Cooperative DYC System Design For Optimal Vehicle Handling Enhancement

Seyed Hossein Tamaddoni, Saied Taheri, Mehdi Ahmadian

Abstract—In many severe maneuvers, the driver-controller interaction seems necessary for maintaining a stable vehicle. This paper introduces a novel cooperative direct yaw control (DYC) design for optimal vehicle stability control in the presence of human driver. The interaction is defined by forming a differential linear quadratic game between the driver who is controlling the steering angle and the controller which is controlling the brake torques. Evaluated by a nonlinear vehicle model, numerical simulations are presented for a vehicle in the standard fishhook test. Preliminary results show the effectiveness of this controller over a commonly used linear quadratic controller.

I. INTRODUCTION

Vehicle direct yaw control system (DYC) as a conventional part of vehicle electric stability control (ESC) aims to help the vehicle follow the cornering line intended by the driver more faithfully and naturally, greatly enhancing its stability. These systems have been developed and recently commercialized by several companies since 1990’s. A comprehensive literature review conducted by Ferguson reveals that ESC can effectively reduce single-vehicle crashes by 30-50% among cars and 50-70% among SUVs. Also, fatal rollover crashes are estimated to be about 70-90% lower with VSC regardless of vehicle type [1].

Direct yaw control (DYC) is one of the most effective methods of active chassis control which can considerably enhance the vehicle stability and controllability [2]. For vehicle control, the yaw moment control is considered as a way of controlling the lateral motion of a vehicle during severe maneuvers using active steering control (ASC), or active differential braking using anti-lock braking system (ABS). The former is the most common way to control the vehicle motion by actively steering the vehicle, which is useful in low lateral acceleration maneuvers (i.e. below 0.3 g) with normal driving conditions [3]. The latter handles the yaw motion of a vehicle by producing an external yaw moment through differential braking [4]. The second approach is suitable for high-g maneuvers wherein the tires are saturated and are no longer capable of producing enough lateral forces by steering to control the vehicle.

Since vehicle stability controllers (VSC) usually work with a human driver co-existing, the overall vehicle performance will depend on its interaction with the human driver, not how well the VSC works by itself [5]. The evaluation of VSC, therefore, needs to be conducted with driver in the loop. For maneuvers with short durations, the driver modeling problem becomes considerably simpler due to the fact that most of the existing VSC perform clearly defined functions such as vehicle sideslip angle regulation, etc. Within short time horizons, it is assumed that human drivers neither demonstrate significant learning or adaptation, nor switching among complex rules. Hence, the driver can be modeled by optimal linear quadratic (LQ) preview control approach [6, 7]. The influences of preview horizon, control horizon and cost function are investigated in [7], and it is found that for the case of long preview and long control horizons, the predictive and LQ approaches will result in identical controllers.

In this paper, game theory is applied to develop an optimal cooperative DYC strategy. In the proposed vehicle stability formation, the driver, who steers the steering wheel, and DYC, which exerts the compensated steering wheel, are considered the dynamic players/agents in the differential game, and Nash optimal linear theory is applied to derive the optimal controller feedback gains that guides the vehicle to more stability and better handling performance.

Fig. 1. Vehicle control/update diagram
For the purpose of this study, it is assumed that the driver response dynamics remain unchanged and the preview window is negligible during short time maneuvers. For cases where the driver dynamics vary with time, one can implement an updating routine such as the one shown in Fig. 1. The inclusion of driver models with preview time in the proposed system of control is the main topic for future research.

The proposed controller is evaluated by a nonlinear vehicle model and compared with the DYC strategy defined in [8] that is based on a simple linear quadratic regulator with no dynamic interaction with the driver. The results and discussion are given in the final sections of this paper.

II. VEHICLE MODELS

A detailed standard nonlinear vehicle model is used in numerical simulations to analyze the response of the controlled vehicle. The model includes nonlinear tire models according to combined sideslip theory [9], nonlinear spring model, nonlinear front steering system, and incorporates the major kinematics and compliance effects in the suspension and steering systems including differential load transfer for each wheel. However, to design the controller, a widely used simplified linear single track vehicle model is considered which captures the essential vehicle steering dynamics. In this respect, the tire forces are assumed to be linear functions of tire slip angle.

