Subspace identification and robust control of an AMB system

H.M.N.K. Balini, Ivo Houtzager, Jasper Witte and Carsten W. Scherer

Abstract—Active magnetic bearing (AMB) systems are non-linear, multi-variable and inherently unstable. Conventionally, LTI models are obtained either from first principles or identification experiments and then used to design/synthesize feedback controllers. In general, identification algorithms are focused on obtaining the current and displacement coefficients for a linearized AMB model, which in-turn is based on first principles modeling. In recent years, it is shown that predictor-based subspace identification algorithms (PBSID) give consistent estimates of the parameters even in the presence of feedback. In view of this, we apply the PBSIDopt method to obtain a linear model of an experimental AMB system. Further, a robust controller against uncertainties in a reduced order model (neglecting flexible dynamics) is synthesized.

I. INTRODUCTION

In recent years, active magnetic bearing systems have emerged as viable alternative to conventional bearings in many industrial applications. Some successful examples are fly-wheel energy storage systems, hard disk drives and high speed milling. Our current research is aimed at supporting the development of a micro-factory (a desktop sized milling machine capable of 3-D micro-machining), [1], [2]. The requirement of high precision in such machines calls for the use of advanced control techniques. Previously, we carried out simulation studies on application of such techniques to AMB systems in [3] and [4]. We used a rigid-body model derived from first principles for our studies. However, such a model differs markedly from a real system. In this paper, we take up the problem of identifying a model for the MBC500, an experimental AMB setup and design a robust controller. In an accompanying paper that is submitted to this conference (ACC 2010), [5], we used this particular model to design and implement high-performance robust and LPV controllers on the MBC500 setup.

Identification and control of AMB spindles is challenging due to the nonlinear, multi-variable and inherently unstable nature of these systems. The nonlinear electro-magnetic force is \( F = \mu_0 \frac{N^2 A I^2}{4G} \), \( \mu_0 \) is the magnetic permeability, \( N \) is the number of coil-turns, \( A, I, G \) denote the surface area, current and air-gap length respectively. Gyroscopic effects are another source of non-linearity, which can however be neglected for long and slender spindles.

Linear models for AMB spindles can be built from models of the constituent subsystems, namely, the electro-magnets, power amplifier, spindle, and the sensors. A second approach is to use the input-output measurements in a closed-loop setting (in the presence of a stabilizing controller). A first principles model for the rigid body dynamics of a horizontally suspended spindle with two magnetic bearings is derived in detail in [6] and [7]. In order to build a practically useful model, it is essential to know the mechanical and electro-magnetic properties very accurately. In general, these properties are hard to measure/calibrate. The linearized expression for the electro-magnetic force around a nominal operating point is given as \( f = k_i i + k_x g \), where \( k_i \) and \( k_x \) are the unknown current and displacement coefficients. These coefficients can be determined experimentally and different procedures are discussed in [8] and [9]. Estimation of these coefficients is often referred to as identification of AMB systems in the literature.

Further, when the mechanical properties of spindle are also unknown, frequency response function (FRF) measurements are used to fit a model. The FRF measurements are obtained from modal analysis experiments (e.g. impact testing) and a nonlinear optimization is carried out to obtain the unknown physical properties, [10] and [11]. First principles models often differ from the FRF measurements due to inaccurate knowledge of the physical parameters and unaccounted system dynamics. In such situations, the analytical model can be adjusted to minimize the discrepancy with measurements. This is known as model reconciliation. It is shown in [12] that model reconciliation can be recast as a control synthesis problem in an \( H_\infty \) framework.

Each of the methods cited in the above references offer valuable engineering insights. However, difficulties exist with their practical usage and eventually lead to unsatisfactory models. Modeling of individual subsystems can be cumbersome and errors are introduced with each of the components. Nonlinear optimization algorithms for estimating the unknown parameters with FRF measurements need an initial guess and return only the local optima. Presence of flexible modes in the spindle dynamics increases the system order and is numerically imposing on the optimization algorithm. Moreover, the FRF measurements of the complete AMB system can only be obtained in a closed-loop setting and hence are influenced by the feedback controller.

