Lyapunov-based Control in Microstepping with a Nonlinear Observer for Permanent Magnet Stepper Motors

Wonhee Kim, Induk Choi and Chung Choo Chung†

Abstract—In this paper, we propose a Lyapunov-based control with a nonlinear observer in microstepping for permanent magnet stepper motors (PMSMs). Given static inputs to a PMSM, stability of the equilibrium points is studied in the sense of Lyapunov stability and their local asymptotic stability is proved using LaSalle’s theorem. From the stability analysis the principle of microstepping is verified. The Lyapunov-based controller was developed to regulate the desired phase current errors. For the implementation of the proposed controller a nonlinear observer is also designed using the Lyapunov method to estimate the full state of PMSM. And we analyze the stability of the closed-loop system. The Lyapunov-based controller and nonlinear observer do not require any transformation so that it could reduce the commutation delay. The proposed method is validated via experiments and its tracking performance was compared with that of the conventional microstepping. We used two position profiles to evaluate the performance of the proposed method. Overall more than a 40% improvement in tracking error was obtained. Furthermore, we achieved improved uniformity in the tracking error during the constant velocity period.

I. INTRODUCTION

Permanent magnet stepper motors (PMSMs) have been used in positioning applications due to their durability, high efficiency and power density, as well as high torque to inertia ratio and absence of rotor winding [1], [2]. However, PMSMs have specific problem when used for precise positioning [3]. Standard PMSMs have a relatively large step sizes, usually 1° of a revolution or 1.8°. Such large step size can cause motor-shaft oscillations at a low speed [4]. Microstepping was invented in 1974 by Durkos for improved resolution and much increased motion stability [5]. Microstepping for two phase stepper motors is the control method in which two sinusoidal inputs shifted 90 degrees are given to the PMSM. Microstepping allows a stepping motor to stop and hold a position between the full and half-step positions. Furthermore, it largely eliminates the jerky character of low speed stepping motor operation and noise at intermediate speeds. In short, it reduces problems with resonance. Although some microstepping controllers offer hundreds of intermediate positions between steps, microstepping does not generally offer great precision, because of both linearity problems and the effects of static friction. However, gearing and its associated backlash problems can be reduced with an improved resolution [4]. These problems of motor dynamics are also minimized by the application of microstepping [6]. Conventional microstepping is open loop control and uses the motor driver, which has proportional and integral (PI) current feedback loops [5], [7], [8].

To improve the position control of PMSM, various feedback control methods have been studied [9], [10], [11], [12]. In [9], a feedback linearization algorithm and field weakening were used to control PMSM. In [10], a nonlinear adaptive control was designed that guaranteed asymptotic tracking of a desired position reference signal when there are unknown parameters. The state feedback position tracking control was proposed for uncertain current-fed PMSMs with non-sinusoidal flux distribution [11]. In [12], the authors proposed a third-order sliding-mode control with which a desired position is accurately tracked. A standard technique known as direct-quadrature (DQ) transformation has been used for commutation in these feedback control of PMSMs, then mechanical dynamics of the motor are linear. However, the commutation is not exact due to time delay of the current measurements and position encoder feedback. The commutation delay causes striking qualitative changes in the behavior of the motor [13], [14]. All of these methods need the full state information of the PMSM. Position can be generally obtained by an additional encoder, and velocity is estimated by observer. Several velocity observer design methods have been proposed [9], [12], [15]. A reduced order observer was proposed to estimate the velocity by phase currents and position in [9]. In [12], a sliding mode observer was studied to estimate the velocity using only position feedback. Chiasson and Novotnak proposed a nonlinear velocity observer that uses phase currents and position [15]. These methods need state or output transformation in order to obtain linear error dynamics. Normally, the phase currents are measured using a motor driver. However, since high frequency noises affect current measurements [9], the estimations of the phase currents are needed.