A. Evaluation Model

In order to study the handling and roll dynamic responses of the vehicle, a nonlinear model of a vehicle is derived which includes longitudinal and lateral translational motions, and roll and yaw motions with rotational dynamics of each of the four wheels [10]. It must be mentioned that the roll dynamics and the suspension compliance properties play an important role in providing a more realistic simulation environment for evaluating the control algorithm that will be developed in this paper.

Based on the vehicle coordinate system, parameters, and external forces depicted in Fig. 2, the nonlinear vehicle model is derived by writing the translational and rotational equations in the vehicle fixed coordinate frame

\[
m(v + v_y) - m_h(v_\phi + 2v_\psi) = (F_{s,LR} + F_{s,FL}) \cos \delta_f - (F_{s,LR} + F_{s,FL}) \sin \delta_f + (F_{s,LR} + F_{s,FL}) + (F_{s,LR} + F_{s,FL}) \cos \delta_f - (F_{s,LR} + F_{s,FL}) \sin \delta_f.
\]

\[
v_i \dot{\psi} + (I_{s,LR} \phi_b - I_{s,FL}) \phi - m_h(v_i \phi + v_y \psi) =
\]

\[
-I_p (F_{s,LR} + F_{s,FL}) \sin \delta_f + I_p (F_{s,LR} + F_{s,FL}) \cos \delta_f - I_p (F_{s,LR} + F_{s,FL}) \sin \delta_f + I_p (F_{s,LR} + F_{s,FL}) \cos \delta_f + I_p (F_{s,LR} + F_{s,FL}) \cos \delta_f - I_p (F_{s,LR} + F_{s,FL}) \sin \delta_f - I_p (F_{s,LR} + F_{s,FL}) \cos \delta_f - I_p (F_{s,LR} + F_{s,FL}) \sin \delta_f - I_p (F_{s,LR} + F_{s,FL}) \cos \delta_f - I_p (F_{s,LR} + F_{s,FL}) \sin \delta_f - I_p (F_{s,LR} + F_{s,FL}) \cos \delta_f - I_p (F_{s,LR} + F_{s,FL}) \sin \delta_f.
\]

Tire forces and moments are calculated based on a widely used semi-empirical tire model based on trigonometric functions known as Magic Formula [9]. Magic-Formula tire
models are considered the state-of-the-art for modeling tire-road interaction in vehicle dynamics applications.

B. Control Model

The commonly used single track bicycle model is considered in this paper [11]. This model captures the needed dynamic information for yaw as well as lateral degrees of freedom. In order to derive the equations, it is assumed that vehicle motion is represented by its global lateral position and velocity, and the yaw angle and yaw rate at the vehicle center of mass as shown in Fig. 3. The state variable vector becomes

\[ x = (y \ y \ \psi \ \dot{\psi})^T \]  

(10)

where as shown in Fig. 3, \( y \) is the global lateral position of the vehicle CG, \( \dot{y} \) is the global lateral velocity in Y direction with respect to a fixed ground coordinate O, and \( \psi, \dot{\psi} \) are yaw angle and yaw rate, respectively.

![Fig. 3. Vehicle control model](image)

For the sake of simplicity, the mathematical model is linearized around the operating conditions

\[ x^* = 0, \ \delta_m = 0, \ \omega_y = 0 \]

Thus, the equation of motion for a constant forward speed is given by:

\[
\begin{align*}
\dot{x}(t) &= Ax(t) + Bu_1 + Bu_2, \\
x(0) &= x_0,
\end{align*}
\]

(11)

with

\[
A = \begin{bmatrix}
0 & 1 & 0 & 0 \\
-C_{af} + C_{ab} & \frac{m_v}{v} & -l_x C_{af} - l_y C_{ab} \\
0 & 0 & 0 & 1 \\
-l_y C_{af} - l_y C_{ab} & \frac{I_y}{v} & -l_x C_{af} - l_y C_{ab} \\
I_y & 0 & 0 & 1
\end{bmatrix}, \\
B_1 = \begin{bmatrix}
0 & 0 & 0 & 0 \\
\frac{C_{af}}{r_c m} & \frac{I_y}{I_z} & 0 & 0
\end{bmatrix}, \quad B_2 = \begin{bmatrix}
0 & 0 & 0 & 0
\end{bmatrix}
\]

where \( u_1 \) is the steering wheel angle (\( \delta_m \)), \( u_2 \) is the compensated yaw moment (\( M_w \)), and \( C_{af}, C_{ab} \) denote the front and rear tire cornering stiffness, respectively, relating linear tire forces to their corresponding linear sideslip angles.

