Leader follower based formation control strategies for nonholonomic mobile robots: Design, Implementation and Experimental Validation

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Abstract—This paper proposes novel formation maintenance strategies for multiple nonholonomic mobile robots based on nonholonomic trajectory tracking techniques and dynamic feedback linearization. It also presents experimental results for formation stability and noise tolerance of the proposed and existing leader-follower based controllers using physical P3AT robots. The research focuses only on the problem of formation maintenance by multiple nonholonomic mobile robots. Two types of formation maintenance controllers are developed by transforming the follower robot’s motion into to separate trajectory tracking tasks and then by applying existing nonholonomic trajectory tracking techniques. A third controller is developed through the use of dynamic feedback linearization. The proposed systems are implemented in physical P3AT type mobile robots and real-world experimental results are shown to compare the formation accuracy and the stability of these controllers.

Index Terms - multi-robot formation control, nonholonomic mobile robots, trajectory tracking

I. INTRODUCTION

Formation control of multiple robots is inspired partly by the necessity of the nature of the tasks and partly by the formation behavior of schools of fishes or flocks of birds [1], where multiple agents combine their senses for efficient food finding or combining their thrust in the liquid or air to move forward as one pack. It’s usage has been explored for search and rescue missions [2],[3], reconnaissance and patrols [4], satellite control and automated highways [5].

The basic formation control problem consists of maintaining desired geometric formations of varied shapes and sizes e.g.: triangular, line, or column. Many types of control techniques have been proposed to tackle the formation maintenance problem. Some techniques involve leader-follower formation control [6],[7],[8], virtual structure approach [9], behavior-based formation control [1] and consensus based formation control [10]. Although centralized, the leader-follower based formation control strategies have been widely exploited for formation control applications owing to their flexibility, simple operation, scalability and lesser computational demand. The research problem addressed in this paper revolves around the leader-follower formation control paradigm.

1) Problem I: Nonlinear control laws for formation control of nonholonomic mobile robots: The formation control can be thought of as a combination of trajectory tracking and posture stabilization [11] of nonholonomic mobile robots. There exists a number of nonlinear time invariant, time varying or discontinuous [12],[11],[13] control techniques proposed and implemented for posture stabilization and trajectory tracking of nonholonomic mobile robots. These numerous nonlinear control techniques have not been exploited for formation control of multiple nonholonomic mobile robots. The extensive literature review also suggests that, out of all the existing leader-follower based control theoretic approaches, the \( l - \psi \) controller proposed by Desai et al.[8] through static feedback linearization has more flexibility and scalability for formation control applications. But since this controllers stabilize not the follower robot’s origins (origin refers to the origin of the robot coordinate system where the calculated robot pose is taken through it’s odometry readings), but an offset from their origins to desired geometric poses leads to: 1.) not fulfilling the real objectives of formation control, which is to stabilize the origins of the robot platform (origin of the follower robot coordinate system) to desired formation locations 2.) The controller may exhibit some form of instability under noisier inputs.

2) Problem II: Comparative study of Leader-follower based control theoretic approaches: The lack of a substantial comparison of different leader-follower based formation control approaches in terms of their stability, rate of convergence and noise tolerance is identified as a potential issue. It is also identified that the real world implementation problems related to platform dynamics, wheel slippage, noises in observation etc. have also not been sufficiently evaluated for the proposed leader-follower based formation controllers through real world experiments.

The key contribution of this paper can be listed as, 1.) development of trajectory tracking type formation maintenance controllers and a dynamic feedback linearized formation maintenance controller. 2.) comparison of trajectory tracking, static feedback linearized and dynamic feedback linearized formation maintenance controllers in terms of formation accuracy, noise tolerance, smoothness of control inputs using P3AT mobile robots.

