criteria are proposed. All the sufficient conditions are established in terms of linear matrix inequalities (LMIs), which can be solved efficiently by using the LMI algorithm. We will also extend the discrete delay decomposition approach to the T–S fuzzy system. These new stability criteria will be include some existing results and be less conservative than some existing results. Two numerical examples are given to illustrate the less conservative of the proposed method.

II. PROBLEM FORMULATION AND PRELIMINARIES

In this section, a class of T–S fuzzy system with time-delay is considered, which is represented by T–S fuzzy model composed of a set of fuzzy implications each of which is expressed by a linear system model. For each \(i = 1, 2, \ldots, r\) \((r\) is the number of plant rules), the \(i\)th rule of this T–S fuzzy model is represented as:

\[
R_i: \text{IF } z_1(t) \in \Gamma_{i1} \text{ and } \cdots \text{ and } z_p(t) \in \Gamma_{ip} \text{ THEN } \dot{x}(t) = A_0x(t) + A_1x(t - d(t)) + B_1u(t)
\]

(1)

where \(x(t) \in \mathbb{R}^n\) constitutes the state vector, \(u(t) \in \mathbb{R}^m\) is the control input, \(R_i, i \in \mathcal{I}_r = \{1, 2, \ldots, r\}\), denotes the \(i\)th fuzzy rule, \(z_h(t), h \in \mathcal{I}_p = \{1, 2, \ldots, p\}\), is the \(h\)th premise variable, \(\Gamma_{ih}, (i, h) \in \mathcal{I}_r \times \mathcal{I}_p\), is the fuzzy set of \(z_h(t)\) in \(R_i\), the system matrices of the \(i\)th rule are denoted by \((A_{0i}, B_{1i}, A_{1i})\), which are assumed known and some constant matrices of compatible dimensions. The time delay is considered to be time-varying and has a lower and upper bound, \(0 < d_1 \leq d(t) \leq d_2\), which is very common in practice.

Using the center-average defuzzifier, product inference, and
singleton fuzzifier, (1) is inferred as
\[
\dot{x}(t) = \sum_{i=1}^{r} \theta_i(z(t)) (A_{0i}x(t) + A_{1i}x(t - d(t)) + B_iu(t))
\]  
(2)
where \( \theta_i(z(t)) = w_i(z(t))/\sum_{i=1}^{r} w_i(z(t)) \), \( w_i(z(t)) = \prod_{j=1}^{i-1} \mu_{A_{0j}}(z_k(t)) \), and \( \mu_{A_{0j}}(z_k(t)) \) is the membership function of \( z_k(t) \) on the compact set \( U_{z_k} \).

Some basic properties are \( \theta_i(z(t)) \geq 0 \) and \( \sum_{i=1}^{r} \theta_i(z(t)) = 1 \). The initial condition of system (1) is given by
\[
x(t) = \varphi(t), \quad \forall t \in [-d_2, -d_1]
\]  
(3)
for all \( \varphi(-) \) a continuous vector-valued initial function.

In this paper, the problem under consideration is to analysis stability of the fuzzy systems with time-varying delays such that fuzzy systems (1) is asymptotically stable.

In this paper, we will attempt to formulate some practically computable criteria to check the asymptotically stability of the system (1). The following lemma is useful in deriving criteria.

**Lemma 1:** For any constant matrix \( W \in \mathbb{R}^{n \times n} \), \( W = W^T \succeq 0 \), scalar \( d_1, d_2 > 0 \), and vector function \( \varphi : [-d_2, -d_1] \to \mathbb{R}^n \) such that the following integration is well defined, then
\[
-(d_2 - d_1) \int_{t-d_1}^{t-d_2} \dot{x}(s)W \dot{x}(s)ds \leq \begin{bmatrix} x^T(t-d_1) & x^T(t-d_2) \end{bmatrix} \begin{bmatrix} -W & W \\ W & -W \end{bmatrix} \begin{bmatrix} x(t-d_1) \\ x(t-d_2) \end{bmatrix}
\]  
(4)

### III. Stability Analysis

In this section, we will give some sufficient conditions for the solvability of the stability and the stabilization problem formulated in the previous section. The following theorem presents a delay-range dependent results in terms of LMIs. and gives the new stability criterion for system (1), with \( u(t) = 0 \), which is dependent not only on the delay upper bound \( d_2 \), but also on the delay range \( d := d_2 - d_1 \).

