Multimodel-Based Techniques for the Identification of the Delay in MIMO Systems

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Abstract—This paper introduces a novel method to identify each individual delay in Multiple Input Multiple Output linear systems with unknown delays. The solution is based on a multimodel scheme consisting of a set of tentative delay estimations along with a switching mechanism aimed at progressively updating them through time according to a figure of merit. Moreover, the set of tentative delays is proved to converge to the actual delays under certain persistent excitation conditions on the input signals. Another contribution of the paper stems from the fact that the presented method can be regarded as an online implementation of an heuristic optimization method known as Pattern Search, very uncommon in control systems applications. Also, numerical examples showing the feasibility and effectiveness of the proposed method are included.

I. INTRODUCTION

The presence of external delays in a dynamical system implies that it does not respond instantly to changes in the input. Under these circumstances the control action is decided in view of previously processed information. This can cause instability and, especially for MIMO systems, constitutes a real challenge for control systems design. There are many examples of systems including delays such as thermal processes, [1] or transport and communication channels, [2], [3], to cite just a few. In [4], [5], many other examples can be found.

One very extended control strategy for systems with external delays is to use a delay compensation scheme, being the Smith Predictor the most extended one. This topology was proposed in the 50’s and has been extended through time to include robustness issues or the possibility to deal with unstable processes [13]. The use of the Smith Predictor facilitates the controller design because it allows to disregard the delay. However, this nice feature assumes the knowledge of the external delay in such a way that if this assumption is not met, the performance of the control system is degraded, causing instability [14] in the worst case.

When the delay is unknown, we need an algorithm to identify the delay of each channel of the MIMO system and then implement a control strategy either in two steps, or in one single step as part of an adaptive controller. With this idea in mind, some researchers have developed algorithms for online identification of delayed SISO systems, see, for instance [6], [7], [8]. In [6], a recursive algorithm for online identification of systems with unknown delays is presented. Later, in [7], a similar work using neural networks is performed whereas in [8] a general identification framework considering unknown time-invariant parameters is introduced for linear time delay systems in the cases of state and output measurements. A disadvantage of these identification schemes is the large number of involved parameters, fact which makes difficult their real-time implementation. Another drawback of some of the works mentioned above is that, being aimed at SISO systems, they are not easily applicable to the MIMO case.

This paper concentrates on the above ideas and introduces a novel online identification scheme for systems with unknown delays which solves the main drawbacks imputable to many current approaches. First, the algorithm is fast, it can be easily implemented and it is conceptually simple. In addition, the proof of identifiability has been derived for it.

The presented approach is based on a multi-model scheme [14],[15]. This way, the basic scheme is composed of a battery of different models operating in parallel. Each model possesses the same rational component but a different value for the delay. A supervisory algorithm compares the mismatch between the actual system and each candidate model and selects, at each time instant, the one that best describes the behaviour of the real system, providing an estimation of the delays of the system.

The main disadvantage of multi-model schemes, normally requiring a large number of fixed models, is compensated by making a relatively small set of candidate models evolve with time according to a so-called Pattern Search Method (PSM). Some PSM works in the field of mathematics can be found in [16], [17], [18] while recent research on this area can be consulted in [9], [10], [11] and [12], being a novelty its application to control systems..

The paper is organized as follows. Section II reviews the problem formulation. Section III proposes an intelligent multi-model identification scheme. In section IV some results of convergence are presented. Simulation examples are presented in Section V. Finally, Section VI summarizes the main conclusions.

Notation: Through the paper, $A > B$ denotes that $a_{ij} > b_{ij}$ for all $i,j$, that is, the element-wise inequality.

II. PROBLEM FORMULATION

Let us consider the following multivariable linear time invariant system:

$$G(s) = (G_{ij}(s)) = (G_{ij}^{df}(s)e^{-h_{ij}s}) \quad (1)$$
where \( i, j \in \mathbb{N}, 1 \leq i \leq m, 1 \leq j \leq n \), \( G(s) \) is an \( m \times n \) transfer function matrix which relates the output vector \( Y(s) \) (size \( m \)) to the input vector \( R(s) \) (size \( n \)), and possibly containing the different delays associated with each input/output pair. The transfer function matrix can be factorized using the Schur product (or component-wise) [19] as

\[
G(s) = G^{df}(s) \bullet D(s)
\]

where \( G^{df}(s) = \left( G^{df}_{ij}(s) \right) \) denotes the part containing the rational component of the system and \( D(s) = (D_{ij}(s)) = (e^{-h_{ij} \tau}) \) is a matrix containing only the delays.

