Lane keeping automation at tire saturation

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Abstract—This paper presents the design and the simulation test of a Takagi-Sugeno (TS) fuzzy lane keeping output feedback controller. The vehicle control law has been developed based on invariant sets and quadratic boundedness theory, based on a common quadratic function. The TS fuzzy model is able to handle elegantly the nonlinear behavior of the vehicle lateral dynamics. The computation of the control law has been achieved using Linear and Bilinear Matrix Inequalities (LMI-BMI) methods. Some design parameters can be adjusted to handle the tradeoff between safety constraints and comfort specifications.

I. INTRODUCTION

Vehicle lateral control has been largely addressed during the last four decades. Precedent work addressed automated lane keeping in highway conditions in which lateral acceleration limits are generally low [8], [10]. The objective was mainly the improvement of the traffic performance by removing the driver completely from the control loop. The second phase of research on lateral control systems addresses lane departure warning and avoidance [16], [14], [9]. The objective is to offer to the driver, who still keeps the responsibility of driving, assistance functions that help to better accomplish the usual lane keeping maneuvers by partial intervention on the steering wheel or on differential wheels brake [13]. In fact, vehicle loss of control and road departure avoidance remain a major goals of car manufacturers and suppliers since these types of accidents still represent a high percentage of the overall accident rate and the number of deaths [1]. However, it appears today that it is necessary to develop fully automated lane keeping systems that are able to handle the complete operating domain of the vehicle lateral forces including the non linear behavior [11]. Two main aspects militate for this evolution: sensor and actuator systems technology are today sufficiently mature to handle complex situations with guaranteed performance; sharing the control with the driver is not really an advantage, in most cases it is better that the controller takes completely the responsibility during the control duration. Therefore, it is important to ensure a good safety level during control intervention. To achieve this, it has been chosen to build an invariant set for the system state and to require that each trajectory that starts inside the invariant set will not exceed it, hence the trajectories will be bounded inside it. The design of the steering control law is thus transformed to provide appropriate invariant sets [18]. The Lyapunov theory and the quadratic Lyapunov functions offer customary ellipsoidal invariants sets [18]. However, while Lyapunov function are easy to construct for linear systems, they are more difficult to find for non linear ones.

Takagi-Sugeno fuzzy systems offer an elegant approach for modeling non linear systems in a wide operating domain. In the literature, control of fuzzy systems has been mainly addressed using parallel distributed control (PDC) concept [6], [17], [5]. State feedback controllers have been first designed with different performance index. When the state is unmeasurable, a fuzzy state observer could be incorporated into the controller design in order to form an output feedback fuzzy control. The fuzzy observer generally uses the same rules as that of the state feedback PDC rules [12]. Sufficient conditions for PDC static output feedback have been also developed for nominal and uncertain fuzzy systems [7]. The dynamic output feedback (DOF) case has been considered in recent work. The DOF formulation considered in this paper presents two main advantages: great flexibility to formulate the stabilization conditions and ability to handle input or state constraint and bounded disturbances. This controller uses the property of quadratic boundedness (invariant set) [2].

Section II gives a description of the developed vehicle lateral dynamics Takagi-Sugeno model of the vehicle in a reference frame related to the lane. Section III addresses the fuzzy output feedback including the requirements concerning the quadratic boundedness, the state constraints and control limitation. Section IV presents the controller synthesis for lane keeping automation. Simulation results for various maneuvers which excite the nonlinear tire dynamics are provided. The conclusions in Section V wrap up the paper.

II. EXACT VEHICLE LATERAL DYNAMICS T-S MODEL

A. Lateral dynamics model

As we are concerned with lateral control, a simple nonlinear model of a vehicle is obtained by neglecting the roll and pitch motions. This model includes the lateral translational motion and the yaw motion (Fig. 1-a). The two wheels of each axle are lumped into one located at its center. This leads to the vehicle bicycle model. The lateral forces between the each tire and the road surface are added at each axle leading to two resulting forces $f_f(\alpha_f)$ and $f_r(\alpha_r)$ at the front and rear wheels of the bicycle model respectively. These forces which will be detailed below are function of the front and rear tires sideslip angle, denoted $\alpha_f$ and $\alpha_r$ respectively.