In contrast to the above methods, subspace identification methods (SIMs) use only the input-output measurements to estimate a state-space model and solely rely on elementary linear algebra techniques (e.g. least squares, matrix factorizations). The single-most obstacle for any identification algorithm in a closed-loop setting is that the noise in input-output measurements is correlated. Neglecting this correlation, reasonable models for AMBs have been identified using open-loop SIMs in [13] and [14]. In [13], the \( 4 \times 4 \) MIMO magnetic bearing system is approximated by means of the simplifying assumption of negligible magnetic interactions and neglecting the flexible modes.
of 4 decentralized SISO models. The well established SIM, N4SID is applied to the input-output measurements for each SISO channel. A SIM for frequency response data of AMB systems is described in [14]. The method relies on the so-called Laguerre network and the 4 × 4 MIMO AMB system is modeled using two 2 × 2 subsystems (by neglecting gyroscopic/electro-magnetic coupling effects).

In the recent past, the problem of noise correlation for input-output measurements in closed-loop systems is rigorously addressed in the literature. An overview of the proposed SIMs is given in [15]. A detailed theoretical analysis on the consistency of estimates for most of these methods is given in [16]. A predictor-based subspace identification (PBSID\textsubscript{opt}) algorithm discussed in [17] and [18] has been shown to be consistent and optimal in comparison to the other algorithms. The PBSID\textsubscript{opt} algorithm used in [19] and in this paper uses a VARX (vector autoregressive with exogenous inputs) model structure. The method follows a three step procedure, namely, estimation of Markov parameters, state-sequence, and finally the state-space matrices.

Our main contribution in this paper is modeling of an AMB system from input-output measurements using the PBSID\textsubscript{opt} algorithm and subsequent design and implementation of a robust controller on an MBC500 experimental setup. The proposed method does not require any prior knowledge of the physical properties of the system and is used to identify a MIMO model which includes the flexible dynamics of the system. The obtained model lays-down a strong foundation for applying advanced control synthesis techniques for an over-all better performance. The identified model for the MBC500 contains the lumped dynamics of the spindle, power-amplifier, actuators and sensors. The model is reduced by neglecting the flexible dynamics of the spindle and this is accounted by means of an additive uncertainty weight. An $H_{\infty}$ controller is then synthesized for robustness against this uncertainty and nominal performance for output disturbance rejection. The resulting model in this paper is used for design and implementation of advanced controllers from $H_{\infty}$ and LPV techniques for higher performance and the details are discussed in [5].

An outline of the PBSID\textsubscript{opt} algorithm is given in Section II. The MBC500 experimental setup used in this research is described in Section III. A comparison of the identified model with measured frequency response functions is carried out in Section IV. The design and implementation of robust controller(s) is given in Section V followed by conclusions in Section VI.

II. THE PBSID\textsubscript{opt} ALGORITHM

In the literature on SIMs, three equivalent state-space system representations are often used, namely, the process, innovation and predictor forms. The innovation and predictor forms contain the Kalman gain $K$ for facilitating optimal state estimates. Let a model of the system that needs to be identified, admit an innovation form,

$$x_{k+1} = Ax_k + Bu_k + Ke_k \quad (1)$$
$$y_k = Cx_k + Du_k + e_k, \quad (2)$$

where $x_k \in \mathbb{R}^n$, $u_k \in \mathbb{R}^r$, and $y_k \in \mathbb{R}^l$ are the state, input and output vectors, and $e_k \in \mathbb{R}^l$ denotes the zero-mean white innovation process. Substituting $e_k = y_k - Cx_k - Du_k$ in the above state expression, the predictor form is obtained.

$$x_{k+1} = \hat{A}_K x_k + \hat{B}_K z_k, \quad (3)$$
$$y_k = Cx_k + Du_k + e_k, \quad (4)$$

where $z_k = [u_k^T \ y_k^T]^T$, $\hat{A}_K = A - BK$ and $\hat{B}_K = [B_K \ K] = [B - KD \ K]$. The objective of the identification algorithm is to estimate the system matrices $A$, $B$, $C$, $D$ and the Kalman gain $K$, from the measured input, output sequences $u_k$, $y_k$ respectively over a time $k = \{0, 1, ..., N - 1\}$. It is well known that SIMs determine the system matrices up to a similarity transformation.