**Theorem 1:** Consider the unforced fuzzy neutral system (\( u(t) = 0 \)). The corresponding system is stable if there exist matrices \( P > 0, Q > 0, Q_1 > 0, Q_2 > 0, Q_3 > 0, R_1 > 0, R_2 > 0, W_{12} > 0, W_{22} > 0 \) \( M_{ei}, N_{ei} \), \( e = 1, 2, 3, 4, 5, 6, 7, i \in \mathcal{I}_R \) such that the following LMIs hold for each \( i \in \mathcal{I}_R \)
\[
\Gamma_i = \begin{bmatrix} \Gamma_{11i} & \cdots & \Gamma_{12i} \\ \cdots & \cdots & \cdots \\ \Gamma_{12i} & \cdots & \Gamma_{22i} \end{bmatrix} \prec 0
\]  
(5)
where
\[
\Gamma_{11i} = \begin{bmatrix} \Gamma_{11i} & \cdots & \Gamma_{12i} \\ \cdots & \cdots & \cdots \\ \Gamma_{12i} & \cdots & \Gamma_{22i} \end{bmatrix} = \begin{bmatrix} [d_2 M_{1i} & d_2 N_{1i} \\ d_2 M_{2i} & d_2 N_{2i} \\ d_2 M_{3i} & d_2 N_{3i} \\ d_2 M_{4i} & d_2 N_{4i} \\ d_2 M_{5i} & d_2 N_{5i} \\ d_2 M_{6i} & d_2 N_{6i} \\ d_2 M_{7i} & d_2 N_{7i} \end{bmatrix}
\]
(6)

**Proof:** For this system, we define the following Lyapunov functional candidate:
\[
V(t) = V_1(t) + V_2(t) + V_3(t) + V_4(t)
\]  
(6)
where
\[
V_4(t) = x^T(t)Px(t)
\]
\[ V_2(t) = \int_{t-d(t)}^{t} x^T(s)Q_1x(s)ds + \int_{t-d_1}^{t} x^T(s)Q_2x(s)ds + \int_{t-d_2}^{t} x^T(s)Q_3x(s)ds \]
\[ V_3(t) = \int_{t-d(t)}^{t} x^T(s)Q_1x(s)ds + \int_{t-d_1}^{t} x^T(s)Q_2x(s)ds + \int_{t-d_2}^{t} x^T(s)Q_3x(s)ds \]
\[ \dot{V}_3(t) = x^T(t-d_1)W_{11}x(t-d_1) - x^T(t-d_1)\frac{dx}{dt}W_{12}x(t-d_1) - x^T(t-d_2)W_{22}x(t-d_2) + 2x^T(t-d_1)\frac{dx}{dt}W_{12}x(t-d_2) - 2x^T(t-d_1)\frac{dx}{dt}W_{22}x(t-d_2) + \frac{dx}{dt}^T(t-d_1)Q_1\frac{dx}{dt}x(t-d_1) - \frac{dx}{dt}^T(t-d_2)Q_2\frac{dx}{dt}x(t-d_2) \]
where

\[ P > 0, Q_1 > 0, Q_2 > 0, Q_3 > 0, \]
\[ W = \begin{bmatrix} W_{11} & W_{12} \\ W_{12} & W_{22} \end{bmatrix} > 0, \quad Q > 0, R_1 > 0, R_2 > 0 \]
and
\[ y^T(s) = \begin{bmatrix} x^T(s) & x^T(s - \frac{dx}{dt}) \end{bmatrix} \]

