A New Battery Model For Use With An Extended Kalman Filter
State Of Charge Estimator

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Abstract—State of charge estimation based on the extended Kalman filter (EKF) has recently received attention for applications that apply rapidly changing current profiles to batteries such as in hybrid electric vehicles. A new battery model is proposed for use with such an estimator in an attempt to further improve its accuracy. The model is based on physical principles underlying the behavior of valve regulated lead acid batteries, taking into account nonlinearities that have been neglected in other models. The model is compared to a linear circuit model consisting of two parallel RC circuits, often used in EKF based state of charge estimation, demonstrating improvement in accuracy.

I. INTRODUCTION

BATTERY state of charge (SOC) is the amount of charge that can be extracted from a battery as a percentage of the battery’s rated amp-hour capacity. Many methods of estimating this quantity have been developed for a variety of applications. References [1, 2] provide excellent reviews of SOC estimators. The recent interest in hybrid electric vehicles has stimulated new research in this area due to difficulties arising from the frequent changes in current exhibited by the load profile in this application. One recently developed successful category of techniques is based on the extended Kalman filter (EKF).

The EKF utilizes a state space model to estimate the SOC. Because of this, EKF based SOC estimators differ from one another most significantly in the battery model. In [3] a hysteresis type model is used for this purpose. In [4-6] a model consisting of a steady state voltage dependent on SOC and current, a hysteresis element, and a voltage gained by filtering the input current through a 4th order transfer function is developed and explored for SOC estimation purposes. A simple two state linear model is used in [7] which is extended to include estimation of the battery capacity for state of health (SOH) monitoring. A model using a detailed description of diffusion is covered in [8].

The accuracy of the estimator is highly dependent on the quality of this model. This paper proposes a new model to be used in an EKF based SOC estimator. The model is based on physical principles underlying the behavior of valve regulated lead acid (VRLA) batteries, includes nonlinearities not accounted for by other models, and remains relatively low-order, using only 3 state variables for efficient use in an EKF.

II. DERIVATION OF MODEL

A. Initial model derivation

In this section we develop the VRLA battery model based on some of the physical principles underlying its operation. Fig. 2 shows a cross section of a typical VRLA battery which includes a PbO2 electrode, a Pb electrode and a separator containing a mixture of water and H2SO4. The primary reaction can be considered the combination of the reaction at the anode, the reaction at the cathode, and the transfer of electrons to and from the positive and negative terminals. In addition to this reaction several other reactions take place including hydrogen evolution, oxygen evolution, oxygen reduction, hydrogen oxidation, and lead corrosion [9]. For the model in this paper we will only be concerned with the main reaction and oxygen evolution reaction. The reactions are given by the following equations:

\[
Pb + H_2SO_4 \leftrightarrow PbSO_4 + 2H^+ + 2e^- \quad (1)
\]

\[
PbO_2 + H_2SO_4 + 2H^+ + 2e^- \leftrightarrow PbO_2 + 2H_2O \quad (2)
\]

\[
H_2O \rightarrow O_2 + 4H^+ + 4e^- \quad (3)
\]

Note that oxygen evolution occurs at very slow rates during times of rest and during discharge, and at more sizable rates during recharge and overcharge. Other than the rates of the reactions described above three additional phenomena are also described in the model. They include ohmic and ionic resistance, diffusion, and double layer capacitance. The ohmic resistance is due to the current flow through the various metallic components, i.e. posts, grid, etc. The ionic resistance describes the voltage drop resulting from the flow of ions across the separator. Diffusion also occurs in the separator, and is due to changes in the sulfuric acid concentration as reactions occur. Finally, the double layer capacity is a buildup of charge just outside of the two electrodes due to polarized molecules attracted to either electrode. This charge separation creates a voltage drop that drives the reaction [10].

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Based on the circuit topology proposed in [11] a model is developed. The equations governing reaction rate and diffusion are taken from [9, 10]. Fig. 1 shows a circuit diagram of the VRLA battery model. The resistance $r_0$ represents the combined ohmic and ionic resistance described above. The capacitances $c_1$ and $c_2$ represent the double layer capacitance of the negative and positive electrodes respectively.

The double layer capacitance may be influenced by changes in the structure of the electrodes, since this will cause a change in the surface area available for reaction. However, this influence is assumed negligible. Therefore $c_1$ and $c_2$ are taken to be constant. Similarly, the resistance $r_0$ is assumed constant in our model.