The major difficulty with closed-loop data (in the presence of feedback) is the correlation between $u_k$, $e_k$. Conventional open-loop SIMs cannot be directly applied in such a scenario. PBSID\textsubscript{opt} circumvents this problem by solving multi-stage least squares problems when the feedback controller does not contain a direct feed-through term. To this end, the input-output data sequences are divided into past and future windows of length $p, f$ respectively ($f \leq p$). These windows are further used to define the stacked vectors,

$$\bar{y}_{k-p} = \begin{bmatrix} y_{k-p} \\ y_{k-p+1} \\ \vdots \\ y_{k-1} \end{bmatrix}, \quad \bar{y}_k = \begin{bmatrix} y_k \\ y_{k+1} \\ \vdots \\ y_{k+f-1} \end{bmatrix}. \quad (5)$$

The stacked vectors $\bar{u}_{k-p}$, $\bar{e}_{k-p}$, $\bar{z}_{k-p}$ and $\bar{z}_k$ are defined in similar fashion. The stacked matrices $Y$, $U$, $X$ and $Z$ are

$$Y = [y_p, y_{p+1}, ..., y_{N-1}], \quad (6)$$
$$U = [u_p, u_{p+1}, ..., u_{N-1}], \quad (7)$$
$$X = [x_p, x_{p+1}, ..., x_{N-1}], \quad (8)$$
$$Z = [\bar{z}_0, \bar{z}_1, ..., \bar{z}_{N-p-1}, \bar{z}_{N-p}]. \quad (9)$$

PBSID\textsubscript{opt} proceeds by first obtaining the Markov parameters (defined below) from a least squares problem. The structure in the Hankel matrices is exploited to obtain an estimate of the observability matrix. This is in turn used to estimate the state-sequence from a low-rank approximation problem. The state-space matrices and the Kalman gain are finally estimated by solving two least squares problems.

A. Estimation of Markov parameters:

The predictor form given by equations (3), (4) can be written as a transfer function in the forward shift operator $z$,

$$y_k = G(z)u_k + (I_z - H(z))y_k + e_k. \quad (10)$$

$$G(z) = D + C(z I^n - \hat{A}_K)^{-1}B_K, \quad (11)$$
$$H(z) = I_z - C(z I^n - \hat{A}_K)^{-1}K. \quad (12)$$
can be replaced by equivalent power series expansions

\[ G(z, \Xi) = \Xi_0 + z^{-1}\Xi_1 + z^{-2}\Xi_2 + \ldots, \quad (13) \]
\[ H(z, \Xi) = I^l - z^{-1}\Xi_1 - z^{-2}\Xi_2 + \ldots, \quad (14) \]

where,

\[ \Xi_i = \begin{cases} D & i = 0 \\ C\bar{A}_K^{-1}B_K & i > 0 \end{cases} \quad \text{and} \quad \Xi'_i = C\bar{A}_K^{-1}K, \quad (15) \]

are the Markov parameters. The above power series expansion is truncated by considering a finite number of terms corresponding to a past window of size \( p \). The approximated one-step ahead predictor is

\[ \hat{y}_{k|k-1} = G_p(z, \Xi)u_k + (I^l - H_p(z, \Xi))y_k. \quad (16) \]
\[ G_p(z, \Xi) = \sum_{i=0}^{p} \Xi_i z^{-i}, \quad H_p(z, \Xi) = I^l - \sum_{i=1}^{p} \Xi'_i z^{-i}, \quad (17) \]

are the finite polynomial matrices defining the so-called VARX model. Note that the one-step ahead predictor given in equation (16) can be re-written as

\[ \hat{y}_{k|k-1} = \begin{bmatrix} z_{k-p} \\
\vdots \\
z_{k-1} \\
u_k \end{bmatrix}_{\Xi}. \quad (18) \]

Define \( \Psi = [\bar{Z}^T \ U^T]^T \) and introduce the stacked matrices \( \bar{Y}, \ \Psi \) in to the one-step ahead predictor equation (18). The unknown Markov parameters can then be estimated by solving the least squares problem

\[ \min_{\Xi} \| Y - \Xi\Psi \|_F^2. \quad (19) \]