The idea of constructing the \( V_3(t) \) in (6) is that first, we keep all the previous term; second, we uniformly divide the time varying delay interval \([-d_2, d_1]\) into two subintervals \([-d_2, -d_1 + \frac{dx}{dt}\)] and \([-d_1 - \frac{dx}{dt}, d_1]\), and then on the subintervals we choose different functionals. For the first term in \( V_3(t) \), notice that
\[ \int_{t-d_1}^{t} x^T(s)Q_1x(s)ds \]
\[ = \int_{t-d_1}^{t-d_1 - \frac{dx}{dt}} x^T(s)Q_1x(s)ds + \int_{t-d_1 - \frac{dx}{dt}}^{t} x^T(s)Q_1x(s)ds \]
\[ \int_{t-d_1}^{t-d_1 - \frac{dx}{dt}} x^T(s)Q_1x(s)ds \]
\[ = \int_{t-d_1}^{t-d_1 - \frac{dx}{dt}} x^T(s)Q_1x(s)ds + \int_{t-d_1 - \frac{dx}{dt}}^{t-d_1} x^T(s)Q_1x(s)ds + 2\int_{t-d_1 - \frac{dx}{dt}}^{t} x^T(s)Q_1x(s)ds \]

Then, the time derivative of \( V(t) \) along the system (1) with \( u(t) = 0 \) is given by
\[ \dot{V}(t) = \dot{V}_1(t) + \dot{V}_2(t) + \dot{V}_3(t) + \dot{V}_4(t) \]

where
\[ \dot{V}_1(t) = 2x^T(t)P \dot{x}(t) \]
\[ \dot{V}_2(t) = x^T(t) (Q_1 + Q_2 + Q_3) x(t) - (1 - d(t))x^T(t-d(t))Q_1x(t-d(t)) - x^T(t-d_1)Q_2x(t-d_1) \]
\[ \dot{V}_3(t) = x^T(t-d_1)W_{11}x(t-d_1) - x^T(t-d_1)\frac{dx}{dt}W_{12}x(t-d_1) - x^T(t-d_2)W_{22}x(t-d_2) + 2x^T(t-d_1)\frac{dx}{dt}W_{12}x(t-d_2) - 2x^T(t-d_1)\frac{dx}{dt}W_{22}x(t-d_2) + \frac{dx}{dt}^T(t-d_1)Q_1\frac{dx}{dt}x(t-d_1) - \frac{dx}{dt}^T(t-d_2)Q_2\frac{dx}{dt}x(t-d_2) \]

On the other hand, the following equations are also true:
\[ -\int_{t-d_2}^{t} \dot{x}^T(s)R_1\dot{x}(s)ds = -\int_{t-d_2}^{t} \dot{x}^T(s)R_1\dot{x}(s)ds \]
\[ \dot{V}_4(t) = d_2 \dot{x}^T(t)R_1\dot{x}(t) - \int_{t-d_2}^{t} \dot{x}^T(s)R_1\dot{x}(s)ds \]
\[ + (d_2 - d_1) \dot{x}^T(t)R_2\dot{x}(t) - \int_{t-d_2}^{t} \dot{x}^T(s)R_2\dot{x}(s)ds \]

In the sequel, the Leibnitz-Newton formula is employed to obtain a delay-range dependent condition. The following equations are true for any matrices \( M_i \) and \( N_i \), \( i = 1, \ldots, r \) with appropriate dimensions
\[ 2\zeta^T(t)M_i \left[ x(t) - x(t-d(t)) - \int_{t-d(t)}^{t} \dot{x}(s)ds \right] = 0 \]
\[ 2\zeta^T(t)N_i \left[ x(t-d(t)) - x(t-d_2) - \int_{t-d_2}^{t-d(t)} \dot{x}(s)ds \right] = 0 \]

where
\[ \zeta^T(t) = \begin{bmatrix} x^T(t) \\ x^T(t-d(t)) \\ x^T(t-d_1) \\ x^T(t-d_2) \\ \dot{x}^T(t) \\ x^T(t-d_1) \end{bmatrix} \]

On the other hand, the following equations are also true:
\[ -\int_{t-d_2}^{t} \dot{x}^T(s)R_1\dot{x}(s)ds = -\int_{t-d_2}^{t} \dot{x}^T(s)R_1\dot{x}(s)ds \]
\[ -\int_{t-d_2}^{t-d(t)} \dot{x}^T(s)R_1\dot{x}(s)ds \]
\[
- \int_{t-d_1}^{t-d_2} \dot{x}(s) R_1 \dot{x}(s) ds = - \int_{t-d_2}^{t-d_1} \ddot{x}(s) R_1 \dot{x}(s) ds
- \int_{t-d_1}^{t-d_2} \dot{x}(s) R_1 \dot{x}(s) ds
\]