Recently Lyapunov-based control in microstepping has been proposed [16]. In the paper, it was shown that, given the static voltage inputs of the phases A and B, the local asymptotic stability of equilibrium points was analyzed using LaSalle’s Theorem. From the analysis, the principle of microstepping was derived in the sense of the Lyapunov method. In addition, for position tracking, the desired current error between the desired voltage input and
current output of each phase was defined. The Lyapunov-based control with full state information guarantees that the desired phase current error converges to zero. Since the proposed nonlinear control method needs the full state information of a PMSM, in this paper, we design a nonlinear observer based on the position measurement and validate its performance experimentally. The nonlinear observer is also designed based on Lyapunov method. The stability of the closed-loop system is studied using a composite Lyapunov function. Since both proposed controller and observer do not require any transformation so that it could reduce the commutation delay. Experiments were executed to evaluate the performance of the proposed method. The algorithm was coded in C Language using a digital signal processor with 20 kHz sampling rate. For position feedback, an incremental optical encoder with 8000 lines/rev was used with quadrature signals. Experimental results showed improvement in position tracking of the proposed method over those of the conventional microstepping. We used two position profiles to evaluate the performance of the proposed method: a sinusoidal profile and a profile for 16 rad move. Overall more than a 40% improvement in tracking error was obtained. Furthermore, improved uniformity in tracking error during constant velocity period was observed.

II. MATHEMATICAL ANALYSIS OF EQUILIBRIUM POINTS

A. Mathematical Model of Permanent Magnet Stepper Motor

A PMSM consists of a slotted stator with two phases and a permanent magnet rotor which has north and south poles. As shown Fig. 1. Detailed descriptions of the operation of PMSMs are given in [17], [18], [19]. The dynamics of a can be represented in the state-space form \( \dot{x} = f(x, u) \) as follows [9], [18], [19]:

\[
\begin{align*}
\dot{\theta} &= \omega \\
\dot{\omega} &= -K_m i_a \sin(N_r \theta) + K_m i_b \cos(N_r \theta) - B \omega / J \\
i_a &= \frac{\nu_a - R i_a + K_m \omega \sin(N_r \theta)}{L} \\
i_b &= \frac{\nu_b - R i_b - K_m \omega \cos(N_r \theta)}{L}
\end{align*}
\]

where \( x = [\theta, \omega, i_a, i_b]^T \) is the state, and \( u = [\nu_a, \nu_b]^T \) is the input. \( \nu_a, \nu_b \) and \( i_a, i_b \) are the voltages [V] and currents [A] in phases A and B, respectively. \( \omega \) is the rotor (angular) velocity [rad/s], \( \theta \) is the rotor (angular) position [rad], \( B \) is the viscous friction coefficient \([N \cdot m \cdot s/rad]\), \( J \) is the inertia of the motor \([Kg \cdot m^2]\), \( K_m \) is the motor torque constant \([N \cdot m/A]\), \( R \) is the resistance of phase winding \([\Omega]\), \( L \) is the inductance of phase winding \([H]\), and \( N_r \) is the number of rotor teeth, respectively. The detent torque and the magnetic coupling between the phases are neglected in this model. The model also neglects the variation in inductance due to magnetic saturation. In addition, an ideal sinusoidal flux distribution was assumed.

B. Analysis of Equilibrium Points

Equation (1) has two voltage inputs in phases A and B. In full stepping mode, clockwise rotation may be obtained by exciting the phases in the sequence A+, B+, A-, B-, A+, ... as table I [17]. To increase position resolution, different voltage inputs were given to both phase A and B. Since equilibrium points depend on the inputs, we divided the equilibrium points into four cases: 1) \( \nu_a = 0 \) and \( \nu_b = 0 \), 2) \( \nu_a = u_a \) and \( \nu_b = u_b \), 3) \( \nu_a = 0 \) and \( \nu_b = u_b \), and 4) \( \nu_a = u_a \) and \( \nu_b = 0 \). Here \( u_a \) and \( u_b \) are nonzero scalar inputs in phases A and B. In this paper, we analyzed case 1 and the general case 4 since the analyses of cases 2 and 3 are similar to that of case 4. For detailed analysis of cases 2 and 3, we refer the reader to [16].

