Explicitly Constrained Generalised Predictive Control Strategies for Power Management in Ambulatory Wireless Sensor Network Systems

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Abstract—This paper develops an explicit generalised predictive control (GPC) strategy for a wireless sensor network (WSN) power control problem that addresses practical constraints typically posed by an ambulatory healthcare problem scenario. An explicit solution of the particular GPC problem that arises facilitates the implementation of traditional on-line optimisation control strategies on a commercial sensor node platform with limited computational performance. The problem is shown to reduce to the evaluation of a piecewise linear function that can offer good performance with existing Mote type devices. The control law is validated experimentally against a number of existing strategies using a scaled, fully IEEE 802.15.4 compliant, testbed that emulates a selection of realistic wireless healthcare scenarios.

Fig. 1. A power-aware sensor network, e.g., in the star topology where the communication runs between a base station and wireless nodes.

I. INTRODUCTION

Power-aware wireless sensor networks (WSNs) that can be organised in a general scenario illustrated in Fig. 1 consist of a number of common features. There will exist a number of (possibly mobile) Mote type sensor nodes. There will also be a remote base station that is used to aggregate the data and to process the data locally or alternatively forward it onto another location within a network topology [1]. Control goals for the resulting sensor networks will inter alia involve optimised battery consumption by the wireless nodes, preservation of sufficient Quality of Service (QoS) among all users, and robust rejection of imperfections within the communication channel. In [2]–[5], it has been shown that dynamic radio power control strategies can play a crucial role in the achievement of good performance in this context. Power control mechanisms based on multiplicative-increase additive-decrease (MIAD), packet error rate (PER), and simple channel model (SCM) have been evaluated under additive white Gaussian noise (AWGN) and Rayleigh environments using the received signal strength indicator (RSSI) sent by sensor nodes as a feedback signal [2], [3]. Recent literature illustrates how distributed power control formulated within a robust control framework, either through the use of quantitative feedback theory (QFT) and/or anti-windup techniques, can achieve robust RSSI tracking and low outage probability [4]. Moreover, the anti-windup approach has been augmented to the QFT based robust power control in order to extensively compensate hardware saturation that is invariably introduced by commercial radio transceivers [5].

The nature of the disturbances that often arise in such a noisy environment, as well as the nature of the performance constraints, exhibit several characteristics suggesting that the use of a predictive control framework [6] may be rewarding. In [7], a power control strategy based on generalised predictive control (GPC) and the inclusion of a suitable T-filter [8] has been investigated and tested in realistic situations that have verified significant improvements in power consumption as well as providing a joint RSSI dynamic that is less sensitive to output disturbances while simultaneously providing an engineering solution that can decouple the effects of the healthcare problem at hand from the environment that problem is required to operate in. Dynamic control is achieved by solving a quadratic programming (QP) problem on-line within a prescribed sampling period implemented within Matlab. To this end, a stable interface between Matlab and TinyOS has been established using TinyOS-Matlab tools written in Java.

In this paper, the GPC formulation considered earlier is extended in a fashion that can be implemented directly using off-the-shelf wireless sensor nodes. The restricted resources of existing sensor node platforms in terms of memory usage and micro-controller capabilities are known to play a critical role as a limiting factor on performance in typical problem scenarios. A particular, reasonably sized constrained GPC controller, synthesised on a commercial sensor node with limited computational performance, is presented that works well in this context. The problem is shown to reduce to an explicit solution of a QP problem, which in turn is recast as a piecewise linear function of the control law presented in [9]. Subsequently the control action is implemented on-line in the form of a lookup table. Exploiting this explicit solution, pre-computed using multi-parametric programming, the controller implementation results in a realtime evaluation of a piecewise linear function that is readily implemented on a Mote based control unit.

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The remainder of this paper is organised as follows. In section II, the dynamic model along with the WSN power control problem formulation is reviewed. In section III, the explicit constrained GPC strategy is described. In section IV, the resulting GPC is experimentally tested using a scaled ambulatory testbed. Some concluding remarks in relation to the specific nature of the problem at hand are also presented in section V.