Adding the terms on the left side of (8)-(12) and using (13)-(14) yield
\[
\dot{V}(t) = \dot{V}_1(t) + \dot{V}_2(t) + \dot{V}_3(t) + \dot{V}_4(t)
= 2x^T(t) P \dot{x}(t) + x^T(t) (Q_1 + Q_2 + Q_3) x(t)
- (1 - \dot{d}(t)) x^T(t) (t - d(t)) Q_1 x(t - d(t))
- x^T(t - d_1) Q_2 x(t - d_1) - x^T(t - d_2)
\times Q_3 x(t - d_2) + x^T(t - d_1) W_{11} x(t - d_1)
- x^T(t - d_1 - \frac{d_r}{2}) W_{11} x(t - d_1 - \frac{d_r}{2})
+ x^T(t - d_1 - \frac{d_r}{2}) W_{22} x(t - d_1 - \frac{d_r}{2})
- x^T(t - d_2) W_{22} x(t - d_2)
+ 2x^T(t - d_1) W_{12} x(t - d_1 - \frac{d_r}{2})
- 2x^T(t - d_1 - \frac{d_r}{2}) W_{12} x(t - d_2)
+ \dot{x}^T(t) \left( \frac{d_r}{2} \right)^2 Q \dot{x}(t)
- \int_{t-d_2}^{t-d_1} \dot{x}^T(s + d_1) \left( \frac{d_r}{2} Q \right) \dot{x}(s + d_1) ds
+ d_2 \dot{x}^T(t) R_1 \dot{x}(t) - \int_{t-d_2}^{t} \dot{x}^T(s) R_1 \dot{x}(s) ds
+ (d_2 - d_1) \dot{x}^T(t) R_2 \dot{x}(t) - \int_{t-d_2}^{t-d_1} \dot{x}^T(s) R_2 \dot{x}(s) ds
+ 2\zeta(t) M_1 \left[ x(t) - x(t - d(t)) - \int_{t-d_1}^{t} \dot{x}(s) ds \right]
+ 2\zeta(t) N_1 \left[ x(t - d(t)) - x(t - d_2) \right]
- \int_{t-d_2}^{t-d_1} \dot{x}(s) ds
\]

Using the Lemma 1, we obtain
\[
\dot{V}(t) \leq \sum_{i=1}^{r} \theta_i(z(t)) \left( \zeta^T(t) \left[ \Gamma + d_2 M R_1^T M \right. \right.
+ (d_2 - d_1) N (R_1 + R_2)^{-1} N^T \zeta(t) \left. \right) \right.
\times [M \zeta(t) + \dot{x}^T(s) R_1] R_1^{-1}
- \int_{t-d_1}^{t} \left[ \zeta^T(t) M + \dot{x}^T(s) R_1 \right] R_1^{-1}
\times [N \zeta(t) + \dot{x}(s) ds]
\times (R_1 + R_2)^{-1} \left[ N^T \zeta(t) + (R_1 + R_2) \dot{x}(s) ds \right]
\]

where
\[
\Gamma = \begin{bmatrix}
\Gamma_{11} & \Gamma_{12} & \Gamma_{13} & \Gamma_{14} & \Gamma_{15} & \Gamma_{16} & \Gamma_{17} \\
* & \Gamma_{22} & \Gamma_{23} & \Gamma_{24} & \Gamma_{25} & \Gamma_{26} & \Gamma_{27} \\
* & * & \Gamma_{33} & \Gamma_{34} & \Gamma_{35} & \Gamma_{36} & \Gamma_{37} \\
* & * & * & \Gamma_{44} & \Gamma_{45} & \Gamma_{46} & \Gamma_{47} \\
* & * & * & * & \Gamma_{55} & \Gamma_{56} & \Gamma_{57} \\
* & * & * & * & * & \Gamma_{66} & \Gamma_{67} \\
* & * & * & * & * & * & \Gamma_{77}
\end{bmatrix}
\]
Since \( R_1 > 0, R_2 > 0 \), then the last two parts in (20) are all less than 0. So, if

\[
\sum_{i=1}^{c} \theta_i(z(t)) \left( \Gamma + d_2 M_i R_i^T M_i^T \right) + (d_2 - d_1) N_i (R_1 + R_2)^{-1} N_i^T < 0,
\]

which is equivalent to (5) by Schur complements, then \( \| V(x(t)) \| < - \epsilon \| x(t) \|^2 \) for a sufficiently small \( \epsilon > 0 \), which ensures the asymptotic stability of system (1). This completes the proof.