II. LEADER-FOLLOWER BASED FORMATION MAINTENANCE CONTROLLERS

This research proposes three types of formation maintenance controllers for the nonholonomic unicycle robots. Two of such controllers are developed through virtual robot path tracking techniques 1.) based on the approximate linearization of the unicycle dynamics described in [11], 2.)
based on a Lyaponov-based nonlinear time varying design described in [12]. The third controller is developed through dynamic feedback linearization. These three controllers will be compared for formation stability between themselves and also with a fourth leader-follower based static feedback linearized formation controller developed in [8]. The real world experimental validation of the proposed and existing control schemas are carried out in P3AT mobile robots. The kinematics of the P3AT mobile robots can be described by the unicycle dynamics given as,

$$
\dot{x} = v \cos \theta, \quad \dot{y} = v \sin \theta, \quad \dot{\theta} = \omega
$$

(1)

Where $$x = (x, y, \theta) \in SE(2)$$ and $$v$$ and $$\omega$$ are the linear and angular velocities respectively. P3AT is a four wheel differential drive mobile robot with 50cm x 49cm x 26cm aluminum body with 21.5cm diameter drive wheels. The P3AT robot also has saturation levels of the linear and angular velocities and linear and angular accelerations.

$$v \leq v_{\text{max}} = 0.6 \text{ms}^{-1}, \quad |\omega| \leq \omega_{\text{max}} = 0.75 \text{rads}^{-1}$$

$$|a| \leq a_{\text{max}} = 0.3 \text{ms}^{-2}, \quad |\alpha| \leq \alpha_{\text{max}} = 0.8 \text{rads}^{-2}$$

$$a$$ and $$\alpha$$ are the linear and angular accelerations respectively. $$v_{\text{max}}, \omega_{\text{max}}$$ are the absolute maximum linear and angular velocities of the robot while $$a_{\text{max}}$$ and $$\alpha_{\text{max}}$$ are the absolute maximum linear and angular accelerations of the robot respectively. In order to preserve the curvature radius originated from $$v$$ and $$\omega$$, a velocity scaling is needed as follows. If the scaled down linear and angular velocities are $$v_s$$ and $$\omega_s$$ respectively, we have:

$$\Lambda = \max\{ |v|/v_{\text{max}}, \quad |\omega|/\omega_{\text{max}}, \quad 1 \}$$

$$v_s = \text{sign}(v)v_{\text{max}}, \quad \omega_s = \omega/\Lambda \quad \text{when} \quad \Lambda = \max\{ |v|/v_{\text{max}}, \quad |\omega|/\omega_{\text{max}} \}$$

$$v_s = v/\Lambda, \quad \omega_s = \text{sign}(\omega)\omega_{\text{max}} \quad \text{when} \quad \Lambda = \max\{ |v|/v_{\text{max}}, \quad |\omega|/\omega_{\text{max}} \}$$

$$v_s = v, \quad \omega_s = \omega \quad \text{when} \quad \Lambda = 1$$

### III. VIRTUAL ROBOT TRACKING BASED FORMATION CONTROLLERS

The nonholonomic motion of the leader-robot results in a path, which can be approximated through an accumulation of straight and circular path segments [14]. The motion of any point fixed in an offset to the origin of the leader robot coordinate system results in a nonholonomic motion. In leader-follower based formation control also, the desired poses of followers can be thought of as fixed in offsets to the origin of the leader-robot coordinate system at any given time. These fixed points mimic virtual robots which the actual followers must track. Tracking such virtual robot paths and their desired velocities require a combination of a nominal feed forward command with a feedback action on the error [15]. Feed forward command generation involves calculating the pose of the virtual robot and it’s linear and angular velocities to which the actual designated follower must reach to.