The following assumptions will be made about the system (1):

Assumption 1. The rational transfer function matrix \( G^{df}(s) \) is known and asymptotically stable.

Assumption 2. The delay between each pair of input/output variables belongs to a known compact interval. That is, there exists two known matrices \( \hat{H} = (\hat{h}_{ij}), \underline{H} = (\underline{h}_{ij}) \in \mathbb{R}^{m \times n} \) such that \( \underline{h}_{ij} \leq h_{ij} \leq \hat{h}_{ij} \) for \( i = 1, 2, \ldots, m, j = 1, 2, \ldots, n \).

Assumption 1 is a typical one for the Smith Predictor and is used in the proposed approach, which is based on a Smith Predictor topology. Assumption 2 will be used in the proposed algorithm to identify the delay, in the following Section III. Thus, the problem to be dealt with is formulated assuming a multivariable plant with known rational part and uncertain but bounded delay matrix.

The aim of the article is the formulation of an identification architecture able to identify the system’s delay provided that the a-priori knowledge about the external delays between the different input and output channels is confined to a matrix interval as stated in Assumption 2. The algorithm, based on a multi-model structure, is described in the following Section III.

III. PROPOSED INTELLIGENT MULTI-MODEL IDENTIFICATION SCHEME

The basic structure of the proposed scheme is outlined in Figure 1. The notation \( \hat{G}(s) = G^{df}(s) \bullet \hat{D}(s) \) denotes the transfer function associated with the nominal delay, \( \hat{D}(s) \).

![Fig. 1. Basic architecture of the identification scheme.](image)

As it can be seen, the scheme is composed of three different elements: a set of candidate models associated with different values for the delays, a figure of merit (performance index) which evaluates the potential behaviour of each model and a switching logic which monitors periodically this index and decides which of the models is the best. The switching mechanism is intended to commute to a suitable delay value which reduces the possible mismatch between the nominal and the actual output of the system. In the following subsections the different elements of the presented architecture are considered in more detail.

A. Set of Nominal models

The proposed architecture is composed of a set of candidate models running in parallel, each one associated with a different delay matrix. These models are designed to be time-varying and adjusted by the system according to its own evolution. This way, the designer is exempted from having to define the models. In order to describe the structure of the set of models, (denoted as \( M \)), and its evolution through time, we consider the following matrices \( H, \hat{H}, \underline{H} \) such that \( \underline{H} \leq \hat{H} = (\hat{h}_{ij}) \leq \underline{H} \), where \( \hat{H}(t) \) is the current nominal delays matrix. Let \( \Delta \hat{H}^{inf}(t) = (\Delta \hat{h}^{inf}_{ij}(t)) \geq 0 \), \( \Delta \hat{H}^{sup}(t) = (\Delta \hat{h}^{sup}_{ij}(t)) \geq 0 \forall t \geq 0 \) and \( \Gamma = (\gamma_{ij}) > 1 \) the reduction factors matrix (the terminology will be clear later). Last of all, the elements of the canonical base of \( \mathbb{R}^{m \times n} \) will be denoted by \( E_{ij} \) (matrices having a one in the \( i,j \)th entry and zeros elsewhere, [19]). Using the above defined matrices we introduce the following ordered sets of nominal models:

\[
M_{i}^{-}(t) = \left\{ \hat{H} - \Delta \hat{h}^{inf}_{ij} E_{ij} \right\}^{n} \quad i = 1, 2, \ldots, m (3)
\]

\[
M_{i}^{+}(t) = \left\{ \hat{H} + \Delta \hat{h}^{sup}_{ij} E_{ij} \right\}^{n} \quad i = 1, 2, \ldots, m (4)
\]

\[
M^{-}(t) = \bigcup_{i=1}^{n} M_{i}^{-}(t) \quad (5)
\]

\[
M^{+}(t) = \bigcup_{i=1}^{n} M_{i}^{+}(t) \quad (6)
\]

\[
M(t) = M^{-}(t) \cup \left\{ \hat{H}(t) \right\} \cup M^{+}(t) \quad (7)
\]

