Control of Systems with Hysteresis via Servocompensation and Its Application to Nanopositioning

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Abstract—Partly motivated by the nanopositioning application in AFM and SPM systems, we consider the problem of tracking periodic signals for a class of systems consisting of linear dynamics preceded by a hysteresis operator, where uncertainties exist in both the dynamics and the hysteresis. A servocompensator is proposed, in combination with an approximate hysteresis inverse, to achieve high-precision tracking. The servocompensator accommodates the internal model of the reference signal and a finite number of harmonic terms. It is shown that, with a Prandtl-Ishlinskii (PI) hysteresis operator, the closed-loop system admits a unique, asymptotically stable, periodic solution, which justifies treating the inversion error as an exogenous periodic disturbance. Consequently, the asymptotic tracking error can be made arbitrarily small when the servocompensator accommodates a sufficient number of harmonic terms. Robustness with respect to uncertainties in the dynamics is also established. Simulation and experimental results are presented to validate the approach and evaluate the role of hysteresis inversion. In particular, experiments on a nanopositioner show that, with the proposed method, tracking can be achieved for a 200 Hz reference signal with 0.6% mean error and 2.3% peak error for a travel range of 20 µm.

I. INTRODUCTION

A growing area of research and development is nanopositioning, which has enabled the development of technologies like Scanning Probe Microscopy (SPM) for use in the fields of biology, materials science, lithography, and others. While nanopositioning has many promising applications, it also presents a number of challenges [1]. One of the most important issues is the control of piezoelectric material-actuated systems, which exhibit coupling of a hysteresis nonlinearity with dynamics. In addition, both the hysteresis and dynamics can vary with the environmental or loading conditions.

Modeling and control of hysteretic systems has received much attention in the literature [2]. For nanopositioning and other smart material-actuated systems, a reasonable model structure is a dynamic system preceded by a hysteresis operator. The hysteresis is often modeled by a Preisach or Prandtl-Ishlinskii (PI) operator [3]. Feedforward inverse control is commonly used in hysteresis control problems [4]–[6], and inversion of both the Preisach and PI operators has been extensively studied [2], [7]. To handle the uncertainty in hysteresis, adaptive control schemes have been proposed [4], [6], [8]. In the work of [4], [6], uncertainty in the plant dynamics is also addressed with the model reference adaptive control framework. These methods suffer from either over-parametrization [4] or from the need for slow adaptation [6]. Adaptive control was also proposed for uncertain discrete time systems with hysteresis [9].

Several robust control methods have also been proposed that do not make use of explicit hysteresis models [10]. High-gain feedback together with a notch-filter was proposed to linearize hysteresis by Zou et al. [11]. In [12], the hysteresis is approximated by a straight line, and an adaptive robust controller is then used to handle the uncertainties including the hysteresis effect. Sliding mode control has been used to achieve robustness to both plant and hysteresis uncertainties [13]. However, sliding mode control at its heart is a high gain method, and thus its performance suffers in the presence of sensor noise.

In this paper we consider the problem of tracking periodic signals for a class of systems consisting of linear dynamics preceded by a hysteresis operator, where uncertainties exist in both the dynamics and the hysteresis. As illustrated in Fig. 1, we propose the use of a servocompensator in combination with an approximate hysteresis inverse to achieve high-precision tracking. Servocompensators [14] based on the internal model principle [15] have shown promise for tracking periodic trajectories and rejecting periodic disturbances. The proposed servocompensator accommodates the internal model of the reference signal and a finite number of harmonic terms.

![Proposed controller structure: a servocompensator/stabilizing controller followed by an approximate inverse compensator for hysteresis.](image)

The key feature of this work is that the effect of hysteresis can be entirely considered as an exogenous periodic disturbance, which can be compensated for by an appropriately designed servocompensator. It is shown that, with a Prandtl-Ishlinskii (PI) hysteresis operator, the closed-loop system admits a unique, asymptotically stable, periodic solution, which justifies treating the inversion error as an exogenous periodic disturbance. Consequently, the asymptotic tracking
error can be made arbitrarily small when the servocompensator accommodates a sufficient number of harmonic terms from the inversion error. The proposed method only requires an approximate model of the hysteresis.