**A. Feedforward Command Generation**

Assuming that the leader robot’s pose at time $$t$$ is $$[x_t, y_t, \theta_t]^T$$ and the velocities being $$[v_t, \omega_t]^T$$. The desired position for the follower can be described as located in an offset of $$o_x$$ units and $$o_y$$ units from the origin to $$X$$ and $$Y$$ directions respectively in the leader robot coordinate system. $$(x_f^T, y_f^T, \theta_f^T)$$ is the desired pose for a follower robot in the Euclidean $$SE(2)$$ coordinate system. E.q.2 is taken from Fig.1.

$$
\begin{pmatrix}
  x_f^T \\
  y_f^T \\
  \theta_f^T
\end{pmatrix} =
\begin{pmatrix}
  \cos \theta_t & -o_x \sin \theta_t & -o_y \cos \theta_t \\
  \sin \theta_t & o_x \cos \theta_t & -o_y \sin \theta_t \\
  0 & 1
\end{pmatrix}
\begin{pmatrix}
  v_t \\
  \omega_t
\end{pmatrix}
$$

(2)

The feed-forward command generation of unicycle robots involve generating desired poses $$(x_f^T, y_f^T, \theta_f^T)$$ and desired velocities $$(v_f^T, \omega_f^T)$$ at a given time $$t$$. The desired poses can be easily taken as shown in Fig.1. The desired linear and angular velocities are taken as in [15], given by,

$$v_f^T = \pm \sqrt{x_f^T + y_f^T} \quad \text{and} \quad \omega_f^T = \frac{y_f^T - x_f^T}{(x_f^T)^2 + (y_f^T)^2}$$

(3)

The $$\omega_f^T$$ is derived through defining $$\theta_f^T$$ as:

$$\theta_f^T = \arctan2(x_f^T, y_f^T) + k\pi \quad k = 0, 1$$

(4)

$$k = 0$$ for forward motion and $$k = 1$$ for backward motion respectively. It is also proven below that the definition of $$\omega_f^T$$ in e.q.3 can be approximated to the actual angular velocity of the leader robot: $$\omega_l$$.

**proof:** Through differentiation of $$\theta_f^T = \arctan2(x_f^T, y_f^T) + k\pi$$ of e.q.4, one gets,

$$\omega_f^T = \frac{y_f^T \dot{x}_f^T - x_f^T \dot{y}_f^T}{(x_f^T)^2 + (y_f^T)^2}$$

(5)

Assuming $$o_x$$ and $$o_y$$ are constants with $$v_l$$ and $$\omega_l$$ both being zero, substitution of the values of e.q.2 and it’s differentiated values of $$\dot{x}_f^T, \dot{y}_f^T, x_f^T$$ and $$y_f^T$$ in e.q.5 results in,

$$\omega_f^T = \omega_l \left( \frac{(o_x^2 + o_y^2)\omega_l - 2o_x v_l \omega_l + v_l^2}{(o_x^2 + o_y^2)\omega_l - 2o_x v_l \omega_l + v_l^2} \right) = \omega_l, \quad v_l \neq 0$$

(6)

Note that $$\omega_f^T$$ is not defined when $$v_l = 0$$. In order to overcome this discontinuity at $$v_l = 0$$, it is proposed that the follower may switch to a posture stabilization control routine [15] at $$v_l = 0$$, to move to the desired pose. Hence the feed forward commands developed are,

$$v_f^T = \pm \sqrt{x_f^T + y_f^T} \quad \text{and} \quad \omega_f^T = \omega_l$$

(7)
Trajectory tracking needs to combine the feed forward commands generated in this section with an action on the feedback...in [12],

\[
\begin{pmatrix}
v_n^t & \omega_n^t
\end{pmatrix} = 
\begin{pmatrix}
-v_f^t \omega_f^t & e_1 \\
-v_f^t \sin e_3 & e_2 \\
\end{pmatrix}
\]

(14)

where \([x_f^t, y_f^t, \theta_f^t]\) is the desired pose and \([x_f^t, y_f^t, \theta_f^t]\) is the actual follower pose at time \(t\) in the Euclidean \(SE(2))\) coordinate system. The current linear and angular velocity commands \([v_f^t, \omega_f^t]\) respectively of the follower are subjected to a nonlinear transformation, where the resulting newer velocity commands \([v_f^t, \omega_f^t]\) of the follower have the following relationship,

\[
\begin{pmatrix}
v_f^t & \omega_f^t
\end{pmatrix} = 
\begin{pmatrix}
v_f^t \cos e_3 - v_f^t & 0 \\
\omega_f^t & \omega_f^t
\end{pmatrix}
\]