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The lateral translation and rotational yaw motion equations written in the vehicle fixed frame take the following form

\[
\begin{bmatrix}
mv (\dot{\beta} + r) \\
J_f \dot{r}
\end{bmatrix} = \begin{bmatrix}
1 & 1 \\
l_f & -l_r
\end{bmatrix} \begin{bmatrix}
f_f(\alpha_f) \\
f_r(\alpha_r)
\end{bmatrix}
\]

(1)

where \(\beta\) is the vehicle sideslip angle and \(r\) is the yaw rate. \(m\) is the vehicle mass while \(J\) is the vehicle moment of inertia. The vehicle center of gravity is located at a distance \(l_f\) from the front axle and a distance \(l_r\) from the rear axle.

Assuming that the angles remain small, the front and the rear sideslip angles are given by:

\[
\alpha_f = \delta_f - (\beta + \frac{l_f}{v} r)
\]

and

\[
\alpha_r = -\beta + \frac{l_r}{v} r
\]

(2)

B. Lateral tire forces model

Several types of models of the forces of tire-pavement interaction have been proposed in the literature [15]. They are usually derived from experimental data, as for the Pajecka model, and have as parameters the adhesion, the speed \(v\) and the normal force \(f_{ni}\). The shape of the lateral force is often similar from one model to another. A first linear domain for small sideslip angle allows to define a slope factor \(c_{yi}\) called the tire cornering stiffness coefficient. When the sideslip angle increases, the tire enters a nonlinear operating zone where the lateral force saturates. The maximum value defines the limit of the vehicle maneuverability, resulting in a loss of controllability that can cause an understeering phenomenon or an unusual oversteering which may surprise the driver. In the sequel, the model HSRI\(^1\) will be used [19]. It allows a simple lateral tire forces formulation which integrates two of the cited parameters: the adhesion \(\mu\) and the normal force \(f_n\):

\[
f_{yi} = c_{yi} f(\lambda) \tan \alpha_i
\]

(3)

where

\[
\lambda = \frac{\mu f_n}{2 c_{yi} |\tan \alpha_i|}
\]

and

\[
f(\lambda) = \begin{cases} 
2 - \lambda \lambda & \text{if } \lambda < 1 \\
1 & \text{if } \lambda \geq 1
\end{cases}
\]

C. Four rules Takagi-Sugeno vehicle fuzzy model

The nonlinear vehicle model is transformed into a four rules Takagi-Sugeno (T-S) fuzzy model according to the values of the front and rear cornering stiff esses:

- if \(|\alpha_f|\) is \(m_1\) and \(|\alpha_r|\) is \(n_1\) then \(f_f = c_{f1} \alpha_f\)
- if \(|\alpha_f|\) is \(m_2\) and \(|\alpha_r|\) is \(n_1\) then \(f_f = c_{f1} \alpha_f\)
- if \(|\alpha_f|\) is \(m_1\) and \(|\alpha_r|\) is \(n_2\) then \(f_f = c_{f2} \alpha_f\)
- if \(|\alpha_f|\) is \(m_2\) and \(|\alpha_r|\) is \(n_2\) then \(f_f = c_{f2} \alpha_f\)

The membership functions \(m_1\) and \(n_i\) \((i = 1, 2)\) are determined by the approximation method of nonlinear function by linear sectors. Coefficients \(c_{fi}\) and \(c_{ri}\) \((i = 1, 2)\) represent the tire cornering stiffness associated to each sector. In fact they represent also the slope of the limits of the sectors which include the tire forces (Fig. 1-b). For example, given two coefficients \(c_{f1}\) and \(c_{f2}\), chosen according to the expected road adhesion and driving conditions, one can determine the membership functions \(m_1(\alpha_f)\) and \(m_2(\alpha_f)\) using the following set of equations:

\[
\begin{cases} 
(c_{f1} m_1(\alpha_f) + c_{f2} m_2(\alpha_f)) \alpha_f = f_f(\alpha_f) \\
m_2 = 1 - m_1
\end{cases}
\]

(4)

which leads to:

\[
m_1 = \frac{f_f - c_{f2} \alpha_f}{(c_{f1} - c_{f2}) \alpha_f}
\]

(5)