A crucial problem that must be addressed in nanopositioning control is that the plant’s behavior can vary greatly under different operating conditions [1]. Specifically, the plant’s resonant frequency is highly variant with the loading conditions. Therefore, we use a robust LQR method [16] to stabilize the closed loop system with respect to uncertainties in the dynamics. This method can directly deal with a plant whose resonant frequency varies over a known range. In addition, this method scales well at higher speeds since the servocompensator’s disturbance rejection is independent of frequency as long as it is properly designed. Simulation results are presented to confirm design robustness. Experiments on a nanopositioner have confirmed the effectiveness of the proposed control scheme. In particular, we have demonstrated tracking of a 200 Hz reference signal with 0.6% mean error and 2.3% peak error for a travel range of 20 μm.

The remainder of the paper is organized as follows. Linear servocompensator theory is first reviewed in Section II. We will also introduce the robust LQR scheme of [16] in this section. Hysteresis modeling and inversion are then discussed in Section III, where we also show the existence and asymptotic stability of periodic solutions of the closed-loop system. We will show that the tracking error is bounded and proportional to the size of the inversion error in hysteresis, and the bound can be reduced by increasing the number of harmonics accommodated in the servocompensator. Simulation and experiments are presented in Section IV. Finally, concluding remarks are provided in Section V.

II. REVIEW OF LINEAR SERVOCOMPENSATOR THEORY

Servocompensators based on the internal model principle were developed in the 1970’s by Davison [14] and Francis [15]. Consider a linear system given by

\[
\begin{align*}
x &= Ax + Bu + Ew \\
e &= y - Cx
\end{align*}
\]  

(1)

where \(x \in \mathbb{R}^n\) is the state, \(u \in \mathbb{R}^m\) is the input, \(y = Cx\) is the output, \(e \in \mathbb{R}^r\) is the tracking error, \(w = H\sigma \in \mathbb{R}^d\) is an exogenous disturbance, and \(y_r = G\sigma \in \mathbb{R}^r\) is the reference trajectory to be tracked. Here \(H\) and \(G\) are real matrices which map \(\sigma\) to \(\mathbb{R}^d, \mathbb{R}^r\) respectively. The vector \(\sigma\) is generated by a linear exosystem,

\[
\dot{\sigma} = S\sigma
\]  

(2)

The following assumptions are made:

**Assumption 1:** \((A, B)\) is stabilizable.

**Assumption 2:** \(\text{eig}(S) \subset \text{cl}(C^+) = \{\lambda \in \mathbb{C}, \text{Re}[\lambda] \geq 0\}\).

**Assumption 3:** \((A, B, C)\) has no zeros at \(\text{eig}(S)\).

The controller is designed as

\[
\begin{align*}
\dot{\eta} &= C^*\eta + B^*e \\
n &= -K_1^*x - K_2^*\eta
\end{align*}
\]  

(3)

where the eigenvalues of \(C^*\) are the same as those of \(S\), and \(B^*\) is chosen as any matrix such that the pair \((C^*, B^*)\) is controllable.

By Theorem 1 of [14], if the gain matrices \((K_1^*, K_2^*)\) can be chosen such that the resulting closed-loop matrix is Hurwitz, then \(e(t) \to 0\) as \(t \to \infty\). With controller (3) applied to the linear plant (1) the closed loop system becomes, for \(\omega = 0\),

\[
\begin{align*}
\dot{x} &= [A + B_2^\Delta C_1^*]x + [B + B_2^\Delta D_1^*]u(t) \\
\gamma &= [\dot{x}, \dot{\eta}]^T, Q = Q^\gamma \geq 0, R > 0
\end{align*}
\]  

(5)

for appropriately defined matrices \(B_2^\Delta, C_1^*, D_1^\Delta\). The matrices \(B_2^\Delta, C_1^*, D_1^\Delta\) are known, and represent knowledge of the range of the parameter uncertainty. The matrix \(\Delta^\Delta\) is unknown and satisfies the bound,

\[
\Delta^\Delta \leq I, \Delta^\Delta^\Delta \leq I
\]

where \(\cdot^T\) denotes the transpose.

With this uncertainty, the closed-loop system (4) becomes

\[
\begin{align*}
\dot{x} &= A\Delta^\Delta x + B\Delta^\Delta y \\
\gamma &= [\dot{x}, \dot{\eta}]^T, Q = Q^\gamma \geq 0, R > 0
\end{align*}
\]  

(6)

Define a cost functional,

\[
J = \int_0^\infty [\gamma^T Q\gamma + Ru^2]dt
\]

(7)

We define new matrices,

\[
\tilde{A} = \begin{bmatrix} A & 0 \\ -B^*C & C^* \end{bmatrix}, \tilde{B} = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}, \tilde{D}_1 = \begin{bmatrix} D_1^0 \\ 0 \end{bmatrix}, \Delta = \begin{bmatrix} \Delta^\Delta \end{bmatrix}
\]

(8)

where each 0 represents an appropriately defined zero matrix.