(9)

Then the error dynamics become,

\[
\dot{e} = \begin{pmatrix}
0 & 0 & 0 \\
-v_f^t & 0 & 0 \\
0 & 0 & 0
\end{pmatrix} e + \begin{pmatrix}
0 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{pmatrix} \begin{pmatrix}
v_f^t & \omega_f^t
\end{pmatrix}
\]

(10)

Here \(e = [e_1, e_2, e_3]^T\). Through linearizing e.q.10 around the reference trajectory one obtains a linear time varying system. If a linear feedback law is defined as:

\[
\begin{pmatrix}
v_f^t \\
\omega_f^t
\end{pmatrix} = 
\begin{pmatrix}
-k_1 e_1 & -k_2 \text{sign}(v_f^t) e_2 \\
-k_3 e_3 & -k_3 e_3
\end{pmatrix}
\]

(11)

Where the choice of gains is (see [13]), \(k_1 = k_3 = 2c_1 (v_f^t)^2 + (\omega_f^t)^2, k_2 = c_2 |v_f^t|\) where \(c_1 \in (0, 1)\) and \(c_2 > 0\), one can substitute the controls of e.q.11 to the linearized system around the desired trajectory of e.q.10 to obtain,

\[
v_f^t = v_f^t \cos(\theta_f^t - \theta_f^t) + k_1 ((x_f^t - x_f^t) \cos(\theta_f^t + (y_f^t - y_f^t) \sin(\theta_f^t))
\]

\[
\omega_f^t = \omega_f^t + k_2 \text{sign}(v_f^t)((y_f^t - y_f^t) \cos(\theta_f^t - (x_f^t - x_f^t) \sin(\theta_f^t)) + k_3 (\theta_f^t - \theta_f^t)
\]

(12)

\([x_f^t, y_f^t, \theta_f^t]\) is the desired pose and \([x_f^t, y_f^t, \theta_f^t]\) is the current follower pose in Euclidean \(SE(2)\) coordinate system at time \(t\). \([v_f^t, \omega_f^t]\) is the desired velocity at time \(t\) and \([v_f^t, \omega_f^t]\) is the follower robot velocity input at time \(t\).

1) Experimental Results: Three P3AT mobile robots were used, two as followers and another as the leader robot. The gains of the followers were taken as \(c_1 = 0.9\) and \(c_2 = 15\). The experiments involved driving two of the P3AT followers for four different velocity courses of the leader robot as given below. Norm of the formation errors and the quality of the follower’s driving (velocity) inputs were recorded for comparison criteria.

- leader moves with constant velocities \((v_f^t, \omega_f^t)\).
- changing angular velocities of the leader \((v_f^t, \omega_f^t)\) while keeping the linear velocity a constant.

- constant angular velocity with a changing linear velocity \((v_f^t, \omega_f^t)\).
- both linear and angular velocities are changing \((v_f^t, \omega_f^t)\).

The formation geometry described above (see Fig.1), in terms of offsets of \(\alpha_0\) and \(\alpha_t\) in the respective X and Y leader robot coordinate system from it’s origin is converted to a new polar geometric system (see Fig.2) for comparison ease,

\[
d_{ls} = \sqrt{(x_l - x_s)^2 + (y_l - y_s)^2}
\]

\[
\beta_{ls} = -\theta_l + \pi + \text{atan2}(y_l - y_s, x_l - x_s)
\]

(13)

subscript \(l\) stands for the leader and \(s\) stands for the follower. \((x_l, y_l, \theta_l)\) is the pose of the leader while \((x_s, y_s, \theta_s)\) is the pose of the follower in the Euclidean \(SE(2)\) coordinate system. Hence the formation errors can be described as