From equations (3)-(7) the time-dependent set of models \( M(t) \) is formed by adding \( \Delta \hat{h}^{sup}_{ij} \) and subtracting \( \Delta \hat{h}^{inf}_{ij} \) to the nominal delay matrix \( \hat{H}(t) \) in each direction of the canonical basis \( \mathbb{R}^{m \times n} \) defined by \( E_{ij} \).

B. Figure of merit

The second element of the proposed scheme is a figure of merit aimed at evaluating the behaviour of each model in the set \( M \) introduced in the previous subsection. The suggested figure of merit is:

\[
J^{(i)}(H) = \int_{t-T_{max}}^{t} \left( y(\tau) - \hat{y}^{(i)}(\tau) \right)^{T} Q \left( y(\tau) - \hat{y}^{(i)}(\tau) \right) d\tau
\]

for \( i = 1, 2, \ldots, 2mn + 1 \), where the integration takes place in the time interval in which all the models act simultaneously without being modified. In addition, \( y(\tau) \) is the
vector output of the plant at the instant \( t = \tau \) while \( \hat{y}^{(i)}(\tau) \) denotes the (vector) output of each different model. \( T_{res} \) is the so-called residence time and defines the window where the different models are compared. The residence time is a positive number large enough be able to distinguish between the different models in parallel but its concrete value does no vary the identification properties of the algorithm. \( Q \) is a symmetric positive definite matrix. Hence, (8) is the integral of a positive-definite quadratic form and, therefore, its global minimum value is zero. In [6], the reader can find alternative figures of merit that can be generalized to the multivariable case straightforwardly. Defining the error between the plant output and the output of the \( i^{th} \) model as

\[
e^{(i)}(\tau) = y(\tau) - \hat{y}^{(i)}(\tau)
\]

and \( \dot{D}^{(i)}(s) = D(s) - \dot{D}^{(i)}(s) \), the Laplace transform of the error is given by

\[
E^{(i)}(s) = \left. \left| Y(s) - \hat{Y}^{(i)}(s) \right| \right| = G^{df}(s) \bullet D(s) - G^{df}(s) \bullet \dot{D}^{(i)}(s) = G^{df}(s) \bullet D(s) \left( 1 - e^{-\hat{h}_{ij} s} \right) = G^{df}(s) \bullet D(s) \left( 1 - \hat{h}_{ij} s \right)
\]

It is readily seen that the error (9) is zero when \( \hat{h}_{ij} = h_{ij} - \tilde{h}_{ij} = 0 \), that is, when the delays of the \( i^{th} \) model are equal to the real ones. According to this fact, the search of the minimum of the quadratic function in (9) leads to an estimation of the real delay matrix. The search of this global minimum is performed by switching between different models and updating the set of candidate models through time as stated in the following subsection.

C. Switching Logic

The switching logic periodically monitors the value of the figure of merit and selects the nominal delay which is the best estimation of the real one. The initial nominal model is selected by the designer among the candidates in \( M \). From that moment onwards, the switching logic can be expressed formally as the Algorithm 1.

The comparisons are carried out in groups of three models (the nominal, plus another additive and subtractive disturbances in the direction of an element of \( E_{ij} \)). These models are compared to each other and differ only in the value of one of the components of the delay. Thus, \( M_c = 1 \) means that the model with the subtractive disturbance has been selected while \( M_c = 2 \) stands for the previous nominal one and \( M_c = 3 \) for the model with the additive disturbance. In this way the element associated with the lowest value of the figure of merit is obtained, while the vector \( v \) is the vector sum necessary to pass the nominal model asset into the best model of the set \( M \).