1) \( \nu_a = 0 \) and \( \nu_b = 0 \): We assumed that zero inputs were given to phase A and B respectively. This is the case that \( \nu_a \) and \( \nu_b \) are 0 in the model, thus (1) becomes

\[
\begin{align*}
\dot{\theta} &= \omega \\
\dot{\omega} &= -K_m i_a \sin(N_r \theta) - B \omega / J \\
i_a &= \frac{\nu_a - R i_a + K_m \omega \sin(N_r \theta)}{L} \\
i_b &= \frac{\nu_b - R i_b - K_m \omega \cos(N_r \theta)}{L}
\end{align*}
\]

The equilibrium points of (2) are as follows:

\[
[\theta, \omega, i_a, i_b] = [0, 0, 0, 0], \quad \forall \theta_0 \in \mathbb{R}^+.
\]

When zero inputs are given, there are infinite number of equilibria. For the stability analysis of the equilibrium points, Jacobian is

\[
\frac{\partial f}{\partial x} \bigg|_{[\theta, \omega, i_a, i_b] = [0, 0, 0, 0]} = \begin{bmatrix}
0 & 1 & 0 & 0 \\
0 & -\frac{\partial}{\partial \omega} & -\frac{\partial}{\partial i_a} S_0 & 0 \\
0 & -\frac{\partial}{\partial \omega} S_0 & 0 & 0 \\
0 & -\frac{\partial}{\partial \omega} C_0 & 0 & 0
\end{bmatrix}
\]

where \( C_0 = \cos(N_r \theta_0) \) and \( S_0 = \sin(N_r \theta_0) \), respectively. Since the Jacobian (4) has one zero eigenvalue, the equilibrium point (3) is locally stable.
2) \( v_a = u_a \) and \( v_b = u_b \): In this case, two static voltage inputs \( u_a \) and \( u_b \) are given to phase A and B. Therefore, the model (1) is given by

\[
\begin{align*}
\dot{\theta} &= \omega \\
\dot{\omega} &= \frac{-K_m i_a \sin(N\theta) + K_m i_b \cos(N\theta) - B\omega}{J} \\
i_a &= \frac{u_a - R i_a + K_m \omega \sin(N\theta)}{L} \\
i_b &= \frac{u_b - R i_b - K_m \omega \cos(N\theta)}{L}.
\end{align*}
\]

Letting \( \tilde{i}_a = i_a - \frac{u_a}{R} \), \( \tilde{i}_b = i_b - \frac{u_b}{R} \), we see that the model (5) may be rewritten as

\[
\begin{align*}
\dot{\theta} &= \omega \\
\dot{\omega} &= \frac{-K_m (\tilde{i}_a + \frac{u_a}{R}) \sin(N\theta) + K_m (\tilde{i}_b + \frac{u_b}{R}) \cos(N\theta) - B\omega}{J} \\
\dot{i}_a &= \frac{-R \tilde{i}_a + K_m \omega \sin(N\theta)}{L} \\
\dot{i}_b &= \frac{-R \tilde{i}_b - K_m \omega \cos(N\theta)}{L}.
\end{align*}
\]

In this case, (6) has the equilibrium points

\[
[\theta, \omega, \tilde{i}_a, \tilde{i}_b] = [\theta_{ab}, 0, 0, 0], \quad \sin(N\theta_{ab}) u_a = \cos(N\theta_{ab}) u_b.
\]

To analyze the stability of the equilibrium points (7), we consider a continuously differentiable function \( V_{ab} \) such that

\[
V_{ab} = \frac{1}{2}(\dot{\theta}^2 + \dot{\omega}^2) + \frac{K_m}{N^2 \omega^2} (2\text{sgn}(u_a - \cos(N\theta)) - (\sin(N\theta))) \geq 0.
\]

Its time derivative along the trajectory (6) is given by

\[
\dot{V}_{ab} = -\frac{R}{2} \omega^2 - \frac{R}{2} (\dot{i}_a^2 + \dot{i}_b^2).
\]

Since \( \dot{V}_{ab} \leq 0 \), by LaSalle’s Theorem we can establish that the equilibrium points \([\theta, \omega, \tilde{i}_a, \tilde{i}_b] = [\theta_{ab}, 0, 0, 0]\) are locally asymptotically stable.

In summary, if static inputs \( u_a : u_b = \cos(Nr\theta^*) : \sin(Nr\theta^*) \) are given to each phase, the position of the rotor goes asymptotically to \( \theta^* \). Therefore, to move the position of the rotor to the desired position \( \theta^d \), the desired inputs are defined as

\[
v_a^d = V_{\text{max}} \cos(N\theta^d), \quad v_b^d = V_{\text{max}} \sin(N\theta^d)
\]

where \( \theta^d \) is the desired position and \( V_{\text{max}} \) is the maximum voltage which can be accepted by the PMSM respectively. These inputs (10) are equivalent to the inputs of conventional microstepping [7].