The following Lemma is adapted from Theorem 1 in [16].

**Lemma 1:** If for some \(\varepsilon = \varepsilon^* > 0, R = R^* > 0\), there exists a unique positive definite solution \(P = P^*\) to the Riccati Equation

\[
[A - \tilde{B}(\varepsilon R + D_1^\Delta D_1^*^{-1}D_1^*C_1^*)^{-1}D_1^*C_1^*P + P[A - \tilde{B}(\varepsilon R + D_1^\Delta D_1^*^{-1}D_1^*C_1^*)^{-1}\tilde{B}^*P + \varepsilon \tilde{P}B_2^\Delta P - \varepsilon \tilde{P}\tilde{B}(\varepsilon R + D_1^\Delta D_1^*^{-1}D_1^*C_1^*)^{-1}\tilde{B}^*P + 1/\varepsilon C_1^*(I - D_1^*(\varepsilon R + D_1^\Delta D_1^*^{-1}D_1^*C_1^*)C_1^* + Q = 0
\]  

(8)
then $\forall \varepsilon \in (0, \varepsilon^*)$, $\forall R \in (0, R^*)$, Eq. (8) has a unique positive definite solution $P$, and the control law
\[
u(t) = - (\varepsilon R + D_1' D_1)^{-1} (\varepsilon B_1 P + D_1' C_1)\gamma \triangleq [K_1, K_2]\gamma \quad (9)
\]
guarantees exponential stability of the closed-loop system (6), when $y_r = 0$.

III. ANALYSIS OF THE CLOSED-LOOP SYSTEM

In this section, we analyze the tracking performance of the proposed composite controller that combines a servocompensator with an approximate hysteresis inverse. We first introduce the hysteresis model used in the paper and its inversion, and then establish the asymptotic stability of the closed-loop system for the tracking error analysis.

A. Hysteresis Modeling and Inversion

In this paper, we will use the Prandtl-Ishlinskii (PI) operator to model hysteresis. The PI operator consists of a weighted superposition of basic hysteretic units called play operators. Each play operator $P_i$ is parameterized by a parameter $r_i$, which represents the play radius or threshold. The output of a play operator $u_r(t)$ for a given monotone continuous input $v(t)$ over $t \in [0, T]$ is given by
\[
u_r(t) = P_r(v; u_r(0))(t) = \max \left\{ \min \{v(t) + r, u_r(0)\}, v(t) - r \right\}
\]

The output $u_r(t)$ also represents the current state of the operator $P_i$. For general inputs, the input signal is broken into monotone segments. The output is calculated from the successive segments, where the last output of one monotone segment becomes the initial condition for the next.

In the interest of practical implementation, we consider only a finite-dimensional PI operator, where the operator can be represented as a finite sum. Consider a PI operator $\Gamma_h$ containing $m + 1$ play operators, where the threshold parameters satisfy $0 = r_0 < r_1 < \cdots < r_m < \infty$. The output of the PI operator is then given by
\[
u(t) = \Gamma_h[v; W_t(0)](t) = \sum_{i=0}^{m} \theta_i P_{r_i}[v; W_t(0)](t)
\]

where $W_t(t)$ represents the state of the play operator $P_i$ at time $t$. The vector $\theta = (\theta_0, \theta_1, \cdots, \theta_m)^\top \geq 0$ represents the weights of each individual play element of the operator. We also define the vector hysteresis operator $\mathcal{P} = (P_{r_0}, P_{r_1}, \cdots, P_{r_m})^\top$, which captures the evolution of the state $W_t(t)$ of the hysteresis under input $v$, i.e.,
\[
W_t(t) = \mathcal{P}[v; W_t(0)](t)
\]

The inverse $\Gamma_h^{-1}$ of a finite-dimensional PI operator can be analytically constructed, which turns out to be another PI operator. The thresholds and weights of the play operators for the inverse PI operator, denoted as $\theta_{inv}, r_{inv}$, can be calculated explicitly in terms of those of the original PI operator [7].