This process is repeated \( m \cdot n \) times at multiples of the residence time, for each element of the matrix. Finally, the vector \( v \) contains the optimum direction of change according to the figure of merit used. This vector is added to the nominal model to obtain the corresponding new nominal model from which a battery of models that will operate in parallel during the next interval of residence is generated again through Eq. (3)-(7). To rebuild the set \( M(t) \), the search patterns \( \Delta h^{sup}_{ij}, \Delta h^{inf}_{ij} \) are adjusted (lines 14 to 25 of Algorithm 1), to ensure the convergence to the true delay taking a reduction factor that depends on the new nominal model value and reduction matrix \( \Gamma \). This guarantees that all the search patterns converge to the same value and hence the scheme tends to a time invariant system which correspond to the estimation of the real delay.

Note that Algorithm 1. can be interpreted as a Pattern Search Algorithm, [9], [10]. In fact, the different delay models define the search pattern and the way in which these models are modified describe the new direction of searching, after evaluating the quality of each pattern. These kinds of algorithms have been used in different fields of Mathematics but have never been applied in on-line control systems design.

Algorithm 1 MIMO case identification algorithm

1: \( \hat{H}(0) \): Initial matrix delay
2: \( \Delta h^{sup}_{ij} \): Initial variation upper bound
3: \( \Delta h^{inf}_{ij} \): Initial variation lower bound
4: \( [\hat{h}_{ij}, \tilde{h}_{ij}]_{ij} : Intervals of uncertainty of delays
5: \( \Gamma = (\gamma_{ij}), \gamma_{ij} > 1 : Matrix reduction factors for \Delta \hat{H}
6: \( T_{res} > 0 \): Residence time
7: \( v \leftarrow 0 \)
8: \( M(0) = M^- (0) \cup \{ \hat{H}(0) \} \cup M^+(0) \)
9: for \( t > 0 \) do
10: if \( t = m T_{res}, m \in \mathbb{N} \) then
11: for \( i, j \mid 1 \leq i \leq m, 1 \leq j \leq n \) do
12: \( M_c \leftarrow \arg \min_{1 \leq k \leq 3} J^{(k)} \)
13: \( v \leftarrow v + (-2 + \text{pos}) \hat{E}_{ij} \)
14: if \( M_c = 1 \) then
15: \( \Delta \hat{h}^{inf}_{ij} \leftarrow \Delta \hat{h}^{inf}_{ij} \left( \frac{1 + \gamma_{ij}}{\gamma_{ij}} \right) \)
16: \( \Delta \hat{h}^{sup}_{ij} \leftarrow \Delta \hat{h}^{sup}_{ij} \left( \frac{1 + \gamma_{ij}}{\gamma_{ij}} \right) \)
17: end if
18: if \( M_c = 2 \) then
19: \( \Delta \hat{h}^{inf}_{ij} \leftarrow \Delta \hat{h}^{inf}_{ij} \left( \frac{1}{\gamma_{ij}} \right) \)
20: \( \Delta \hat{h}^{sup}_{ij} \leftarrow \Delta \hat{h}^{sup}_{ij} \left( \frac{1}{\gamma_{ij}} \right) \)
21: end if
22: if \( M_c = 3 \) then
23: \( \Delta \hat{h}^{inf}_{ij} \leftarrow \Delta \hat{h}^{inf}_{ij} \left( \frac{1}{\gamma_{ij}} \right) \)
24: \( \Delta \hat{h}^{sup}_{ij} \leftarrow \Delta \hat{h}^{sup}_{ij} \left( \frac{1 + \gamma_{ij}}{\gamma_{ij}} \right) \)
25: end if
26: end for
27: end if
28: \( \hat{H}(t) = \text{proj}_{[\hat{h}_{ij}, \tilde{h}_{ij}]_{ij}} (\hat{H}(t) + v) \)
29: \( M(t) = M^- (t) \cup \{ \hat{H}(t) \} \cup M^+ (t) \)
30: end for
IV. RESULTS ON CONVERGENCE OF THE IDENTIFICATION SCHEME

This section states the convergence results of Algorithm 1, guaranteeing the identification of the real delay matrix. Basically, the proof of the identification properties is performed in two steps. First, the conditions under which (8) has a unique global minimum for $\hat{H} = H$ are stated. Having established that the real delay is the only global minimum of the figure of merit, it will be shown that the proposed Algorithm 1 is able to asymptotically find the global minimum of function $J(\hat{H})$ and as it is unique in $h_{ij} = \hat{h}_{ij}$ the delay is identified.