III. CONTROLLER AND OBSERVER DESIGN

A. Lyapunov Based Controller

In the previous section, given \( v_a^d \) and \( v_b^d \) in phase A and B, we showed that the state of PMSM (1) locally converges to an equilibrium point \( x_e = [\theta^d, 0, v_a^d/R, v_b^d/R] \), i.e.

\[
\begin{align*}
\lim_{t \to \infty} \theta(t) &= \theta^d \\
\lim_{t \to \infty} \omega(t) &= 0 \\
\lim_{t \to \infty} i_a(t) &= \frac{v_a^d}{R} \\
\lim_{t \to \infty} i_b(t) &= \frac{v_b^d}{R}.
\end{align*}
\]

Given sufficiently small \( r \geq 0 \), since \( f(x,u) \) in (1) is locally Lipschitz in \( x \), \( \forall x \in W = \{ x \in \mathbb{R}^d \mid |\omega| \leq \omega_{\text{max}} \} \), where \( x_e \) is an equilibrium point, the dynamics of the PMSM have an unique solution [20]. Therefore, if \( i_a(t) \) and \( i_b(t) \) converge to \( \frac{v_a^d}{R} \) and \( \frac{v_b^d}{R} \), respectively, then the position of the rotor \( \theta(t) \) converges to the desired position \( \theta^d \). Therefore, if the relationships (12)

\[
\begin{align*}
i_a &= \frac{v_a^d}{R} \\
i_b &= \frac{v_b^d}{R},
\end{align*}
\]

are satisfied, then the position of the rotor moves to the desired position. Now, we design a Lyapunov-based feedback controller satisfying the relationships in (12). Let us define the phase desired current errors as

\[
\begin{align*}
e_a &= v_a^d - R i_a \\
e_b &= v_b^d - R i_b,
\end{align*}
\]

and the Lyapunov candidate function, \( V_1 \) as

\[
V_1 = \frac{1}{2} e_a^2 + \frac{1}{2} e_b^2.
\]

From (1) and (25), differentiating \( V_1 \) with respect to time yields

\[
\begin{align*}
\dot{V}_1 &= e_a (v_a^d - R i_a) + e_b (v_b^d - R i_b) \\
&= e_a (v_a^d - R i_a + K_m \omega \sin(N\theta)) \\
&\quad + e_b (v_b^d - R i_b - K_m \omega \cos(N\theta)).
\end{align*}
\]

To make (26) negative definite, we design a control law for the inputs \( v_a \) and \( v_b \) such as

\[
\begin{align*}
v_a &= (R i_a - K_m \omega \sin(N\theta)) + \frac{1}{R} (v_a^d + \rho e_a) \\
v_b &= (R i_b + K_m \omega \cos(N\theta)) + \frac{1}{R} (v_b^d + \rho e_b)
\end{align*}
\]

where \( \rho \) is a positive control gain. Then we get

\[
\dot{V}_1 = -\rho e_a^2 - \rho e_b^2.
\]

Therefore, the \( e_a \) and \( e_b \) go to zero as \( t \to \infty \). In section II, given a static desired position, the convergence of the equilibrium point is proved. Notice that the Lyapunov-based control law (16) only guarantees local exponential stability of the desired currents \( v_a^d = \frac{v_a^d}{R} \) and \( v_b^d = \frac{v_b^d}{R} \) required for microstepping. We assume that the desired position trajectory \( \theta^d(t) \) and its derivative with respect to time \( \dot{\theta}^d(t) \) are bounded. If the desired velocity \( \dot{\theta}^d(t) \) is below the convergence rate of the phase desired currents by control law (16), the position tracks the desired position trajectory while the position keeps inside the region of attraction of the desired position. Then the control law (16) results in bounded tracking.

B. Nonlinear Observer Design

In the previous section, the proposed controller design assumed that the full states are measurable. Now we assume that the only position is measurable by an additional encoder. In order to estimate the velocity and the phase currents, a nonlinear observer is proposed as such

\[
\begin{align*}
\dot{\theta} &= \omega + l_1 (\theta - \hat{\theta}) \\
\dot{\omega} &= \frac{-K_m i_a \sin(N\theta) + K_m i_b \cos(N\theta) - B\omega}{J} \\
&\quad + l_2 (\theta - \hat{\theta}) \\
\dot{i}_a &= \frac{v_a - R i_a + K_m \omega \sin(N\theta)}{L} + l_3 (\theta - \hat{\theta}) \\
\dot{i}_b &= \frac{v_b - R i_b - K_m \omega \cos(N\theta)}{L} + l_4 (\theta - \hat{\theta})
\end{align*}
\]
where $l'_i$ s are observer gains. The estimation errors are defined as
\[ \dot{\hat{\theta}} = \dot{\theta} - \hat{\theta}, \]
\[ \ddot{\hat{\omega}} = \omega - \ddot{\theta}, \]
\[ \dot{\hat{i}}_a = i_a - \dot{i}_a, \]
\[ \dot{\hat{i}}_b = i_b - \dot{i}_b. \]