As is often the case, the exact values of the vector $\theta$ are considered unknown in our work. Instead, an estimate $\hat{\theta}$ is used in the inversion. This implies that the inversion output $v$ is given by $v(t) = \Gamma_h^{-1}[u_d; W_t(0)](t)$, or equivalently, $u_d(t) =$ $\hat{\Gamma}_h[v; W_t(0)](t)$. Here $u_d$ is the desired trajectory for the output of $\Gamma_h$. Since $\Gamma_h$ and $\hat{\Gamma}_h$ share the input $v(t)$, their states $W_t(t)$ are the same. Therefore, comparing $u_d$ and $u$ by using (12) in (11) results in the expression
\[
u_d(t) - u(t) = \hat{\theta} W(t) + d(t)
\]
where $\hat{\theta} = \hat{\theta} - \theta$, and $d(\cdot)$ represents a small, bounded error in modeling the hysteresis with the PI operator. In order to simplify the presentation, we make the following assumption:

Assumption 4: The inversion error satisfies (13) with $d(\cdot) = 0$.

Remark 1: The results of this paper will still hold if $d(\cdot) \neq 0$ as long as $d(\cdot)$ is bounded.

B. Asymptotic Stability of the Closed-loop System

Recall Fig. 1. Note that the controller is a composite of a servocompensator/stabilizing controller and an approximate inverse compensator of hysteresis. The servocompensator (3) is designed to accommodate the internal model of the periodic reference signal and its dominant harmonics, where the harmonics are induced by the error in hysteresis compensation. To apply the linear servocompensator theory as outlined in Section II, however, we need to establish that the inversion error can indeed be treated equivalently as a matched exogenous disturbance.

We assume that the uncertainty in the plant takes the form of (5). We define $\mathcal{B} \triangleq (B + B^* \Delta^* D_1')$ for ease of notation. We design $u_d$ to act as a stabilizing input. Under feedback (9) (with $u$ replaced by $u_d$) and using (13), Eq. (6) becomes
\[
\dot{\gamma} = \left[ A + B^* \Delta^* C_1 - B \bar{K}_1 - \gamma \bar{K}_2 \right] \gamma + \left[ \mathcal{B}(-\hat{\theta} W(t)) \right] B^* \gamma(t)
\]

Note that the vector $W(t)$ is governed by the following hysteresis operator:
\[
W(t) = \mathcal{W}[u_d; W_t(0)](t) \triangleq \mathcal{P} \circ \hat{\Gamma}_h^{-1}[u_d; W_t(0)](t)
\]

where $u_d$ is given by
\[
u_d = [K_1, K_2]\gamma
\]

Eqs. (14), (15), and (16) form a complete description of the closed-loop system, which clearly shows the coupling of an ordinary differential equation with a vector hysteresis operator. The system can be analyzed with a perturbation technique introduced in [17], where the nominal system is obtained by letting $\hat{\theta} = 0$ in (14). From the linear servocompensator theory, for a $T-$periodic reference trajectory $y_r$, the solution $\gamma(t)$ of the nominal system converges exponentially to a (unique) periodic solution $\gamma_T$.

Define $u_{sr} = [K_1, K_2] y_t$ and $y_T = \hat{\Gamma}_h^{-1}[u_{sr}; W_0(t)]$. Note that $u_{sr}$ is $T-$periodic, and $y_T$ is also $T-$periodic (after a duration of $T$ seconds, to be precise). For any continuous function $z$, define the oscillation function $\text{osc}$ by
\[
\text{osc}_{[t_1, t_2]}[z] = \sup_{t_1 \leq \tau, \sigma \leq t_2} |z(\tau) - z(\sigma)|
\]
Assumption 5: $\text{osc}[^0,T][u_T] > 2r_{\text{inv},m}$, and $\text{osc}[^T,2T][v_T] > 2r_m$, where $r_{\text{inv},m}$ and $r_m$ are the largest play thresholds for $f^{-1}_h$ and $G_h$, respectively.

Theorem 1: Let the reference trajectory be $T$–periodic. Let Assumptions 1–5 hold. Then there exists some $\varepsilon^* > 0$, such that, for any initial condition $(\gamma(0), W(0))$ and any $\hat{\gamma}$ with Euclidean norm $||\hat{\gamma}|| \leq \varepsilon^*$, the solution of the closed-loop system (14), (15), and (16) will converge asymptotically to a unique periodic solution.