In practice, the two functions \(m_1\) and \(m_2\) are obtained with the numerical values: \(c_{f1} = 1.1 c_f\) and \(c_{f2} = 0.7 c_f\). It is important to outline that this sector representation is an exact approximation of the non linear system. The membership functions \(n_1\) and \(n_2\) for the rear tire forces are obtained by the same procedure. Finally, one can write:

\[
\begin{cases} 
f_f = [(h_1 + h_3) c_{f1} + (h_2 + h_4) c_{f2}] \alpha_f \\
f_r = [(h_1 + h_2) c_{r1} + (h_3 + h_4) c_{r2}] \alpha_r
\end{cases}
\]

(6)

with \(h_1 = m_1 \times n_1\), \(h_2 = m_2 \times n_1\), \(h_3 = m_1 \times n_2\) and \(h_4 = m_2 \times n_2\). In order to have the front and the rear sideslip angle as state vector components, let us define the state...
\( \ddot{x} = [\alpha_f, \alpha_r, \delta_f]^T \) and the control input \( u = \dot{\delta}_f \), the fuzzy system takes the form:

\[
\dot{\bar{x}} = \sum_{i=1}^{4} h_i(\alpha_f, \alpha_r) \bar{A}_i\bar{x} + \bar{B}u
\]  

(7)

where

\[
\bar{A}_i = \begin{bmatrix} a_{11i} & a_{12i} & a_{13} \\ a_{21i} & a_{22i} & a_{23} \\ 0 & 0 & 0 \end{bmatrix}, \bar{B} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}
\]  

(8)

and

\[
\begin{align*}
a_{11i} &= -\frac{r}{l_f + l_r} - \frac{1}{v} \left( \frac{1}{m} + \frac{1}{\tau_f} \right) c_{fi}, \\
a_{12i} &= \frac{r}{l_f + l_r} - \frac{1}{v} \left( \frac{1}{m} - \frac{1}{\tau_r} \right) c_{ri}, \\
a_{21i} &= -\frac{v}{l_f + l_r} - \frac{1}{v} \left( \frac{1}{m} - \frac{2}{l_r} \right) c_{fi}, \\
a_{22i} &= \frac{v}{l_f + l_r} - \frac{1}{v} \left( \frac{1}{m} + \frac{2}{l_f} \right) c_{ri}, \\
a_{13} &= \frac{1}{l_f + l_r} - \frac{1}{v} \left( \frac{1}{m} + \frac{2}{l_f} \right) c_{fi}, \\
a_{23} &= \frac{1}{l_f + l_r} - \frac{1}{v} \left( \frac{1}{m} - \frac{2}{l_f} \right) c_{ri}.
\end{align*}
\]  

D. Additional dynamics for lane keeping

For lane keeping purposes, the model equation (7) has to be expanded with the dynamics of the relative yaw angle and the lateral displacement with respect to the lane centerline. Let \( \psi_l = \psi - \psi_t \) be the yaw angle error which is the angle between the vehicle orientation and the tangent to the road. The road reference curvature \( \rho_{ref} \) is defined by \( \psi_l = \rho_{ref} \bar{v} \). We have the following equality:

\[
\psi_l = r - v \rho_{ref} = \frac{v}{l_f + l_r} (-\alpha_f + \alpha_r + \delta_f) - v \rho_{ref}
\]  

(9)

Denoting by \( l_f \) the look-ahead distance, the equations giving the evolution of the measurement of the lateral offset \( y_l \) from the centerline at sensor location is obtained by

\[
\begin{align*}
\dot{y}_l &= v (\beta + \psi_l) + l_f r \\
&= -\frac{l_f}{l_f + l_r} \alpha_f + \frac{l_f}{l_f + l_r} \alpha_r + v \psi_l + \frac{l_f}{l_f + l_r} \delta_f
\end{align*}
\]  

(10)

In the following, it will be assumed that the only variables available for feedback are \( y_l \) and \( \psi_l \) which are measured using a video sensor. The fuzzy model is finally discretized at a sample time of 0.005s. The final fuzzy model is of the form:

\[
\begin{align*}
x(t+1) &= \sum_{i=1}^{4} h_i(\alpha_f, \alpha_r) A_i x(t) + B_i u(t) + E w(t) \\
y(t) &= C x(t) + D w(t)
\end{align*}
\]  