Practical considerations during an identification experiment often restrict the class of excitation signals. In addition, presence of feedback contributes to rendering the stacked matrix \( \Psi \) ill-conditioned. Due to these limitations the Markov parameters are sensitive to perturbations on the measured data. A standard trick to solve an ill-posed least squares problem is to include a regularization quantity to the cost function as

\[ \min_{\Xi} \left[ \| Y - \Xi\Psi \|_F^2 + \rho^2\| \Xi \|_F^2 \right]. \quad (20) \]

The regularization quantity \( \rho \) can be chosen from well established methods and we use the Tikhonov method in combination with the generalized cross validation (GCV) technique.

B. Estimation of state-sequence:

The state vector \( x_k \) at time instant \( k \) can be obtained by driving equation (3) with input sequence \( z_{k-p}, \ z_{k-p+1}, \ldots, z_k \), and initial condition \( x_{k-p} \) at time instant \( k - p \),

\[ x_k = \bar{A}_K x_{k-p} + \bar{K}z_{k-p}, \quad (21) \]

where \( \bar{K} = \begin{bmatrix} \bar{A}_K^{-1}B_K & \bar{A}_K^{-2}B_K & \ldots & \bar{A}_K B_K & B_K \end{bmatrix} \).

In view of the truncation of the power series expansion of the transfer functions to contain only the first \( p \) terms, it is reasonable to approximate \( \bar{A}_K^i = 0 \) for all \( i \geq p \). The matrix \( \bar{A}_K \) is then said to be deadbeat with degree \( p \).

\[ x_k = \bar{K}z_{k-p}. \quad (22) \]

A key ingredient to the success of \( \text{PBSID}_{\text{opt}} \) is the observation that the product of the observability matrix and the state vector is

\[ \hat{\Gamma}_x = (\bar{\Gamma}\bar{K})z_{k-p}, \quad (23) \]

where,

\[ \bar{\Gamma} = \begin{bmatrix} C^T \ (\bar{A}_K)^T \cdots \ (\bar{A}_K^{l-1})^T \end{bmatrix}^T. \quad (24) \]

Note that equation (23) can be used to estimate the state vector \( x_k \), provided \( \bar{\Gamma} \) is known. Taking the matrix product and exploiting the fact that \( \bar{A}_K^i = 0 \) for all \( i \geq p, \)

\[ \bar{\Gamma} \tilde{K} = \begin{bmatrix} C\bar{A}_K^{p-1}B_K & C\bar{A}_K^{p-2}B_K & \cdots & \bar{B}_K \\
0 & \bar{A}_K^{p-1}B_K & \cdots & \bar{A}_K B_K \\
\vdots & \vdots & \ddots & \vdots \\
0 & \cdots & 0 & \bar{A}_K^{l-1}B_K \end{bmatrix}. \quad (25) \]

Further, observe that the entire matrix \( \bar{\Gamma} \tilde{K} \) can be constructed from only the elements of the first block row (in equation (25)), which is equivalent to \( \bar{C} \bar{K} \). This is precisely equal to a partitioned block of the Markov parameter estimated in the previous step. Hence

\[ \Xi = [\bar{C}\bar{K} \ D], \quad (26) \]

can be used to construct \( \hat{\Gamma} \bar{K} \) as

\[ \hat{\Gamma} \bar{K} = \begin{bmatrix} \Xi(:,1 : pm) \\
[0_{l \times m} \Xi(:,1 : (p-1)m)] \\
[0_{l \times 2m} \Xi(:,1 : (p-2)m)] \\
\vdots \\
[0_{l \times (f-1)m} \Xi(:,1 : (p-f+1)m)] \end{bmatrix}, \quad (27) \]

where \( m = r+l \). Using the stacked matrices defined earlier, an estimate of the state-sequence \( \bar{X} \) can be obtained by solving low-rank approximation problem

\[ \min_{\bar{X}} \| (\hat{\Gamma} \bar{K}) \bar{Z} - \hat{\Gamma} \bar{X} \|_F, \quad (28) \]

where \( \hat{\Gamma} \) represents an estimate of the observability matrix. A standard procedure to compute the low-rank approximant \( \hat{\Gamma} \bar{X} \) satisfying (28) is to use the singular value decomposition

\[ (\hat{\Gamma} \bar{K}) \bar{Z} \approx [U \ U_\perp] \begin{bmatrix} \Sigma_n & 0 \\
0 & \Sigma \end{bmatrix} \begin{bmatrix} V \\
V_\perp \end{bmatrix}. \quad (29) \]