(19)

Then error dynamics are given by
\[ \dot{\hat{\theta}} = -l_1\dot{\hat{\theta}} + \ddot{\theta}, \]
\[ \ddot{\hat{\omega}} = -l_2\ddot{\hat{\omega}} + \omega - \ddot{\theta}, \]
\[ \dot{\hat{i}}_a = -l_3\dot{\hat{i}}_a + [\ddot{\theta} + K_m\ddot{\omega} + \omega], \]
\[ \dot{\hat{i}}_b = -l_4\dot{\hat{i}}_b + [\ddot{\theta} + K_m\ddot{\omega} + \omega]. \]

(20)

For the proof of the stability of error dynamics (20), the Lyapunov candidate function $V_2$ is defined as
\[ V_2 = \frac{1}{2}(\dot{\theta}^2 + \dot{\theta}^2 + \dot{\theta}^2 + \dot{\theta}^2). \]

(21)

The derivative of $V_2$ with respect to time is given by
\[ V_2' = \dot{\hat{\theta}}(-l_1\dot{\hat{\theta}} + \dot{\theta}) + \frac{l_2}{2} \dot{\hat{\omega}}^2 + \frac{l_3}{2} \dot{\hat{i}}_a^2 + \frac{l_4}{2} \dot{\hat{i}}_b^2. \]

(22)

If we define that $l_1$ is positive, $l_2$ is $\frac{1}{l_1}$, $l_3$ is zero, and $l_4$ is zero, the derivative of (21) becomes negative definite
\[ V_2' = -l_1\dot{\theta}^2 - \frac{l_2}{l_1} \ddot{\omega}^2 - \frac{l_3}{l_1} \ddot{i}_a^2 - \frac{l_4}{l_1} \ddot{i}_b^2. \]

(23)

Since (23) is negative definite and (21) is radially unbounded, the equilibrium point of the error dynamics (20) is globally asymptotically stable [20]. Therefore, the estimates $\hat{\theta}$, $\hat{\omega}$, $\dot{\hat{\theta}}$, and $\dot{\hat{i}}_b$ are guaranteed to converge to $\theta$, $\omega$, $\dot{\theta}$, and $\dot{i}_b$ for arbitrary initial conditions $\hat{\theta}(0)$, $\hat{\omega}(0)$, $\dot{\hat{\theta}}(0)$, and $\dot{\hat{i}}_b(0)$ respectively.

IV. CLOSED-LOOP STABILITY

In control law (16), the actual values of the phase currents and velocity are replaced with the estimations (19) as follows:
\[ v_a = (R\dot{i}_a - K_m\dot{\omega}sin(N\theta)) + \frac{l_2}{l_1}(v''_a + \rho\dot{\hat{\theta}}), \]
\[ = (R\dot{i}_a - K_m\dot{\omega}sin(N\theta)) + \frac{l_2}{l_1}(v''_a + \rho\dot{\hat{\theta}}), \]
\[ \dot{\hat{\theta}} = -l_1\dot{\hat{\theta}} + \ddot{\theta}, \]
\[ \ddot{\hat{\omega}} = -l_2\ddot{\hat{\omega}} + \omega - \ddot{\theta}, \]
\[ \dot{\hat{i}}_a = -l_3\dot{\hat{i}}_a + [\ddot{\theta} + K_m\ddot{\omega} + \omega], \]
\[ \dot{\hat{i}}_b = -l_4\dot{\hat{i}}_b + [\ddot{\theta} + K_m\ddot{\omega} + \omega]. \]

(24)