(12)

where \( x = [\alpha_f, \alpha_r, \delta_f, \psi_l, y_l]^T \) and \( y = [\psi_l, y_l]^T \). The disturbance \( w(t) = \rho_{ref} \in \mathbb{R} \) is \( \mathcal{E} = \{ w \in \mathbb{R} \mid w^T Q w \leq 1 \} \) is bounded. Matrices \( A_i \) and \( B_i \) can be easily derived from equations (8), (9) and (10). This discrete time fuzzy system is characterized by common \( B_i, E \) and \( C \) matrices for all the sub-models:

\[
E = \begin{bmatrix} 0 & 0 & 0 & -v \\ 0 & 0 & 0 & 0 \end{bmatrix}, C = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.
\]

This property simplifies drastically the stability and performance conditions as only simple summations are involved. The matrix \( D \) is zero for the nominal system.

III. Dynamic output feedback fuzzy controller

In the following, we seek a dynamic fuzzy controller of the form:

\[
x_c(t+1) = \sum_{i=1}^{4} h_i(\alpha_f, \alpha_r) A_i x_c(t) + B_i y(t) \\
u(t) = C_c x_c(t) + D_c y(t)
\]  

(13)

where \( x_c \in \mathbb{R}^5 \) is the controller state; \( \{A_i, B_c, C_c, D_c\} \) are matrices to be designed.

This controller still uses the parallel distributed compensation (PDC) concept of the fuzzy system control. In this concept, each control rule is distributively designed for the corresponding rule of a T-S fuzzy model. Linear control theory can then be used to design controllers for each of the consequent part of the fuzzy system while ensuring the same properties for the fuzzy system.

As pointed out in [2], \( D_c \) is an important parameter for stabilization, and the controller structure is able to handle constraints on the input and the state. By combining (12) and (13), the augmented closed-loop fuzzy model is given by

\[
\dot{x}(t+1) = \sum_{i=1}^{4} h_i(\alpha_f, \alpha_r) \Phi_i \bar{x}(t) + \Gamma w(t).
\]  

(14)

where \( \bar{x} = \begin{bmatrix} x^{T} & y_l \end{bmatrix}^{T} \), \( \Phi_i = \begin{bmatrix} A_i + B_D C_c & B_c A_c^i \end{bmatrix} \) and \( \Gamma = \begin{bmatrix} B_D D + E \\ B_c D \end{bmatrix} \).

Let \( \Phi_2 = \sum_{i=1}^{4} h_i(\alpha_f, \alpha_r) \Phi_i \), the closed loop system takes the form:

\[
\dot{x}(t+1) = \Phi_2 \bar{x}(t) + \Gamma w(t).
\]  

Finally, the control input \( u(t) \) is given by:

\[
u(t) = \begin{bmatrix} D_c C_c & C_c \end{bmatrix} \bar{x}(t) + D_c Dw(t)
\]  

(15)

A. Invariant set and output feedback PDC control

Assume that there exists a quadratic function \( V(\bar{x}) = \bar{x}^T P \bar{x} \), where \( P \) is a symmetric, positive definite matrix that satisfies the condition [4]:

\[
V(\bar{x} + \bar{x}) \leq V(\bar{x}), \forall \bar{x}, w \text{satisfying (14)}, w^T Q w \leq 1, V(\bar{x}) \geq 1.
\]  

(16)

Consider the reachable set \( \Lambda \) defined by:

\[
\Lambda \triangleq \{ \bar{x}(T) | \bar{x}, w \text{satisfying (14)}, \bar{x}(0) = 0, w^T Q w \leq 1, T \geq 0 \}.
\]  

(17)

The set \( \mathcal{E}_P \) is defined by:

\[
\mathcal{E}_P = \{ \bar{x}(t) \in \mathbb{R}^10 | \bar{x}(t)^T P \bar{x}(t) \leq 1 \},
\]  

(18)

is an invariant set for the system (14) with \( w \in \mathbb{R} \), \( w^T Q w \leq 1 \). This means that every trajectory that starts inside \( \mathcal{E}_P \) remains inside it for \( t \to \infty \). The existence of such a function \( V(\bar{x}) \) means that the set \( \mathcal{E}_P \) is an outer approximation of the reachable set \( \Lambda \). \( \mathcal{E}_P \) is also an outer approximation of the reachable set

\[
\Lambda^* \triangleq \{ \bar{x}(T) | \bar{x}, w \text{satisfying equation (14)}, \bar{x}(0) \in \mathcal{E}(P), w^T Q w \leq 1, T \geq 0 \}.
\]  

(19)

In this section the control law and the invariant set \( \mathcal{E}_P \) are
synthesized. This is achieved using BMI (Bilinear Matrix Inequalities) optimization method such that the system without the disturbance is asymptotically stable and at the same time, the reachable set for an initial state values inside the invariant set is contained in this invariant set.