Sketch of the proof. Assumption 5 indicates that the tracking error analysis. From the

C. Tracking Error Analysis

From Theorem 1, given a composite controller (servocompensator and inverse hysteresis compensator), the closed-loop signals, including the inversion error $\theta^TW(t)$, converge to $T$–periodic signals that depend only on the reference signal $y_r$. Therefore, in analysis, one can essentially treat the inversion error as a matched, exogenous disturbance, as illustrated in Fig. 2.

We can now make use of the periodicity of $W(t)$ in the tracking error analysis. From the $T$–periodicity of $W(t)$, we can write

$$\alpha(t) = \hat{\theta}^TW(t) = c_0 + \sum_{k=1}^{\infty} c_k \sin \left( \frac{2\pi kt}{T} + \phi_k \right)$$

which can be broken into compensated and uncompensated parts. Define $\alpha_c$ as the set of all $k$’s such that the internal model of $\sin(2\pi kt/T)$ is accommodated in $C^*$, and then define

$$\alpha_c = c_0 + \sum_{k \in \alpha_c} c_k \sin \left( \frac{2\pi kt}{T} + \phi_k \right)$$

$$\alpha_d = \sum_{n \notin \alpha_c} c_k \sin \left( \frac{2\pi kt}{T} + \phi_k \right)$$

Since the hysteresis inversion error is now effectively an exogenous disturbance, we can use the superposition principle to analyze the steady-state tracking error. Write

$$e(t) = y_r(t) - Cx = y_r(t) - (y_1(t) + y_2(t) + y_3(t))$$

where $y_i, i = 1, \cdots, 3$, will be defined shortly. First, consider the signal $y_1(t)$, which arises when $\alpha_c, \alpha_d = 0$. By Theorem 1 of [14] and Lemma 1, this part of the response will approach $y_r$ as $t \to \infty$. Next, we consider the output response when $y_r, \alpha_d = 0$, which we will define as $y_2(t)$. Since $\alpha_c$ is comprised entirely of signals whose internal models are contained in $C^*$, Theorem 1 in [14] states that the response of this system will asymptotically track the reference trajectory, which in this case is zero. Finally, we consider the output when $y_r, \alpha_c = 0$, which is denoted $y_3(t)$. In this case, $\alpha_d$ will not be accommodated in the servocompensator design. Since the closed-loop system is a stable linear system, the response from this portion is bounded. From these discussions, the tracking error under the proposed control scheme will be of the order of $\alpha_c$, which can be made arbitrarily small by accommodating a sufficient number of harmonics in the servocompensator design.

Remark 2: This tracking error bound is valid for any modeling uncertainty or matched disturbance that does not cause the system to become unstable. In such a case, $\alpha_d(t)$ would also contain any matched disturbance that is not in $\alpha_c$. This is shown by Davison in [14].

IV. SIMULATION & EXPERIMENTAL RESULTS

A. Simulation Results

We use simulation to verify the robustness of the proposed control scheme to plant uncertainties. A first harmonic servocompensator was used for this study. The dynamics model used in simulation was based on that identified for a commercial nanopositioning stage (Nano-OP65 with NanoDrive controller, Mad City Labs Inc.), which is shown in Fig. 3. For numerical conditioning purposes, the simulated time $t$ is scaled from the physical time $\tau$ with $\tau = 2.14484 \times 10^4 \tau$. The hysteresis was modeled by a PI operator with five play elements, and an approximate inversion was used, with weights perturbed 10% from the true values. The plant model is

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ -1 + r_1 & -1.180 + r_2 \end{bmatrix} x + \begin{bmatrix} 0 \\ .4476 + r_3 \end{bmatrix} u(t)$$

where $r_1, r_2 \in [-0.2, 0.2], r_3 \in [-0.3, 0.3]$. Gains were selected via (8) and (9). The time-scaled reference trajectory was chosen to be $y_r = \sin(2 \pi t)$. We will quantify the performance by using Mean tracking error, computed by taking the average of $|e(t)|$ over one period of the reference input, then dividing by the reference amplitude.

![Fig. 3. Nanopositioning stage used in experimentation. Position feedback is provided by a built-in capacitive sensor.](image-url)
The inversion of the hysteresis operator was carried out as described in [7]. For comparison purposes, a proportional-integral (PI) controller was also implemented in combination with the feedforward hysteresis inversion, where the proportional and integral gains were chosen to be 2 and 5000, respectively. The two relative error metrics we use for comparison are the mean tracking error, as discussed above, and peak tracking error, computed by taking $\max|e(t)|$ once the system has reached steady state, then dividing by the reference amplitude.