The derivative of $V_2$ with respect to time is given by
\[ V_2' = d(e_a\dot{e}_a + e_b\dot{e}_b), \]
\[ + (1 - d)(-l_1\dot{\theta}^2 - \frac{l_2}{l_1} \ddot{\omega}^2 - \frac{l_3}{l_1} \ddot{i}_a^2 - \frac{l_4}{l_1} \ddot{i}_b^2), \]
\[ = d(e_a\dot{e}_a + e_b\dot{e}_b), \]
\[ + (1 - d)(-l_1\dot{\theta}^2 - \frac{l_2}{l_1} \ddot{\omega}^2 - \frac{l_3}{l_1} \ddot{i}_a^2 - \frac{l_4}{l_1} \ddot{i}_b^2), \]
\[ = d(e_a\dot{e}_a + e_b\dot{e}_b), \]
\[ + (1 - d)(-l_1\dot{\theta}^2 - \frac{l_2}{l_1} \ddot{\omega}^2 - \frac{l_3}{l_1} \ddot{i}_a^2 - \frac{l_4}{l_1} \ddot{i}_b^2), \]
\[ = d(e_a\dot{e}_a + e_b\dot{e}_b), \]
\[ + (1 - d)(-l_1\dot{\theta}^2 - \frac{l_2}{l_1} \ddot{\omega}^2 - \frac{l_3}{l_1} \ddot{i}_a^2 - \frac{l_4}{l_1} \ddot{i}_b^2), \]
\[ = d(e_a\dot{e}_a + e_b\dot{e}_b), \]
\[ + (1 - d)(-l_1\dot{\theta}^2 - \frac{l_2}{l_1} \ddot{\omega}^2 - \frac{l_3}{l_1} \ddot{i}_a^2 - \frac{l_4}{l_1} \ddot{i}_b^2), \]
\[ = d(e_a\dot{e}_a + e_b\dot{e}_b), \]
\[ + (1 - d)(-l_1\dot{\theta}^2 - \frac{l_2}{l_1} \ddot{\omega}^2 - \frac{l_3}{l_1} \ddot{i}_a^2 - \frac{l_4}{l_1} \ddot{i}_b^2). \]

(26)

Since the actual control inputs $v_a$ and $v_b$ are (24), $\dot{V}_c$ becomes
\[ \dot{V}_c = d(-p_1^2 - p_2^2) + d(e_a\dot{e}_a + e_b\dot{e}_b), \]
\[ + (1 - d)(-l_1\dot{\theta}^2 - \frac{l_2}{l_1} \ddot{\omega}^2 - \frac{l_3}{l_1} \ddot{i}_a^2 - \frac{l_4}{l_1} \ddot{i}_b^2). \]

(27)

The vectors are defined as follows
\[ \psi_1 = [e_a \ e_b]^T, \]
\[ \psi_2 = [\theta \ \omega \ \dot{\hat{\theta}} \ \dot{\hat{i}}_a \ \dot{\hat{i}}_b]^T. \]

(28)

Applying (28) to (27) gives us
\[ \dot{V}_c = -[\psi_1 \ \psi_2]^T H [\psi_1 \ \psi_2], \]
\[ \psi_1 = [e_a \ e_b]^T, \]
\[ \psi_2 = [\theta \ \omega \ \dot{\hat{\theta}} \ \dot{\hat{i}}_a \ \dot{\hat{i}}_b]^T. \]

(29)

where
\[ H = \begin{bmatrix} H_1 & H_2 \\ L & H_3 \end{bmatrix}, \]
\[ H_1 = \begin{bmatrix} d & 0 \\ 0 & d \end{bmatrix}, \]
\[ H_2 = \begin{bmatrix} \frac{\partial R}{\partial R} & \frac{\partial R}{\partial L} & \frac{\partial R}{\partial C} \\ 0 & \frac{\partial L}{\partial R} & \frac{\partial L}{\partial C} \end{bmatrix}, \]
\[ H_3 = \begin{bmatrix} (1 - d)l_1 & 0 & 0 & 0 \\ 0 & (1 - d)l_1 & 0 & 0 \\ 0 & 0 & (1 - d)l_1 & 0 \\ 0 & 0 & 0 & (1 - d)l_1 \end{bmatrix}. \]

If $H$ is positive definite, $\dot{V}_c$ is negative definite. Since $H_1$ is positive definite, by Schur complement [21], we see that
\[ H_1 \ H_2^T H_1^{-1} H_2 > 0. \]