Note that $J^* = 0$ for $\hat{H} = H$ and $J^* \geq 0$ as its integrand is positive. Therefore the global minimum is given by $J = 0$ while the real delay satisfies $J^* = 0 \ \forall t \in [0, +\infty)$.

The problem is that there may exist another values for $\hat{H}$ different from the actual value $H$ for which $J^*(\hat{H}) = 0$. In this way, the beginning of the proof starts establishing the conditions under which $h_{ij} = \hat{h}_{ij}$ is the unique global minimum of $J^*(\hat{H})$ given by (8). It will be proved that a different estimate of $\hat{h}_{ij}$ makes $J^*(\hat{h}) > 0$ if the reference signals meet certain requirements. Indeed, if $J(\hat{H}) = \int_{-\infty}^{\infty} e^{T} Q e d\tau$ (extended across the real axis) with $Q = Q^T > 0$ (positive definite), then there exists a nonsingular matrix $L$ such that $Q = L^T L$ (for example through a Cholesky decomposition) so that

$$J(\hat{H}) = \int_{-\infty}^{\infty} e^{T} L^T L e d\tau = \int_{-\infty}^{\infty} |L e|^2 d\tau$$

The integral is identically null when $Le = 0$ i.e. $e = 0 \ \forall t > 0$ since $L$ is nonsingular. Since $e$ is a vector $e_i = 0 \ \forall t > 0 \ i = 1, 2, \ldots, n$, and therefore

$$e_i(t) = \sum_{j=1}^{n} \left[ \varphi_j(t - h_{ij}) - \varphi_j(t - \hat{h}_{ij}) \right] = 0$$

with $\varphi_j = \mathcal{L}^{-1} \left[ G^d_{ij}(s) R_j(s) \right]$, for $j = 1, 2, \ldots, n$.

Each term in brackets identically cancels when $\hat{h}_{ik} = h_{ik}$. The difficulty of the multivariable case lies in the fact that the signals sum can be compensated for some combination of terms different from those shown in brackets. Under these circumstances, the minimum of $J(\hat{H})$ is unique provided the following condition on the reference signal.

Assumption 3. Let the reference signals $r_1(t), r_2(t), \ldots, r_n(t)$ be continuous and set $T_{res} > 0$. Consider the following conditions where each signal $r_i(t)$ have to satisfy

$$r_i(t) \neq r_i(t - \lambda)$$

and

$$r_i(t) - r_i(t - \lambda_i) \neq \sum_{j=1, j \neq i}^{n} \left( r_j(t) - r_j(t - \lambda_j) \right)$$

where $\forall i, \lambda_1, \lambda_2, \ldots, \lambda_n \in [h, \bar{h}]$ with $\bar{h} = \max \{ h_{ij}, h = \min h_{ij}, j \neq i \}$ and $t \in I \subseteq [kT_{res}, (k+1)T_{res}]$ $k \in \mathbb{N}$, for, at least, one connected interval, $I$ of positive measure.

Hence, the only possibility of (11) and (12) to be identically zero is when $\hat{h}_{ij} = 0$. This could be regarded as the translation to the delay problem of the persistent excitation condition in the case of parametric estimation. Basically, the interpretation of (14) is that the reference signals must not be periodic and the different sums between them cannot be equal to other reference signal or a delayed version of it. Despite Eq. (14) appears to be very complicated, it is easy to obtain a set of reference signals fulfilling it by adding sinusoids with incommensurable frequencies and different amplitudes and phases. In this way, Eq. (14) is easy to fulfill in practice. Hence, the convergence theorem can be formulated as follows.

Theorem 1. Consider the delay system given by (1) satisfying Assumptions 1 and 2. Then, the Algorithm 1 can identify the real delay when the reference signal $r(t)$ satisfies Assumption 3 and $\Gamma$ is sufficiently close to unity.

Note that Theorem 1 requires $\Gamma$ being sufficiently close to unity. However, simulation results have shown, that in practical applications a finite value for it is enough. The proof is omitted for reasons of space.