The mean and maximum tracking errors for different controllers and reference signals are shown in Table II. From the table, the tracking error under the PI controller is about 15-fold and 10-fold of that under the servocompensator, when the mean and peak errors are considered, respectively. Even the simple first harmonic servocompensator significantly outperforms the PI controller in both error metrics. Including the 2nd and 3rd harmonics of the reference further reduces this error, which in the case of a 50Hz reference averaged in the tens of nanometers for a 40$\mu$m travel range. Experimental results for the 50 Hz cases are shown in Figs. 5 and 6.

An interesting result is that the performance gain from the higher order servocompensator was larger at 50 Hz than at 200Hz. Looking at Figure 7, we can see the significant presence of many high order harmonics in the tracking error signal. We conjecture that this is due to the higher harmonics of the reference being affected by the resonant peak of the plant, making them more significant when compared to the same harmonics at 50 Hz. This idea is supported by fourier analysis of the two error signals.

| Perturbation | Tracking error | Mean $|e(t)|$ | $\max|e(t)|$ |
|--------------|----------------|-------------|-------------|
| none         | $2.75 \times 10^{-4}$ | 4.227% | 7.632% |
| (0.1269, 0.1475, -0.2493) | $2.10 \times 10^{-4}$ | 14.224% | 23.230% |
| (-0.0401, -0.0961, 0.1800) | $2.90 \times 10^{-4}$ | 0.75% | 2.18% |
| (-0.0274, 0.1643, -0.1909) | $2.09 \times 10^{-4}$ | 0.23% | 1.48% |
| (-0.0945, -0.1418, -0.2184) | $1.87 \times 10^{-4}$ | 0.8% | 5.67% |
| (-0.1420, 0.1412, 0.0732) | $2.70 \times 10^{-4}$ | 0.6% | 2.3% |

In this paper we have proposed a new approach to the problem of tracking periodic signals in systems with hysteresis. The control scheme combines a servocompensator with hysteresis inversion, and its performance in the presence of uncertainties in both hysteresis and dynamics is justified through rigorous analysis of the closed-loop system. This control method shows promise for SPM applications due to its high tracking performance, excellent scaling with frequency, and robustness to uncertainties common in SPM applications. Experimental and simulation results have been presented to validate the proposed control scheme.

Note that the design of the servocompensator requires certain knowledge of the reference signal. In particular, the

Table I shows the mean tracking error for the unperturbed case and other five cases with parameter perturbations. It can be seen that the tracking performance is almost the same despite the perturbation on the plant parameters.

**TABLE I**

| Perturbation | Tracking error | Mean $|e(t)|$ | $\max|e(t)|$ |
|--------------|----------------|-------------|-------------|
| none         | $2.75 \times 10^{-4}$ | 4.227% | 7.632% |
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| (-0.1420, 0.1412, 0.0732) | $2.70 \times 10^{-4}$ | 0.6% | 2.3% |

**B. Experimental Results**

Experiments were conducted on a piezo-actuated nanopositioner (Fig. 3) to examine the performance of the proposed control scheme. The hysteretic behavior was experimentally characterized with a quasi-static input. As shown in Fig. 4, the hysteresis loop was not odd-symmetric. Following [7], a superposition operator with 9 deadzone elements was used in conjunction with a PI operator with 8 play elements to model the asymmetric hysteresis. From a theoretical perspective, the results of [17] hold for the asymmetric operator, and so the theoretical results above hold as long as the modeling error can be made small. The discrepancy between the model prediction and the actual measurement was within 1 $\mu$m for a travel range of 60 $\mu$m. The plant dynamics was identified based on the frequency response obtained with small-amplitude sinusoidal inputs.

In the tracking experiments sinusoids of 50 and 200 Hz were utilized for reference signals, with the travel distance being 40 $\mu$m for 50 Hz, and 20 $\mu$m for 200 Hz due to safety concerns for the stage. Control and measurement were facilitated by a dSPACE platform. The stabilizing gains were chosen in the same manner as in the simulation, and a high-gain observer [18] was used for state estimation.
control parameters vary with the frequency of the reference. Adaptive servocompensators have been studied by several researchers [19]. A worthwhile extension of the presented work would be to exploit advances in adaptive servocompensators and design a controller that can adapt to the frequency of the reference and disturbance signals, while retaining the robustness properties of the current design.

REFERENCES