(30)

is equivalent to
\[ H_3 - H_1^T H_1^{-1} H_2 > 0. \]

This can be rewritten as
\[ (1 - d) \begin{bmatrix} l_1 & 0 & 0 & 0 \\ 0 & \frac{\partial R}{\partial R} & 0 & 0 \\ 0 & 0 & \frac{\partial R}{\partial L} & 0 \\ 0 & 0 & 0 & \frac{\partial R}{\partial C} \end{bmatrix} > 0, \]
\[ -\frac{\partial R}{\partial L} \begin{bmatrix} L_R \ \ L_R \ \ L_R \ \ L_R \end{bmatrix} > 0, \]
\[ \begin{bmatrix} L_R \ \ L_R \ \ L_R \ \ L_R \end{bmatrix} > 0. \]

where $L_R = L - R$, $S = K_m \sin(Nr \theta)$, and $C = K_m \cos(Nr \theta)$. It is clear that there always exist $d > 0$ and $\rho > 0$ to satisfy (31).
TABLE II
PMSM, CONTROLLER, AND OBSERVER PARAMETERS

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L$</td>
<td>10 mH</td>
<td>$R$</td>
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<td>$J$</td>
<td>$3 \times 10^{-3}$ kg·m²</td>
<td>$K_m$</td>
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<td>$l_2$</td>
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</tr>
<tr>
<td>$l_3$</td>
<td>0</td>
<td>$l_4$</td>
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</tbody>
</table>

V. EXPERIMENTAL RESULTS

Experiments were performed to evaluate the performance of the proposed microstepping control. For the experiments, the Lyapunov-based control (16) and the nonlinear observer (18) algorithms were coded in C Language using TMS320C6713 DSP (manufactured by TI Co.) and was applied to a stepper motor PK266-01B (manufactured by Oriental Motor co.). For position feedback, an incremental optical encoder (8000 lines/rev) was used. Quadrature signals were used to obtain $\times 4$ resolution. A picture of the experimental setup is shown in Fig. 2. The sampling rate was 20 kHz and 16-bit A/D converters and 16-bit D/A converters were used. Two L292 PWM motor drivers from ST Microelectronics were used for the conventional microstepping [7]. In each driver, a PI current feedback loop is embedded and its 0 dB gain crossover frequency was 630 Hz. For the proposed control method, two MSK4223 PWM motor drivers (from M.S. Kennedy Co.) were used but the optional PI current feedback was not used. Given the parameters of PMSM, controller, and observer in Table II, for $d = 0.3$ the eigenvalues of $H$ are $6120.19, 6120.11, 397.91, 397.89, 350, \text{ and } 0.25$ regardless of $\theta$. Thus these parameters satisfy the condition of closed-loop stability (31).

We used two position profiles to evaluate the performance of the proposed method: a sinusoidal profile and a profile for a 16 rad move. The sinusoidal profile had an amplitude of 1 rad with 0.5Hz as illustrated in Fig. 3. The profile for the 16 rad move is illustrated in Fig. 4. The position tracking errors for each case are shown in Figs 5 and 6, respectively. In Fig. 5, we see that applying the Lyapunov-based control provided more than 40% improvement in tracking performance over that of the conventional microstepping.
VI. Conclusions

In this paper, we proposed Lyapunov-based control with a nonlinear observer in microstepping for PMSMs and validated its performance via experiments. We had shown the local asymptotic stability of the equilibrium points using LaSalle’s Theorem. From the analysis, the principle of microstepping was proved in the sense of Lyapunov method. The Lyapunov-based control with full state information had been shown that it guarantees the exponential convergence of the desired phase current error. In this paper, thus we designed a nonlinear observer based on the measurement of position. The nonlinear observer was also designed based on Lyapunov method. It was also proved shown that the estimation error converges exponentially to zero by the Lyapunov method. And the stability of the closed-loop system was studied using the composite Lyapunov function. Both proposed controller and observer do not require any transformation so that the commutation delay could be reduced. Experiments were executed to evaluate the performance of the proposed microstepping control. Experimental results showed improvement in position tracking of the proposed method over the conventional microstepping.

VII. Future Works

Actually, since there exists the mechanical errors of the phase resistance of PMSM, the exact resistances are unknown. And the phase resistance may vary during operation [10]. Thus, both the phase A and B have different resistance. Therefore, the adaptive algorithm is under development to estimate the phase resistance.

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References