V. SIMULATION EXAMPLES

This section shows simulations obtained by applying the proposed identification scheme. A complete knowledge of the delay-free part of the plant is assumed. The rational component of the considered plant is given by:

$$G^d(s) = \begin{bmatrix} \frac{12}{s^2+4+6} & \frac{2}{s^2+5+2.5} \\ \frac{1}{s^2+5+2.3} & \frac{1}{s^2+0.5+0.2} \end{bmatrix}$$

and the matrix associated with the real delay in the plant is given by:

$$H = \begin{bmatrix} 3.4 & 4.4 \\ 5.4 & 2.4 \end{bmatrix}$$

The input signals selected for the implementation of the simulations presented here have been:

$$r_1(t) = 2 \sin(\pi/4 t) + 4 \sin(\pi/6 t) + 8 \sin(\pi/7 t) + 2 \sin(\pi/3 t)$$

$$r_2(t) = 2 \sin(\pi/3 t) + 4 \sin(\pi/2 t) + 8 \sin(\pi/5 t) + 2 \sin(\pi/6)$$

This choice of the reference signals satisfy Eq. (13) and (14) showing that these conditions are easy to satisfy in practice.

Table I sets out the parameters concerning the Algorithm 1. In this case, we do not refer to a specific component within the matrices $\Delta \hat{H}_{\sup/\inf}$ since for the parameters used for the Algorithm 1, all entries of the matrices $\Delta \hat{H}_{\sup/\inf}$ and $\Gamma = (\gamma_{ij})$ are equal, so all quantities $\Delta \hat{h}_{ij}$ evolve adopting exactly the same values.

Figure 2 shows the evolution of the error $(y - y_m)$ channel two, can be seen as actually tends to zero as the system identifies the channel delay. Figure 3 shows the evolution of the matrix $\hat{H}(t)$, up positions in some of the delay values identified:
Now assume a modeling error of 6% in the rational component of the system, which is reasonable if it is associated with errors in measurement data. Thus, the rational component of the plant is given by:

\[
G_{df}(s) = \begin{bmatrix}
0.96s^2 + 1.04s + 0.18 & 1.92 \\
1.05s^2 + 2.03s + 2.46 & 2.12 \\
\end{bmatrix}
\]

The delay matrix obtained through simulation is:

\[
\hat{H} = \begin{bmatrix}
3.42 & 4.35 \\
5.42 & 2.43 \\
\end{bmatrix}
\]

From Figure 4 show the evolution through time of the estimated delays and is clear the correct identification of the delay. In conclusion, the scheme is actually able to identify delays in the system even with uncertainty in the parameters of the plant. Comparing Equations (16) and (20), there is great similarity in the results. Note that in both simulations the value of \( \Gamma \) is 1.15 which is actually not very close to unity but the system works correctly.

In the next subsection will show some simulations that permit to verify the robustness of the proposed scheme.

### A. Robustness Issues

The results shown in the previous section assumed an exact knowledge of the delay-free rational part of the plant, but the reality is that most systems possess modeling or instrumentation errors. Hence, simulations were made to observe the performance of the proposed scheme under small discrepancies between the model and the true plant. The matrix associated to the true plant existing delay is still given by Eq. (16).

### VI. CONCLUSION

In this work, an algorithm to identify the delay in MIMO system is presented. The algorithm is based on a Multi-model Scheme composed of different models of the system, each one with a different delay, operating in parallel. Then, a switching logic selects, for each time interval, the model that best describes the behavior of the system. The set of models is updated through time following a Pattern Search based Algorithm, which is a novel application of these algorithms in Control. The convergence properties of the algorithm to the real delay are proved and simulation examples show the effectiveness of the proposed approach. Moreover, the algorithm is showed to provide good identification results even in the presence of parametric mismatch in the rational component of the system, being able to identify the real delay under these circumstances.

<table>
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<th>( T_{res} )</th>
<th>( \Delta H(0) )</th>
<th>( H(0) )</th>
<th>( \gamma_{ij} )</th>
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<td>5 5</td>
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TABLE I

VALUES OF THE PARAMETERS OF INTEREST USED FOR THE SIMULATION
REFERENCES