Fig. 2. Formation geometry in the new coordinates system

\[e_d = d_{ls} - d_{ls}^d, e_\beta = \beta_{ls} - \beta_{ls}^d, e_\theta = \theta_l - \theta_s, (d_{ls}^d, \beta_{ls}^d, \theta_l^d)\]

are the desired and \((d_{ls}, \beta_{ls}, \theta_s)\) are the current values for the formation variables. The resulting formation errors for an example velocity course by the application of approximate linearization based formation control are depicted in Fig.3. The follower-1’s desired formation geometry is \((d_{ls}^d = 1m, \beta_{ls}^d = \frac{\pi}{2}\), \(\theta_{ls} = 0\)) and the other depicted follower-2’s desired formation variables values are \((d_{ls}^d = \sqrt{8}m, \beta_{ls}^d = \frac{\pi}{2}\), \(\theta_{ls} = 0\)). It is observed that the distance-\(d_{ls}\) and the bearing-\(\beta_{ls}\) errors converge almost to zero in both followers, while the relative orientation difference-\(\theta_{ls}\) error stays small; bounded around zero. When the desired bearing is \(\pm \frac{\pi}{2}\), the relative orientation error \(\theta_{ls}\) goes almost around zero while for other desired bearing values, the \(\theta_{ls}\) error stays bounded around zero. The velocity profiles for these two followers are depicted in Fig.4.

Fig. 3. Formation errors for two follower robots with approximate linearization based controller

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\((v_l^f, \omega_l^f)\) are the current linear and angular velocities of the follower robot. \((v_l^a, \omega_l^a)\) are the desired linear and angular velocities. \(k_2 > 0\) while \(k_1(v_l^f, \omega_l^f)\) and \(k_3(v_l^a, \omega_l^a)\) being continuous positive gain functions. \((e_1, e_2, e_3)\) are defined as in e.q.8. \((v_l^g, \omega_l^g)\) are the the new linear and angular velocity inputs to the followers. Equation.14 becomes a controller based on a Lyapunov function of,

\[
V = \frac{1}{2}(e_1^2 + e_2^2) + \frac{1}{2} \omega_l^2 \quad \text{with} \quad \dot{V} = -k_1k_2e_1^2 - k_3e_2^2 \leq 0
\]

When \(v_l^f, \omega_l^f\) and its derivatives are bounded and if \(v_l^f \to 0\) and \(\omega_l^f \to 0\) as \(t \to \infty\), the above controls in e.q.14 globally asymptotically stabilizes the origin \(e \equiv 0\) [12]. Using similar choices for the gains as in the approximate linearized formation controller, \(k_1 = k_3 = 2c_1 \sqrt{(v_l^f)^2 + (\omega_l^f)^2}\) where \(c_1 \in (0, 1)\) and \(k_2 = b > 0\), the application of the controls of e.q.14 to the error dynamics of e.q.10 results in,

\[
\begin{align*}
\dot{v}_l^f &= v_l^f \cos(\theta_l^f - \theta_l^a) + k_1 (x_l^f - x_l^a) \cos \theta_l^a + (y_l^f - y_l^a) \sin \theta_l^a \\
\dot{\omega}_l^f &= \omega_l^a + k_2 v_l^f \frac{\sin(\theta_l^a - \theta_l^f)}{\theta_l^a - \theta_l^f} ((y_l^f - y_l^a) \cos \theta_l^a - (x_l^f - x_l^a) \sin \theta_l^a) + k_3 (\theta_l^a - \theta_l^f)
\end{align*}
\]

(15)

\((x_l^f, y_l^f, \theta_l^f)\) is the desired pose and \((x_l^a, y_l^a, \theta_l^a)\) is the current follower pose in Euclidean SE(2) coordinate system at time \(t\). \((v_l^f, \omega_l^f)\) is the desired velocity at time \(t\) and \((v_l^a, \omega_l^a)\) is the follower robot velocity inputs at time \(t\).