To illustrate the relation between the Theorem 1 and previous results in [19], we present the following result.

**Theorem 2:** Consider the system described by (1) and (3). For a given scalar \( d_1 > 0, d_2 > 0 \), if there exist real matrices \( P \geq 0, Q_1 > 0, Q_2 > 0, Q_3 > 0, R_1 > 0, \) and \( R_2 > 0 \) such that \( \Xi_{ii} \leq 0 \), then there exist real matrices \( P > 0, Q_1 > 0, Q_2 > 0, Q_3 > 0, Q \geq 0, R_1 > 0, \) and \( R_2 > 0 \), and \( W_{11} = W_{11}^T, W_{12}, W_{22} = W_{22}^T \) such that \( W \geq 0 \) and \( \Gamma_{ii} < 0 \).

**Proof:** if there exist real matrices \( P > 0, Q_1 > 0, Q_2 > 0, Q_3 > 0, R_1 > 0, \) and \( R_2 > 0 \) such that \( \Xi_{ii} < 0 \), then there exists a sufficiently small scalar \( \epsilon_1 > 0 \) such that

\[
\Xi_{ii} + \text{diag} \{ \epsilon_1 I, 0, 0 \} < 0.
\]

Choosing \( \epsilon_2 \) such that \( 0 < \epsilon_2 < \epsilon_1 \), one can see the following is true

\[
\Xi_{ii} + \text{diag} \{ \epsilon_1 I, 0, 0 \} + \text{diag} \{ 0, -\epsilon_2 I, 0 \} < 0
\]

from which we have

\[
\begin{bmatrix}
\Xi_{ii} + \epsilon_1 I & * & * \\
0 & \epsilon_2 I - \epsilon_1 I & * \\
\Xi_{ii} & 0 & -\epsilon_2 I + \Xi_{22}
\end{bmatrix}
\]

Similarly, one can also choose a sufficiently small scalar \( \epsilon_3 > 0 \) such that

\[
\begin{bmatrix}
\Xi_{ii} + \epsilon_1 I & * & * \\
0 & \epsilon_2 I - \epsilon_1 I & * \\
\Xi_{ii} & 0 & -\epsilon_2 I + \Xi_{22}
\end{bmatrix}
\]

\[
\begin{bmatrix}
d_1^2 \epsilon_2 A_{0i}^T A_{0i} & 0 & 0 & \frac{d_2^2}{2} \epsilon_3 A_{0i}^T A_{0i} \\
0 & \epsilon_2 I - \epsilon_1 I & * & 0 \\
0 & * & 0 & \frac{d_2^2}{2} \epsilon_3 A_{0i}^T A_{0i} \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
-\epsilon_3 I & \epsilon_3 I & 0 \\
* & -\epsilon_3 I & 0 \\
* & * & 0
\end{bmatrix} \leq 0
\]

Then, we have

\[
\begin{bmatrix}
\Xi_{ii} + \epsilon_1 I & * & * \\
0 & \epsilon_2 I - \epsilon_1 I & * \\
\Xi_{ii} & 0 & -\epsilon_2 I + \Xi_{22}
\end{bmatrix}
\]

\[
\begin{bmatrix}
\frac{d_1^2}{4} \epsilon_2 A_{0i}^T A_{0i} & 0 & 0 & \frac{d_2^2}{2} \epsilon_3 A_{0i}^T A_{0i} \\
0 & \epsilon_2 I - \epsilon_1 I & * & 0 \\
0 & * & 0 & \frac{d_2^2}{2} \epsilon_3 A_{0i}^T A_{0i} \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
-\epsilon_3 I & \epsilon_3 I & 0 \\
* & -\epsilon_3 I & 0 \\
* & * & 0
\end{bmatrix} \leq 0
\]

from which, in view of Schur complement, we deduce that \( W \geq 0 \), and \( \Gamma_{ii} < 0 \) by setting \( W_{11} = \epsilon_1 I, W_{12} = 0, W_{22} = \epsilon_2 I \) and \( Q = \epsilon_3 I \).