II. POWER CONTROL PROBLEM FORMULATION

It is assumed that the problem is posed as a sensor network of one receiver/base station and \( n \) transmitters/sensor nodes, labeled \( i = 1, \ldots, n \). Throughout the paper, the logarithmic (e.g. dB or dBM) value of a variable \( x \) is denoted by \( \bar{x} \), namely, \( \bar{x} = 10 \log_{10} x \).

The closed-loop power control system of WSNs under study based on the distributed control scenario [10] requires only local signal-to-interference-plus-noise ratio (SINR) measurements taking into account the channel conditions. The SINR experienced at the corresponding connected base station to sensor node \( i \) is given by

\[
\gamma_i(t) = \frac{g_i(t)p_i(t)}{\sum_{l=1}^{i-1} g_l(t)p_l(t) + \sum_{l=i+1}^{n} g_l(t)p_l(t)},
\]

where \( g_i \) is the channel gain between the base station and sensor node \( i \), \( p_i \) is the power level expended to send the signal by sensor node \( i \), and \( I_i \) is the interference plus noise power at the base station.

The RSSI of the Tmote Sky sensor nodes [11] has been shown to have a strong relationship with the SINR in this problem setting, [2]. Moreover, the use of RSSI as a performance measure for communication for power control purposes is now a feature of the literature, [2]–[5], [7]. This relationship is once again exploited here to deliver prespecified performance objectives in terms of packet error rate (PER) as outlined in [12].

An estimate of the SINR in dB from the RSSI in dBM established in [2] is given by

\[
\bar{\gamma}_i(t) \approx \bar{r}_i(t) - C - 30,
\]

where \( \bar{r}_i \) denotes the RSSI of sensor node \( i \), \( C \) represents measurement offset and the term 30 accounts for the conversion from dBM to dB.

A simplified closed-loop interpretation of the distributed RSSI-based (dB) power control algorithm under study is depicted in Fig. 2. Note that the integrator in the sensor node nominally includes a delay of one sample instant.

At time \( t \), the base station measures the RSSI. In order to track a RSSI target \( \bar{r}_i \), the measurement is compared to the target at the base station to get the RSSI tracking error \( \bar{e}_i \). The control error will be fed into the power controller, thereby generating the power control update command \( \bar{u}_i \). Then the power \( \bar{p}_i \) to transmit data packets is calculated by the distributed power control mechanism:

\[
\bar{p}_i(t) = \bar{p}_i(t-1) + \bar{u}_i(t-1).
\]

This allows the system dynamic to be represented by using a transfer function model,

\[
G(z^{-1}) = \frac{z^{-1}}{1 - z^{-1}}.
\]

Before the updated power control signal will be transmitted to each node via the channel, it is coded to the power level \( \bar{p}_i \) by the radio transceiver. This hardware constraint is a fact of life for any commercial transceiver chipset. The transmitted signal with this power level is then delivered through the channel, wherein it is necessarily corrupted by highly time varying uncertain interference, noise, and channel gain. In this work, these parameters are treated as an unstructured output disturbance signal \( \bar{d}_i \).

III. EXPLICIT GPC FORMULATION FOR THE WSN POWER CONTROL SYSTEM

A power control process described by (2) facilitates the explicit generalised predictive power control algorithm presented in this section. The work here extends [7] to explicitly take into account the issue of practical implementation of GPC on a resource restricted practical sensor node.

In the classical GPC context, a process is modelled by the controlled auto-regressive integrated moving average (CARIMA) form, as an input-output paradigm that takes into account the influence of disturbance (in this context real channel variation due to environmental factors) through the use of a so called T-filter [13]. The CARIMA model generates control increments, that enable offset free predictions of the steady-state where a systematic inclusion of the disturbance model is included,

\[
A(z^{-1})\bar{r}(t) = b(z^{-1})\Delta(z^{-1})\bar{u}(t) + T(z^{-1})\bar{d}(t).
\]