III. CONTROL ALGORITHM DESIGN

A. Control Concept

Direct Yaw Control (DYC) system aims at improving vehicle stability and handling performance. Based on Wong’s findings (2000), vehicle stability is guaranteed provided that the controller can keep the vehicle yaw rate close to the desired value that can be dynamically calculated based on the driver’s steering input and vehicle forward speed:

\[
\psi_{desired} = \frac{v_i}{(l_x + l_y)(1 + k_u v_i^2)} \delta_y \]

(12)

where \( k_u \) or so-called understeering coefficient is a positive constant which is a measure of vehicle stability.

B. Controller Design

Consider the two-player infinite-horizon linear quadratic differential game described in (11). For each player, i.e., the driver (\( u_1 = \delta_m \)) and the controller (\( u_2 = M_w \)), a quadratic cost function is given by

\[
J_i = \int_0^\infty \left( x^T Q x + u_1^T R_1 u_1 + u_2^T R_2 u_2 \right) dt,
\]

(13)

where all weighting matrices are constant and symmetric with \( Q \geq 0, R_1 = D_1^T D_1 \geq 0 \) and \( R_2 = D_2^T D_2 > 0 \), and \( i = 1,2 \) is the player number.

The Nash equilibrium for differential games is defined such that it has the property that there is no incentive for any unilateral deviation by any one of the players [12]. In other words, at Nash equilibrium, the player who chooses to change his/her strategy cannot improve his/her payoff.

Restricting the optimal solution to the class of linear feedback strategies:

\[
\Gamma_i^* = \{ u_i \in \Gamma_i | u_i(x,t) = G_i(t)x^* \},
\]

(14)

there exists a generically unique linear feedback Nash equilibrium given by coupled Riccati equation [12, 13].

\[
A^* K_i + K_i A + Q_i - K_i S_i K_i - K_i S_i K_i
\]

(15)

\[
A^* K_i + K_i A + Q_i - K_i S_i K_i - K_i S_i K_i
\]

(16)

where

\[
S_i = B_i^T R_i^* B_i, \quad S_i = B_i^T R_i^* B_i
\]

Then the pair of strategies

\[
(u_1^*, u_2^*) = (-R_i^* B_i^* K_i(x(t)), -R_i^* B_i^* K_i(x(t)))
\]

(17)

are linear feedback Nash equilibriums.

IV. SIMULATION AND RESULTS

Computer simulations are carried out to verify the effectiveness and robustness of the control algorithms. Therefore, the presented controller is evaluated using the
nonlinear model with the control objective to guarantee handling performance in the standard fishhook test maneuver using the following desired states:

$$x_{\text{desired}} = (y_{\text{desired}} \quad 0 \quad 0 \quad \psi_{\text{desired}})^T,$$

where the desired lateral position is given as a function of longitudinal position in Fig. 4. The test is defined by a predetermined cone placement in the road and the maneuver is carried out on a dry surface.

Initial vehicle conditions are set to:

$$x(t_0) = (0 \quad 0 \quad 0 \quad 0)^T.$$  

The optimal strategies defined in (17) are computed for a sedan at the standard nominal speed $v_s = 60 \text{kph}$. The vehicle parameter values are given in Table I.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vehicle mass</td>
<td>1418 (kg)</td>
</tr>
<tr>
<td>Vehicle yaw moment of inertia</td>
<td>1819 (kg m^2)</td>
</tr>
<tr>
<td>Steering gear ratio</td>
<td>19.5</td>
</tr>
<tr>
<td>Front axle distance from CG</td>
<td>1.012 (m)</td>
</tr>
<tr>
<td>Rear axle distance from CG</td>
<td>1.568 (m)</td>
</tr>
<tr>
<td>Front cornering stiffness</td>
<td>112000 (N/rad)</td>
</tr>
<tr>
<td>Rear cornering stiffness</td>
<td>84000 (N/rad)</td>
</tr>
</tbody>
</table>

To evaluate the performance of the proposed Nash-based control algorithm, a common linear quadratic regulator (LQR) is also developed based on what is presented in [8] and evaluated in such a way that driver’s and controller’s gains are optimized independently for the system using the parameters provided below:

$$Q_x = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0.1 & 0 & 0 \\ 0 & 0 & 0.1 & 0 \\ 0 & 0 & 0 & 0.01 \end{bmatrix},$$

$$Q_u = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0.1 & 0 & 0 \\ 0 & 0 & 10 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$R_x = 10, \quad R_{\psi} = 0, \quad R_{y} = 10^{-3}, \quad R_{\Delta y} = 10^{-5}. $$

Using the same $Q$ and $R$ matrices, the final state feedback gains for both strategies are calculated.