IV. NUMERICAL EXAMPLES

**A. Example 1.**

Consider system (1) with

\[
A_1 = \begin{bmatrix}
-2 & 0 \\
0 & -0.9
\end{bmatrix}, \quad A_2 = \begin{bmatrix}
-1.5 & 1 \\
0 & -0.75
\end{bmatrix},
\]

\[
A_{d1} = \begin{bmatrix}
-1 & 0 \\
-1 & -1
\end{bmatrix}, \quad A_{d2} = \begin{bmatrix}
-1 & 0 \\
1 & -0.85
\end{bmatrix}
\]

When considering the time-varying delay case, Table 1 lists the maximum allowed time-delay \( d_2 \) for asymptotically stability. Table 1 shows the comparison results between [13], [19] and Theorem 1 in this paper. From Table 1, it can be found that by using the method in this paper, we can find that the proposed method can reduce the conservativeness of the result for T-S fuzzy system with fast interval time-varying delay when compared with [13], [19]. The merit of the proposed method becomes more obviously with the increase of \( d_r \).

**TABLE I**

<table>
<thead>
<tr>
<th>Maximum allowed time-delay ( d_2 ) using criteria in [13], [19] and this paper</th>
</tr>
</thead>
<tbody>
<tr>
<td>( d_2 )</td>
</tr>
<tr>
<td>[13]</td>
</tr>
<tr>
<td>[19]</td>
</tr>
<tr>
<td>Our results</td>
</tr>
</tbody>
</table>

**B. Example 2.**

Consider the following system with time-varying delay:

\[
A_1 = \begin{bmatrix}
-3 & 1 \\
1 & -1
\end{bmatrix}, \quad A_2 = \begin{bmatrix}
-2 & 1 \\
1 & 0
\end{bmatrix},
\]

\[
A_{d1} = \begin{bmatrix}
0.1 & 0 \\
0.2 & -0.5
\end{bmatrix}, A_{d2} = \begin{bmatrix}
0.1 & 0.2 \\
0 & -0.5
\end{bmatrix}
\]

When considering the time-varying delay case, Table 2 lists the maximum allowed time-delay \( d_2 \) for asymptotically stability. Table 1 shows the comparison results between [20], [19] and Theorem 1 in this paper. From Table 2, it can be found that by using the method in this paper, we can find that the proposed method can reduce the conservativeness of the result for T-S fuzzy system with fast interval time-varying delay when compared with [20], [19].
TABLE II
MAXIMUM ALLOWED TIME-DELAY $d_2$ USING CRITERIA IN [20, [19] AND THIS PAPER

<table>
<thead>
<tr>
<th>$d_1$</th>
<th>0</th>
<th>0.3</th>
<th>0.6</th>
<th>0.9</th>
<th>1.2</th>
<th>1.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>[19]</td>
<td>1.734</td>
<td>1.827</td>
<td>1.938</td>
<td>2.118</td>
<td>2.299</td>
<td>2.490</td>
</tr>
<tr>
<td>Our results</td>
<td>1.874</td>
<td>1.977</td>
<td>2.187</td>
<td>2.348</td>
<td>2.547</td>
<td>2.777</td>
</tr>
</tbody>
</table>

V. CONCLUSIONS

In this paper, an appropriate type of of Lyapunov-Krasovskii functionals method has been constructed to study the stability analysis for $T$–$S$ fuzzy systems with time-varying delay in a range. A novel method for stability analysis has been provided by introducing some free-weighting matrices and employing the range of the time-varying delays. These conditions are expressed in terms of LMIs. By comparing the proposed results with the previous ones through two numerical examples, it is shown that the derived criteria are less conservative than some previous ones. The numerical results seem to suggest that the proposed methods may improve the results in some existing papers.

REFERENCES