Here, \( \bar{d}(t) \) is the disturbance input, the operator \( \Delta \) is defined as \( \Delta(z^{-1}) = 1 - z^{-1} \), and \( A(z^{-1}) = a(z^{-1})\Delta(z^{-1}) \). In this work, CARIMA model transference from incremental power control update command \( \Delta\bar{u}(t) = \bar{u}(t) - \bar{u}(t-1) \) to the RSSI output \( \bar{r}(t) \) is cast as a channel model that attempts to describe the effects of channel fading, interference, and noises. \( A(z^{-1}) \) and \( b(z^{-1}) \) are polynomials for a strictly proper system. Also, \( T(z^{-1}) \), T-filter, is treated as a design polynomial or a tuning parameter because it has effects on loop sensitivity thereby improving the disturbance rejection properties of the design [8].

It has been shown in [7] that \( T \) plays a crucial role in the compensation for uncertain output channel phenomena. The impact of \( T \) on the sensitivity function for output disturbance rejection is plotted in Fig. 3, where the dotted line represents \( T = 1 \) (no prefilter, denoted as GPC), the dashed line...
The GPC formulation provides the control action $\Delta \hat{u}(t)$ as a function of $\hat{x}_r$ implicitly defined by (7). By treating $\hat{x}_r$ as a vector of parameters, the optimisation problem (7) can be solved off-line, with respect to all the values of $\hat{x}_r$ of interest, and make this dependence explicit. Note that in the context of a CARIMA model, the vector $\hat{x}_r$ represents the system state. The problem (7) can be possibly represented into an equivalent multi-parametric quadratic programming (mpQP) problem [9],

$$\min_{\hat{u}} J = \frac{1}{2} \hat{u}^T Q \hat{u} + \hat{b}^T C_x^T \hat{u}; \ s.t. \ A \hat{u} \leq \hat{b} + S_y \hat{f}_r,$$

with the augmented vector of parameters now redefined as $\hat{f}_r \in \mathbb{R}^{\dim(x_r)+1} = [\hat{x}_r; \ f^r(t+N_k)]^T$ by assuming that the value of the RSSI target is constant over $N_y$ in order to keep the dimension of $\hat{f}_r$ as low as possible to avoid the rapid growth of size in the explicit solution, and $Q \in \mathbb{R}^{N_u \times N_u} = 2(G^T G + \lambda I), C_0 \in \mathbb{R}^{N_x \times \dim(x_r)+1} = 2G^T F_x - 1 N_x$.

Also, the condensed form of the constraint in (8) can be described as

$$A \in \mathbb{R}^{q \times N_u}, \ b \in \mathbb{R}^q = \begin{bmatrix} 1 \bar{N}_f \bar{f}_{\max} \\ -1 \bar{N}_f \bar{f}_{\min} \end{bmatrix},$$

$$S_y \in \mathbb{R}^{q \times \dim(x_r)+1} = \begin{bmatrix} -F_x & 0 \bar{N}_v \\ F_x & 0 \bar{N}_v \end{bmatrix}.$$  

Here, $q$ is $2N_u$, when considering only the performance constraint (5) and $I$, $1\bar{N}_f$, and $0 \bar{N}_v$ represent an identity matrix of suitable dimensions, a column vector of ones of dimension $N_y$, a column vector of zeros of dimension $N_y$, respectively.

The mpQP problem (8) can be solved off-line resulting in piecewise affine optimal control functions of the given vector of parameters [9], i.e., the $\hat{f}_r$-space is divided into several convex regions within which each one single affine or linear control law is valid, represented as

$$\Delta \hat{u}(t) = f(\hat{f}_r(t)) = \begin{cases} F_1 \hat{f}_r(t) + g_1, & \text{if } \hat{f}_r(t) \in \bar{\Theta}_1 \\ F_2 \hat{f}_r(t) + g_2, & \text{if } \hat{f}_r(t) \in \bar{\Theta}_2 \\ \vdots & \vdots \\ F_{N_y} \hat{f}_r(t) + g_{N_y}, & \text{if } \hat{f}_r(t) \in \bar{\Theta}_{N_y} \end{cases}$$

with a polyhedral partition $\mathcal{P} = \{\bar{\Theta}_1, \ldots, \bar{\Theta}_{N_y}\}$, where the polyhedral sets are represented by linear inequalities (hyperplanes),

$$\bar{\Theta}_i = \{\hat{f}_r(t) \in \mathbb{R}^{\dim(x_r)+1} | H_i \hat{f}_r(t) \leq k_i \}, \ i = 1, \ldots, N_y. \hspace{1cm} (10)$$