1) Invariant set - quadratic boundedness: According to the previous considerations, the closed loop linear system \( \dot{x}(t + 1) = \Phi_c \bar{x}(t) + \Gamma w(t) \) is strictly quadratically bounded with a common Lyapunov matrix \( P > 0 \) for all allowable \( w(t) \in \mathcal{E}_Q \), for \( t > 0 \), if \( \Phi_c \bar{x}(t) + \Gamma w(t) \rightleftharpoons \bar{x}^T P \bar{x} \), this implies \( \Phi_c \bar{x}(t) + \Gamma w(t) \rightleftharpoons \bar{x}^T P \bar{x} \), for all \( w \in \mathcal{E}_Q \). The corresponding condition is obtained using the \( S \)-procedure and invoking the Schur complement, using that the satisfaction of \( w \in \mathcal{E}_Q \) and \( \bar{x}^T P \bar{x} \geq 1 \) implies \( w^T Q w \leq \bar{x}^T P \bar{x} \). The implication

\[
\text{Let us define } P = \begin{bmatrix} P_1 & P_2^T \\ P_2 & P_3 \end{bmatrix} \text{ and } P^{-1} = \begin{bmatrix} M_1 & M_2^T \\ M_2 & M_3 \end{bmatrix}.
\]

Assuming that \( P_1 \) and \( P_2 \) are full rank matrices, let us define \( T_1 = \begin{bmatrix} I & M_1 \\ 0 & M_2 \end{bmatrix} \) and \( T_2 = \begin{bmatrix} I & P_1 \\ 0 & P_2 \end{bmatrix} \). By pre- and post-multiplying the left-hand side of (20) with \( diag \{ T_1^T, I, T_2^T \} \) and \( diag \{ T_1, I, T_2 \} \), respectively, and applying

\[
\begin{align*}
\hat{D}_c &= D_c \hat{C}_c = D_c CM_1 + C_c M_2 \hat{B}_c = P_1 B D_c + P_1^2 B_c \\
\hat{A}_c &= P_1 A_c M_1 + P_1 B D_c C_c M_1 + P_1^2 B_c C_c M_1 \\
&+ P_1 B_c C_c M_2 + P_1^2 A_c M_2
\end{align*}
\]

one can obtain

\[
\sum_{i=1}^{4} h_i(\alpha_i, \alpha_c) Y_i \geq 0
\]

where

\[
Y_i = \begin{bmatrix}
(1 - \alpha_i) P_1 & * & * & * \\
(1 - \alpha_i) I & (1 - \alpha_i) M_1 & * & * \\
0 & 0 & \alpha Q & * \\
A_i + B D_c C & A_i M_1 + B \hat{C}_c & B \hat{D}_c D + E & M_1 * \\
P_1 A_i + \hat{B}_c C & \hat{A}_c & \hat{B}_c D + P_1 E & I \ P_1
\end{bmatrix}
\]

Notice that the matrices \( M_1, M_2, P_1 \) and \( P_2 \) verify:

\[
M_1^T P_2 = I - M_1 P_1.
\]

In addition, it is possible to handle constraints on the control signal and the state:

\[
-\bar{u} \leq u(t) \leq \bar{u}, \quad -\bar{\Psi} \leq \Psi(x(t + 1)) \leq \bar{\Psi}, \quad \forall t \geq 0
\]

where \( \bar{u} > 0, \bar{\Psi} := [\Psi_j, \ldots, \Psi_q]^T \) with \( \Psi_j > 0, j = 1, \ldots, q, \Psi \in \mathbb{R}^{q \times q} \).

Notice that the bounds are provided separately on each state variables or as combination of state variables.