1) Experimental Results: The gains and the desired geometric poses are chosen as similar to the above experiment. This behavior of the controller is similar to the "Approximate linearization based formation controller" above. Here also the relative orientation error stays bounded for all the bearing values except for \(\pm \frac{\pi}{2}\) where the error goes to zero.

IV. Static and Dynamic Feedback Linearized Formation Controllers

This section presents two other formation controllers whose error coordinates are transformed to the new coordinate system as shown in Fig.2. One such controller is developed in this research through dynamic feedback linearization while a fourth static feedback linearized formation controller with the new error coordinates, developed in [8] is used for comparison purposes. Differentiation of the formation variables of e.q.13 results in,

\[
\begin{pmatrix}
\dot{d}_{ls} \\
\dot{\beta}_{ls} \\
\dot{\theta}_{ls}
\end{pmatrix}
= \begin{pmatrix}
\cos \gamma_{ls} & 0 & 0 \\
\sin \gamma_{ls} & 0 & 0 \\
0 & 0 & -1
\end{pmatrix}
\begin{pmatrix}
v_l \\
\omega_l \\
\omega_l
\end{pmatrix}
+ \begin{pmatrix}
\cos \beta_{ls} & 0 & 0 \\
-sin \beta_{ls} & 0 & 0 \\
0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
\nu_l \\
\omega_l \\
\omega_l
\end{pmatrix}
\]

(16)

where \(\theta_{ls} = \theta - \theta_s\) is the relative orientation between the leader and follower. \(\gamma_{ls} = \theta_{ls} + \beta_{ls}\), while \(\nu_l = [v_l \quad \omega_l]\) is the exogenous input by the leader robot to the system. \(\nu_s = [v_s \quad \omega_s]\) is the follower’s driving inputs. The decoupling matrix (decouples control variables from state variables) in this context is singular.

A. Static Feedback Linearized Formation Controller

To overcome the singularity of e.q.16, [8] proposes a static feedback linearization by shifting the current output state vector to an offset from the origin of the robot coordinate system. The control input to the followers are given by,

\[
u_{ls} = G_1^{-1}(k[\nu_{ls} - z_{ls}] - F_1\nu_l)
\]

(17)

\[
G_1 = \begin{pmatrix}
\cos \alpha_{ls} & p_s \sin \alpha_{ls} \\
-sin \alpha_{ls} & p_s \cos \alpha_{ls}
\end{pmatrix}
F_1 = \begin{pmatrix}
-\cos \beta_{ls} & 0 \\
\sin \beta_{ls} & 0
\end{pmatrix}
\]

where \(\gamma_{ls} = \theta_{ls} + \beta_{ls}\) and \(p_s\) is the offset along the \(x\) coordinate of the robot described earlier. \(z_{ls} = [\nu_{ls} \quad \beta_{ls}]^T\) is the system output with the above offset \(p_s\) and \(\beta_{ls} = \theta - \theta_s\) is the new relative orientation between the leader and follower. \(\nu_l = [v_l \quad \omega_l]\) is the exogenous input by the leader robot to the system while \(\nu_s = [v_s \quad \omega_s]\) is the follower’s driving inputs. \(k = [k_1 \ k_2]^T > 0\) are the controller gains, while \(z_{ls} = [\nu_{ls} \quad \beta_{ls}]^T\) are the desired relative distance and bearing of the follower robot from the leader robot.