The matrix \( \Sigma_n \) contains the \( n \) largest singular values which corresponds to the order of the system. The orthogonal matrix \( V \) contains the corresponding row space. By setting the remaining singular values to zero, the observability matrix and the state-sequence can be estimated as

\[ \hat{\Gamma} = U \Sigma_n^{1/2}, \quad (30) \]
\[ \bar{X} = \Sigma_n^{1/2} V. \quad (31) \]
C. Estimating the State space matrices:

The state-sequence $\hat{X}$ estimated above can be used along with the input and output data matrices $Y$, $U$ to estimate the state-space matrices by solving the following two least squares problems.

- Minimize
  \[
  \left\| Y(:,1:N-p-1) - \begin{bmatrix} C & D \end{bmatrix} \begin{bmatrix} \hat{X}(:,1:N-p-1) \\ U(:,1:N-p-1) \end{bmatrix} \right\|_F^2,
  \tag{32}
  \]
  over the variable $[C \ D]$.

- Minimize
  \[
  \left\| X(:,2:N-p-1) - \begin{bmatrix} A & B & K \end{bmatrix} \begin{bmatrix} \hat{X}(:,1:N-p-1) \\ U(:,1:N-p-1) \\ E(:,1:N-p-1) \end{bmatrix} \right\|_F^2,
  \tag{33}
  \]
  over the variable $[A \ B \ K]$. The data matrix $E = Y - C\hat{X} - DU$ is computed using a solution to the first problem.

Note that the literature is abundant of methods to estimate the system matrices from the estimated observability matrix.

III. SYSTEM DESCRIPTION

The MBC500 experimental setup used in this research is shown in Figure 1. It consists of a horizontally suspended spindle supported by two magnetic bearings at its ends. The system is provided with 4 decentralized SISO controllers which can be switched on/off. The input-output measurements for identification are obtained by stabilizing the system with the internal controllers switched on. The system contains 4 input-ports for each of actuators and 8 output-ports (4 for the position sensors and 4 for control input measurement). These input-output ports can be connected to a digital processor via A-D and D-A converters. The dSPACE DS1103 processor board is used for data-acquisition and later on for robust controller implementation.

A schematic view of the magnetic bearing-spindle-sensor assembly is shown in Figure 2. A pair of opposing electromagnets, and position sensors are located in the horizontal and vertical planes on either ends of the spindle. The system dynamics in the horizontal and vertical planes are assumed to be decoupled (neglecting gyroscopic/electro-magnetic coupling). Hence, the entire AMB setup can be modeled by two $2 \times 2$ MIMO subsystems corresponding to each plane. Robust controllers are designed for each of these subsystems.

IV. EXPERIMENTAL RESULTS

The choice of excitation signal in an identification experiment greatly determines the outcome of the identification algorithm. In view of the recommendations made in the literature, we use a pseudo random binary sequence (pbrs-signal) to excite the horizontal and vertical plane dynamics in separate experiments. The point of excitation within the closed-loop interconnection is shown in Figure 3. A pbrs-signal is known to contain infinite frequency components and is therefore passed through a low-pass filter which is cut-off at 800 Hz (5026 rad/s). This filtered pbrs-signal is used for excitation and data acquisition is done at a sampling frequency of 10 kHz. The acquired data is checked for outliers and then down-sampled by a factor of 6 to carry out the PBSID$_{opt}$ algorithm.
The order of the system \( (n) \) is determined by the SVD of \( (\tilde{\Gamma} \tilde{K}) \bar{Z} \), and the state-sequence matrix \( \hat{X} \) is subsequently obtained. The matrix \( (\tilde{\Gamma} \tilde{K}) \) is built from the Markov parameters using equation (27). The singular values of the data matrix \( (\tilde{\Gamma} \tilde{K}) \bar{Z} \) for each of the horizontal and vertical planes are shown in Figure 4. Clearly, the values are separated into two sets from the \( 8^{th} \) singular value onwards. The system order is therefore taken as \( n = 8 \).