For a pre-specified scalar \( \eta \in (0, 1] \), the quadratic boundedness property ensures that if \( \hat{x}(0) \in \mathcal{E}_P \), then \( \hat{x}(t) \in \mathcal{E}_P \), \( \forall t \geq 0 \), thus \( \forall w(t) \in \mathcal{E}_Q \)

\[
\max_{t \geq 0} \| u(t)^2 \| = \max_{t \geq 0} \left\| \begin{bmatrix} D_c & C_c \\ \eta P & 0 & Q \end{bmatrix} \right\| \leq \max_{t \geq 0} \left\| \begin{bmatrix} D_c & C_c \\ D_c D \end{bmatrix} \right\| \eta P \left\| \begin{bmatrix} 0 \\ 0 \end{bmatrix} \right\|^{-1/2} \left\| \begin{bmatrix} \eta P \ 0 \end{bmatrix} \right\|^{-1/2} \leq \bar{u}^2
\]

By applying the Schur complement, the condition (26) is satisfied if

\[
\begin{bmatrix}
\eta P_1 & * & * \\
0 & \eta I & \eta M_1 & * \\
0 & 0 & Q & * \\
\sqrt{2} D_c C & \sqrt{2} \hat{C}_c & \sqrt{2} \hat{D}_c D & \bar{u}^2
\end{bmatrix} \geq 0.
\]

A similar procedure can be applied for the constraints on the state variables. One can achieve from the convex properties (29):

\[
\sum_{t=1}^{\infty} h_i(\alpha_i, \alpha_c) \tilde{Y}_i \geq 0, \quad \forall t \geq 0,
\]

\[
\tilde{Y}_i = \begin{bmatrix}
\eta P_1 & * & * \\
0 & \eta I & \eta M_1 & * \\
0 & 0 & Q & * \\
\tilde{\Psi}_i & \eta \tilde{\Psi}_i & \tilde{\Psi}_i & \Xi
\end{bmatrix}
\]

and \( \tilde{\Psi}_{i1} = \sqrt{2} \Psi (A_i + B \hat{D}_c C), \tilde{\Psi}_{i2} = \sqrt{2} \Psi (A_i M_1 + B \hat{C}_c), \tilde{\Psi}_{i3} = \sqrt{2} \Psi (B \hat{D}_c D + E) \).

IV. CONTROL SYNTHESIS FOR AUTOMATIC LANE KEEPING

A. Control objectives

From the vehicle point of view, a lane keeping maneuver requires the controller to reject lateral acceleration and yaw rate disturbances caused by changes in the radius of curvature. In fact, in this configuration, the reference curvature is an external input for the system. Under this disturbance, the main objective of the steering law is to perform lane keeping maneuver in a wide operating domain.
while providing the smallest possible overshoot with respect to the lane centerline. More generally, the state variables should not exceed the bounds of a "safety zone", namely \( |\alpha_f| \leq \alpha_f^M, |\alpha_r| \leq \alpha_r^M, |\delta_f| \leq \delta_f^M, |\psi_f| \leq \psi_f^M \) and \(|\gamma_f| \leq \gamma_f^M\). In addition, it has been shown in [18] that having the front wheels in a central lane strip of width \(2d\) corresponds to the space between two hyperplanes \(|\gamma_f + (l_f - l_s)\psi_f| = \frac{2d-a}{2}\), where \(a\) is the half vehicle axle length and \(2d\) is assumed greater than \(a\). Thus, the state vector \(x\) has to be confined to a hypercube \(L(Z^M)\) defined by the above bounds. Finally, the control input, the steering angle rate, has to be bounded \(|\delta_f| \leq \delta_f^M\). According to the equation (25), \(\bar{u} = \delta_f^M\), while \(\Psi = [\alpha_f^M, \alpha_r^M, \psi_f^M, \gamma_f^M, 2d-a]^T\) and
\[
\psi = \begin{bmatrix}
0, 0, 0, (l_f - l_s), 1
\end{bmatrix}.
\]

B. Dynamic output feedback synthesis

The PDC output feedback controller was synthesized with the following numerical values:
\[
\alpha = 0.02, \quad \eta = 0.02, \quad \bar{u} = 100 \text{deg/s}, \quad \alpha_f^M = 13 \text{deg}, \quad \delta_f^M = 8 \text{deg}, \quad \psi_f^M = 7 \text{deg}, \quad y_f^M = 0.3m, \quad d = 1m.
\]
The achieved \(Q\) is \(10^4\), which ensures that the constraints are verified for a disturbance of a magnitude less than \(10^{-2}1/m\) at the considered longitudinal speed of \(20m/s\).