1) Simulation Results: For the same experiment run above, the formation errors for the two followers are shown in Fig.7. The velocity profiles for these two followers with nonlinear static feedback linearization are depicted in Fig.8. As can be seen from Fig.7, relative orientation error stays bounded as proved in [8]. Angular velocity of the follower-2 seems quite noisy when compared to the angular velocities, generated from previous controllers for the same follower.
Fig. 7. Formation errors for two follower robots with static feedback linearized control

Fig. 8. Velocity profiles for the two followers with static feedback linearized control

B. Dynamic feedback linearized formation controller

The singularity of the decoupling matrix of e.q.16, is removed through the dynamic extension [16,17,18] such that the linear velocity \( v_s \) of the follower robot is taken as a dynamic state while the first integrator of \( v_s \) is taken as a control variable for the follower along with \( \omega_s \).

\[
\dot{\xi}_s = v_s, \quad \ddot{\xi}_s = a_s \tag{18}
\]

\( a_s \) is the linear acceleration of the follower and \( \xi_s \) is the dynamic extension to \( v_s \). Substituting the new variables to e.q.16 and differentiating results in,

\[
\dot{z}_{ls} = G_2(z_{ls}, \theta_l, \dot{\theta}_l)\dot{u}_l + F_2(z_{ls}, \theta_l, \dot{\theta}_l)\dot{u}_l + L \tag{19}
\]

\( \theta_l = \theta - \theta_s \) and \( z_{ls} = [d_{ls}, \beta_{ls}]^T \) is the system output and \( \dot{u}_l = [a_l, \omega_l] \) is the exogenous input by the leader robot where \( a_l \) is the linear acceleration of the leader and \( \omega_l \) is the angular velocity. \( \dot{u}_l = [a_s, \omega_s] \) is the follower’s driving inputs and \( a_s \) is it’s linear acceleration while \( \omega_s \) is the angular velocity. \( G_2, F_2 \) and \( L \) are given as,

\[
G_2 = \begin{pmatrix} \cos \gamma_s & \xi_s \sin \gamma_s \\ -\sin \gamma_s & \xi_s \cos \gamma_s \end{pmatrix}, \quad F_2 = \begin{pmatrix} -\cos \beta_{ls} & \xi_s \sin \gamma_s \\ \sin \beta_{ls} & -\xi_s \cos \gamma_s \end{pmatrix}, \quad L = \begin{pmatrix} \xi_{ds} \sin \gamma_s + \nu_l \beta_{ls} \sin \beta_{ls} - \nu_s \beta_{ls} \sin \beta_{ls} & \xi_{ds} \cos \gamma_s + \nu_l \beta_{ls} \cos \beta_{ls} - \nu_s \beta_{ls} \cos \beta_{ls} - \omega_l \end{pmatrix}
\]

In spite of the dynamic extension there exists a potential singularity, when \( \xi_s = v_s = 0 \). It is a structural singularity to nonholonomic unicycle type mobile robots[15]. In order to overcome this singularity, this research uses only a naive approach that resets the state of \( \xi_s \), once the velocity of the axle falls below a lower threshold. This can be accomplished through imposing a constraint for the followers as : the linear velocity of the follower \( v_s > |v_s^{\text{lower}}| \) where \( v_s^{\text{lower}} \) is the lower threshold, a smaller positive value. If \( v_s \) falls in between \( -v_s^{\text{lower}} \) and \( +v_s^{\text{lower}} \) from any feedback controls, then \( v_s \) is reset to \( -v_s^{\text{lower}} \) or \( +v_s^{\text{lower}} \) depending on which side (negative or positive) the follower velocity decreased from. Thus it results in a bounded velocity input with isolated discontinuities with respect to time. Through nonlinear dynamic feedback linearization of e.q.19 one gets,

\[
\hat{u}_l = G_2^{-1}(C - F_2\dot{u}_l - L)
\]

Here \( C \) is given by \( C = [c_1, c_2]^T \).

\[
\dot{z}_{ls} = \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = \begin{pmatrix} \beta_{ls}^d + k_1(d_{ls}^d - d_{ls}) + k_2(d_{ls}^d - d_{ls}) \\ \beta_{ls}^d + k_1(\beta_{ls}^d - \beta_{ls}) + k_2(\beta_{ls}^d - \beta_{ls}) \end{pmatrix}
\]

\[
\dot{z}_{ls} = [d_{ls}, \beta_{ls}]^T \text{ are the desired relative distance and bearing of the follower robot from the leader robot.} \tag{20}
\]

1) Experimental Results: The same experiment is performed for this controller with gains of: \( k_i = 0.9 \) for \( i = (1, \ldots, 4) \). The formation errors over time for the two followers are shown in Fig.9. The initial starting velocities of both follower robots are 0.01 m/s. Again it is seen that the relative orientation error stays bounded. It is also observed that the linear and angular velocities are much smoother than the command velocities obtained by the previously experimented controllers.