The performance weight \( W_\text{SS} \), corresponding to the horizontal and vertical planes respectively, is chosen as an additive uncertainty. It is desired to synthesize a robustly stabilizing controller against this uncertainty. The uncertain plant model for \( i = 1, 2 \) corresponding to the horizontal and vertical planes respectively:

\[
G_{\text{unc}} = G_i + \Delta, \quad \|\Delta\|_\infty < 1, \quad (34)
\]

For the horizontal plane dynamics, Figure 6 shows the identified flexible model by the dash-dotted line, the nominal plant model \( G_i \) by the solid line and the additive uncertainty weight \( W_\Delta \) by the dashed line. An \( H_\infty \) control synthesis problem is posed to meet the following objectives.

- Robust stability
  \[
  \|W_\Delta K_i (I + G_i K_i)^{-1}\|_\infty \leq 1 \quad (35)
  \]
- Nominal performance
  \[
  \|W_y (I + G_i K_i)^{-1}\|_\infty \leq 1 \quad (36)
  \]

The performance weight \( W_y = \frac{0.67s + 600}{s + 0.6} I \) is chosen as an upper bound on the closed-loop sensitivity. The interconnection shown in Figure 7 is used to synthesize an \( H_\infty \) controller.

V. ROBUST CONTROL DESIGN

We first synthesized an \( H_\infty \) controller using the identified \( 8^{th} \) order model. The resulting controller is found to be unstable and could not be implemented. Hence, a reduced order model is obtained by neglecting the flexible dynamics. The neglected dynamics is accounted for by means of an additive uncertainty. It is desired to synthesize a robustly stabilizing controller against this uncertainty. The uncertain plant model for \( i = 1, 2 \) corresponding to the horizontal and vertical planes respectively:

\[
G_{\text{unc}} = G_i + \Delta, \quad \|\Delta\|_\infty < 1, \quad (34)
\]
Fig. 7. Interconnection for controller synthesis
(K_i, i = 1, 2) for both the horizontal and vertical planes. The controller K_i is synthesized with H_\infty performance index \gamma taking values of 3.14 and 3.41 for the horizontal and vertical plane dynamics respectively. The synthesized controllers are implemented on the dSPACE DS1103 processor in closed-loop with the MBC500 setup. The controllers are implemented at a sampling frequency of 25 kHz and successfully support levitation of the spindle. The spindle displacements on one-end with the H_\infty controllers and with the internal controllers, at a rotational speed of 750rpm, are shown in Figure 8. It is well-known that rotating spindles are subject to periodic disturbances with frequency equal to the rotational speed (due to unbalanced mass). Clearly, the synthesized controllers exhibit better disturbance rejection property. Note that the internal controllers do not contain integral action and hence the displacements are centered at a non-zero location.

Fig. 8. Displacement of the spindle at sensor locations

VI. CONCLUSIONS

The necessary steps from data acquisition to model identification using PBSID_{opt} leading to successful design and implementation of H_\infty controller have been elaborated in this research. The PBSID_{opt} algorithm overcomes the problem of noise correlation in input-output measurement of closed-loop systems by dividing the identification problem into three subproblems. It relies solely on matrix factorization and least squares techniques. It does not require any prior knowledge of the parameters. The order of the system is estimated from the singular value distribution of a particular data matrix. The obtained estimates of system matrices are known to be consistent from theory. These features are fully exploited in identifying a model for the MBC500 experimental setup. We demonstrated that the identified model can be readily used for synthesis of H_\infty controllers. This lays down a strong foundation for our continuing research on implementing high performance LMI based controllers, [5].

ACKNOWLEDGMENT

We gratefully acknowledge the support of MicroNed, a consortium to nurture micro-systems technology in The Netherlands.

REFERENCES

ization of a miniature spindle test setup with active magnetic bearings,” in Advanced intelligent mechatronics, 2007 ieee/lasme international conference on, Zurich, Switzerland, Sept. 2007.
cients of active magnetic bearings,” in Proc. of 8th International Symposium on Magnetic Bearings, Mito, Japan, August 2002.