C. Simulation tests in nominal conditions

Analysis of the controllers is now conducted for switching from manual to automatic control while the vehicle is cruising at \(20m/s\) at a lateral displacement of \(0.1m\). The vehicle enters then at \(t = 2\) sec a sharp curve of \(100m\) radius of curvature which excite the tire nonlinear dynamics. In fact at the considered speed of \(20m/s\), the steady state lateral acceleration is of \(0.4g\).

Results are shown on the eight plots of figures 3 and 4. Figure 3-a shows that the front (solid line) and the rear (dashed line) sideslip angles are less than \(12\)deg. However, it is evident that the tire enter the nonlinear region as this is depicted on figures 4-c and 4-d. The relative yaw angle maximum value is quite high but is still under \(7\)deg. The lateral displacement at the sensor location is under \(0.2m\) (Fig IV-C-c). The contribution of each of the four sub-models of the fuzzy system is visible through the values of the \(h_i\) \((i = 1, \ldots , 4)\) shown in the figure 4-b. Finally the steering angle value and rate shown on figures 3-d and 4-a verify the constraints.

D. Robustness face to sideslip angles estimation errors

The implementation of the fuzzy controller requires the knowledge of the front and the rear sideslip angles. It is now established that these variables can be well estimated using observers. However, as the output controller use explicitly these variables, some robustness properties should hold face to estimation errors. In the following, simulation tests are conducted under the less favorable situation: a \(5\%\) under estimation of the rear tire sideslip angle and the over estimation of the front tire sideslip angle. In addition the road curvature is subject to white noise with a power of \(20\%\) of curvature magnitude. Figures 5 and 6 clearly show that even if the coefficients \(h_i\) are disturbed, the effect on the state variables are well filtered. However, the limits on several variables are exceeded without violating safety thresholds.

V. CONCLUSIONS AND FUTURE WORKS

In this paper the design and the test of an output feedback fuzzy PDC lane keeping control law have been described. The main goal of the proposed controller is to perform
the maneuver for a wide vehicle lateral dynamics. The control law for the automatic steering provides an ellipsoidal invariant set. This invariant set presents two main advantages: a bounded overshoot of the state variables with respect to the road curvature disturbance as well as a bounded worst displacement at the front part of the vehicle on the lane with respect to the centerline. Simulation tests have shown that the controller is able to perform well even for high curvatures values that cause tire forces saturation. The controller implementation is also robust face to rough estimation of the sideslip angles. Future works intend a practical study on a prototype vehicle.

Fig. 4. Robustness of the fuzzy dynamic output feedback: Front and rear tire sideslip angle, relative yaw angle, lateral displacement, steering angle, for automatic driving in sharp curve

Fig. 5. Robustness of the fuzzy dynamic output feedback: steering angle rate, coefficients $h_i$, front and rear tire forces, for automatic driving in sharp curve

**Table I**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_f$ front cornering stiffness</td>
<td>40000 N/rad</td>
</tr>
<tr>
<td>$G_f$ rear cornering stiffness</td>
<td>35000 N/rad</td>
</tr>
<tr>
<td>$I_y$ vehicle yaw moment of inertia</td>
<td>2454 kg m$^2$</td>
</tr>
<tr>
<td>$l_f$ distance from CG to front axle</td>
<td>1.22 m</td>
</tr>
<tr>
<td>$l_r$ distance from CG to rear axle</td>
<td>1.44 m</td>
</tr>
<tr>
<td>$l_{ls}$ look-ahead distance</td>
<td>5 m</td>
</tr>
<tr>
<td>$m$ total mass</td>
<td>1600 kg</td>
</tr>
<tr>
<td>$f_{s_{f,2}}$ normal force on front tires</td>
<td>2.22 kN</td>
</tr>
<tr>
<td>$f_{s_{r,4}}$ normal force on rear tires</td>
<td>1.88 kN</td>
</tr>
<tr>
<td>$v$ longitudinal velocity</td>
<td>20 m/s</td>
</tr>
<tr>
<td>$\mu$ adhesion</td>
<td>[0.1]</td>
</tr>
</tbody>
</table>

**References**