Therefore, the power controller implementation on wireless sensor nodes requires evaluation of a piecewise linear function (9)-(10) at each sampling instant.

IV. EXPERIMENTAL SETTINGS AND RESULTS

A number of experiments have been performed to test the efficacy of the suggested power control scheme using a range of Tmote Sky sensor nodes. This embedded platform uses an IEEE 802.15.4 compliant transceiver [7]. Fig. 4 illustrates a typical problem scenario. In this testbed, a Tmote is connected to a personal computer (PC) acting as the base.
station or coordinator via the USB port. In addition, there are four other Tmote sensor nodes that are wirelessly connected to, (and randomly located around), the base station.

The data flow within the WSN testbed is depicted in figure 5. In this work, power control algorithms are directly implemented (via nesC) on the Tmote that is identified a priori as the coordinator. An interface between Matlab and TinyOS has been established using stable bridging tools written in Java for data management purposes. On receipt of data packets from sensor nodes at time \( t \), the coordinator takes RSSI measurements and then performs the aforementioned power control strategy, resulting in a defined power level for the next sampling instant that is then forwarded (via the wireless channel) to the appropriate sensor node.

Three test scenarios are considered for each power control law. In each scenario stationary sensor nodes are randomly placed in different positions over the course of four experiments. In addition, a selection of fully autonomous MIABOT Pro miniature mobile robots [15] are used to provide a controlled, ambulatory dimension to the experiment. Note that, for consistency, the trajectories along which the mobile robots move remain the same for each of the four experiments that are conducted for each scenario. The duration of each experiment performed is 140 sec with a sampling interval of 1 sec.

In the IEEE 802.15.4 standard [12], a minimum requirement is that packets received at -85 dBm SINR shall have a maximum of 1% PER. As a result using the relationship (3), a target RSSI value of -55 dBm is selected for tracking thereby guaranteeing a PER of < 1%.

A. Performance Criteria

For each experimental scenario, three criteria are used to evaluate the performances of the various designs:

- **Average power consumption:**
  \[
  \bar{P}_i = \frac{1}{N} \sum_{t=1}^{N} p_i(t),
  \]  
  \( N \) is the total number of samples.

- **Outage probability:**
  \[
  \bar{O}_i(\%) = \text{Prob}(\bar{r}_i < \bar{r}^{\text{th}}) = \frac{\text{the number of times that } \bar{r}_i \leq \bar{r}^{\text{th}}}{\text{the total number of iterations}} \times 100,
  \]  
  (12)
  where the RSSI threshold \( \bar{r}^{\text{th}} \) is chosen to be -57 dBm, a minimum value required for signal reception to occur.

- **Standard deviation of the RSSI tracking error:**
  \[
  \bar{\sigma}_e_i = \left( \frac{1}{N} \sum_{t=1}^{N} [\bar{r}(t) - \bar{r}(t)]^2 \right)^{\frac{1}{2}}.
  \]  
  (13)

These performance criteria have been calculated for all nodes in each test scenario. The average of the results is the final metric taken for analysis purposes in this comparative study.

B. Benchmark Comparison with Existing Control Laws

In this subsection, the explicit GPC formulated in section III is compared to two existing control strategies that have previously been shown to perform well, namely power control with \( \mathcal{H}_\infty \) [16] and with adaptive step-size [17]. For illustrative results that assess the efficacy of these existing strategies, please consult [4].