V. FORMATION CONTROLLER COMPARISON

Formation errors and the quality of the command inputs of the four controllers described above, are considered for
comparison. The same set of experiments with different leader velocity profiles are performed for the different formation controllers explained above. The formation controllers developed above are listed as controller 1 to 4.

- controller 1 - Approximate linearized trajectory tracking type formation controller
- controller 2 - Lyapunov based trajectory tracking type formation controller
- controller 3 - Static feedback linearized \( d - \beta \) type formation controller
- controller 4 - Dynamic feedback linearized \( d - \beta \) type formation controller

The resulting formation RMS errors per follower robot per unit time is given in Table I. Individual \( d_{ls}^{error} \), \( \beta_{ls}^{error} \), \( \theta_{ls}^{error} \), as well as holistic \( \frac{1}{3} \sum_{i=1}^{3} \text{error} \) errors are given for each velocity profile of the leader. Lastly the average individual and holistic RMS error component are given for each controller.

### Table I

RMS FORMATION ERROR VALUES FOR THE DIFFERENT FORMATION CONTROLLERS DEVELOPED ABOVE

<table>
<thead>
<tr>
<th>Velocity profile-1</th>
<th>( d_{ls}^{error} )</th>
<th>( \beta_{ls}^{error} )</th>
<th>( \theta_{ls}^{error} )</th>
<th>Holistic error</th>
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<tbody>
<tr>
<td>controller 1</td>
<td>0.0533</td>
<td>2.5741</td>
<td>18.3964</td>
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<td>24.7626</td>
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<tr>
<th>Average Error for all the above velocity profiles</th>
<th>( d_{ls}^{error} )</th>
<th>( \beta_{ls}^{error} )</th>
<th>( \theta_{ls}^{error} )</th>
<th>Holistic error</th>
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<tbody>
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### VI. Conclusion

From these results it’s sufficient to conclude that the static feedback linearized controller outperforms its counterparts with a narrow margin. It minimizes \( \beta_{ls}^{error} \) to a much lesser value than others but suffers from a high \( \theta_{ls}^{error} \) value. Another flaw of this controller is that the given formation control law does not stabilize the origin* (considered as the origin of the robot coordinate system of the robot where the robot pose is calculated from its odometry readings) of the follower robot, instead an offset point from the origin to desired formation values. Thus the comparison of these values may be somewhat controversial. On the other hand, both approximate linearized and Lyapunov based nonlinear trajectory tracking type controllers keep the \( \theta_{ls}^{error} \) at a possible minimum. The approximate linearized formation controller also tries to keeps \( d_{ls}^{error} \) at much more lower values throughout the experiment than the others. The Lyapunov based nonlinear formation controller has shown better performance with varying \( v_l \) and \( \omega_l \). The dynamic feedback linearized controller is next best to the static feedback linearized controller. Observing the velocity command inputs of the followers, it can be concluded that both 1.) approximate feedback linearized, and 2.) nonlinear Lyapunov based controllers exerts much oscillation in their respective linear velocities but quite stable in rendering the angular velocities. Static and dynamic feedback linearized controllers renders a much smoother linear velocity profile for all the followers. But the static feedback linearized controller has more noisy angular velocity profile (Highly oscillating), whereas as the dynamic feedback linearized controller has a very smooth angular velocity profile across all followers, more smoother than its counterparts.

### References