It must be mentioned that the gain $G_{u}$ for global lateral position is negligible and will be zeroed out since the vehicle controller has no information regarding the desired lateral position. This is due to the fact that the desired lateral position is decided upon by the driver during the performed maneuver and cannot be known to the controller or the states.

Fig. 5 shows the simulation results of the vehicle performance with Nash and LQR optimization strategies for the standard Fishhook test. Fig. 5b indicates that while in both cases the vehicle yaw rates are following their corresponding desired values to provide the desired handling performance, the maximum value of yaw rate is lower with Nash controller compared to the common LQ regulator.

Fig. 6 shows the time history of vehicle sideslip angle of the front and rear axles for cases with Nash and LQR controllers. Fig. 6 illustrates that both Nash and LQR controllers limit the vehicle lateral instability satisfactorily with Nash controller performing better on side slip minimization, hence, providing more stability margin to the vehicle compared to the linear quadratic regulator.
Fig. 6. Time history of sideslip angle

Fig. 7 reports the driver’s steering angle and the compensated yaw moment required to maneuver the vehicle for the Fishhook test using Nash strategy as well as LQR approach. The maximum value of 900 N.m for the required yaw moment can be generated by a differential right-left braking strategy and seems affordable for a common sedan by the current Anti-lock Braking System (ABS) technology.

Fig. 7. Time history of control inputs for certain driver model: (left) steering wheel angle, and (right) compensated yaw moment

For better insight into the system performance, the cost sub-functions of the driver and the controller defined in (13) are plotted separately in Fig. 8. The final value of the cost functions can be found by integrating the sub-functions over time which is tabulated in Table II.

Fig. 8. Comparison of cost functions: (right) driver cost function, (left) controller cost function

Fig. 8 indicates that the Nash strategy results in less value of objective (cost) function as compared to the linear quadratic regulator (LQR). Therefore, in cases where the driver dynamics are known to the controller (by offline/online identification methods), the control designer can take advantage of the extra significant piece of information (driver estimated response) to form a differential cooperative game and obtain the Nash optimal feedback gains in order to accomplish the control task.

V. CONCLUSION

Based on Game Theory, a novel cooperative optimal control strategy is introduced. Using the definition of a linear quadratic differential game, the driver’s steering input and the controller’s compensated yaw moment are defined as two dynamic players of the game “vehicle stability”, and their corresponding control efforts are optimized through Nash optimal strategy.

As known by vehicle dynamists, driver workload and response time are considered vital in the safe outcome of

| TABLE II |
| COEFFICIENT VALUES |
| unit \ strategy | Nash | LQR |
| driver | 9.7363 × 10^4 | 1.6206 × 10^5 |
| controller | 9.7347 × 10^4 | 1.6204 × 10^5 |
accident avoidance maneuvers. Throughout this study, the driver is part of the simulation loop and decides the steering wheel angle $\delta_{sw}$ which can be approximated by the controller through an offline or online update routine. Taking advantage of the extra significant piece of information by estimating driver response to the road, game theory shows that the proposed controller is more optimal than the common linear quadratic regulators (LQR) even in the presence of small deviations between the actual driver steering angle and the estimated angle. In case of significant angle error, the driver response dynamics can be updated online or an additional steering angle can be added to the existing driver input via active steering control procedures.

In the game system presented in this paper, the driver’s input and the controller optimal solution are plotted and compared with common LQR’s. The simulation results show that the proposed ‘Game Theory’-based controller not only stabilizes the system due to its activation based fast response, but also involves the controller more in stabilizing the vehicle compared to common LQR controllers hence reducing the driver’s workload.

An advantage of the proposed controller not to be overlooked is the stability aspect. The side slip control in the case with the proposed controller is indicative of improved vehicle handling stability. Although not demonstrated in this work, more extensive vehicle testing is likely to show that a closed-loop yaw control system based on Nash strategy not only improves vehicle performance, but also increases driver confidence in an emergency maneuver, because the controller takes more responsibility when Nash strategy is applied. However, based on the existing subtleties in workload distribution between the driver and the controller, the proposed controller needs to be carefully studied in the future.

REFERENCES