1) Power control with explicit GPC: The mpQP problem (8) uses \( N_x = 3, N_u = 3, \lambda = 10^{-5} \), and the parameter space for which the explicit solution can be computed is bounded by

\[
-57 \leq \bar{r}(t - i) \leq 0.1, \quad i = 0, 1,
-20 \leq \Delta \bar{u}(t - 1) \leq 20,
-55.1 \leq \bar{r}^d(t - 1) \leq 0.1.
\]

Note that these limits on \( \bar{r} \) and \( \bar{r}^d \) correspond to RSSI threshold and to target RSSI values, respectively.

To obtain the requisite explicit piecewise linear controller, the multi-parametric quadratic solver in the Hybrid Toolbox [18] has been used. The problem is solved using an active-set approach. The mpQP solution was computed in 0.485 sec (6 regions were examined), and the resulting controller is defined over \( N_P = 4 \) regions that require a memory usage of only 18624 bytes (Note that Tmote Sky nodes provide 48 Kbytes of memory [11], thereby suggesting that there still exists plenty of space for the development of user applications on the node).
2) Power control with $\mathcal{H}_\infty$: In [16], robust power tracking control with fixed state feedback gain $K$ is solved via an $\mathcal{H}_\infty$ optimal tracking problem formulation in the presence of uncertain interference plus noise and channel gain. With this method, a resulting optimal $\mathcal{H}_\infty$ gain $K$ of 1 is obtained.

3) Power control with adaptive step-size: In this algorithm [17], the power control mechanism is given by

$$p_i(t) = \hat{p}_i(t) + \bar{\delta}_i(t) \text{sign}(\hat{e}_i(t)), \quad (14)$$

where the parameter $\delta_i$ is adaptively updated based on the system data and desired specification according to

$$\bar{\delta}_i(t) = [\alpha \bar{\delta}_i^2(t-1) + (1-\alpha)\bar{\sigma}_i^2(t)]^{1/2} \quad (15)$$

where $\alpha$ is the forgetting factor, assumed to be 0.95, and $\bar{\sigma}_i$ is the sampled standard deviation of power control tracking error during the updating interval and is selected as $\bar{\sigma}_i(t) = |i^2 - \hat{p}_i(t)|$.

The dynamic information including the variations of the noisy wireless channel from the experiments is summarised by the plots in Fig. 6. Note that the coded power levels of the Tmote are expressed as integer values between 3 and 31, where 3 corresponds to the lowest power (-25 dBm), and 31 to maximum transmission power (0 dBm) [19]. Fig. 6 provides sample test results for a scenario where two mobile nodes and two stationary nodes using the aforementioned control laws are considered. Although Fig. 6 does illustrate the difficulty in comparing the respective designs using transient datasets, it can be observed that an explicit GPC approach results in fewer transient spikes in the required transmission power. In addition, the results are aggregated for the performance criteria (11, 12, 13) in Fig. 7 and Table I. Note that (unsurprisingly) average performance degrades when increased levels of mobility are introduced. It is clear from Fig. 7(a) that the WSN transmission power control using explicit GPC performs best with a significant improvement in power consumption over all test scenarios ranging from 17.94-31.22% and 62.11-70.79% with respect to $\mathcal{H}_\infty$ and adaptive step-size approaches. Similarly inspection of both signal reception and tracking performance from Fig. 7(b) and 7(c), underscores the fact that an explicit GPC approach exhibits smaller outage probabilities and RSSI tracking errors than the other schemes overall conditions.

V. CONCLUDING REMARKS

This paper has presented a GPC based power control algorithm that is capable of being implemented on existing commercially available wireless sensor node devices. The explicit form of the GPC controller for the constrained WSN system under consideration corresponds to a piecewise linear control law that is readily implemented notwithstanding the tightly constrained resources that obtain in such devices. The proposed scheme performs well when compared with existing robust and adaptive power control algorithms and is far more efficient from an implementation perspective. In further work that is underway, the results presented here will be extended to address the fundamental challenge of
maximising network coverage area in this problem setting. This circumstance necessitates the introduction of a hand-off/routing protocol that is power-aware, thereby motivating a challenging controller design problem that is inherently hybrid in nature.

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