A reaction at one of the electrodes can be modeled as a nonlinear resistance in series with a voltage source, both dependent on sulfuric acid concentration and temperature. In this case, the voltage source corresponds to the equilibrium voltage of the reaction. The nonlinear elements $\rho_1$, $\rho_2$, and $\rho_3$ represent the reactions of the negative electrode, the positive electrode, and oxygen evolution at the positive electrode respectively.

The current source $\gamma_3$ is caused by the reduction of oxygen at the negative terminal, which, due to fast transport through the separator, occurs at the same rate as the oxygen evolution reaction.

The circuit in Fig. 1 can be described by the following set of differential equations:

$$\dot{v}_1(t) = \frac{i_1(t) - \gamma_1(t) - \gamma_3(t)}{c_1}$$

$$\dot{v}_2(t) = \frac{i_2(t) - \gamma_2(t) - \gamma_3(t)}{c_2}$$

$$v_f(t) = n_c(v_1(t) + v_2(t) + r_0 \cdot i_f(t))$$

Here $\gamma_1$, $\gamma_2$, and $\gamma_3$ are reaction rates for the negative and positive electrodes and the oxygen evolution reaction respectively. These reaction rates are modeled as currents $\gamma_1$, $\gamma_2$, and $\gamma_3$ through $\rho_1$, $\rho_2$, and $\rho_3$ in the circuit model of Fig. 1. The parameter $n_c$ is a factor included to allow for multiple cells. The equations for the reaction rates are provided in [10], and are given by

$$\gamma_1(t) = i_1(t) \left( \frac{a_{1F}}{RT} (v_1(t) - v_2(t)) \right) - e^{\frac{(1-a_1) a_{1F}}{RT} (v_1(t) - v_2(t))}$$

$$\gamma_2(t) = i_2(t) \left( \frac{a_{2F}}{RT} (v_2(t) - v_f(t)) \right) - e^{\frac{(1-a_2) a_{2F}}{RT} (v_2(t) - v_f(t))}$$

$$\gamma_3(t) = i_3(t) \left( \frac{a_{3F}}{RT} (v_3(t) - v_f(t)) \right) - e^{\frac{(1-a_3) a_{3F}}{RT} (v_3(t) - v_f(t))}$$

where the constants $i_0$ represents the exchange current of each reaction, $\alpha_i$ (i = 1, 2, 3) is the transfer coefficient, which varies between 0 and 1, $n$ is the number of electrons involved in the reaction (in this case 2), $F$ is the Faraday constant (96485 C/mol), $R$ is the gas constant (8.315 J/°K mol), $T$ the temperature in Kelvin, and $V_0$ is the equilibrium voltage for each reaction. The exchange current is influenced by the concentration of reactants and products involved in the reaction, which therefore is related to the SOC. The exchange current is given by

$$i_0(t) = i_{00} \theta_0(t) (1 - \theta_0(t))^\alpha_0$$

where $i_{00}$ is a constant and $\theta_0$ is a quantity roughly proportional to the concentration of reactants and products at the surface of the electrodes.

The equilibrium voltage for each reaction is given by

$$V_0(t) = V_{00} + \frac{RT}{nF} \left( A_0 \ln \theta_0(t) - B_0 \ln (1 - \theta_0(t)) \right)$$

where $V_{00}$ is a constant, and $A$ and $B$ are integer valued constants referring to the number of reactant and product species respectively in the discharge reaction.

The concentration of sulfuric acid is accounted for in this model using $\theta$. The concentration gradient across the separator is governed by the diffusion equation

$$\frac{\partial \theta(x,t)}{\partial t} = D \frac{\partial^2 \theta(x,t)}{\partial x^2} \quad \forall x \in (0,1)$$

and the boundary conditions

$$\hat{\theta}(0,t) = \frac{1}{c_{cap}} i_n(t) + D \frac{\partial \theta(x,t)}{\partial x}$$

$$\hat{\theta}(2x_0,t) = \frac{1}{c_{cap}} i_n(t) - D \frac{\partial \theta(x,t)}{\partial x}$$

where $x$ is the distance into the separator from the negative electrode, $D$ is the diffusion constant, $x_0$ is the distance from one electrode to the center point of the separator, and $c_{cap}$ is the capacity of the battery in Amp-
seconds. This boundary condition is based on the assumption that the reactions occur on the surface of the electrodes and that the changes in concentration at points 0 and 2 are due to the combination of the reaction, and the flux of the electrolyte towards the electrode.

This equation may be approximated by discretizing in space. Each point used for this discretization process will result in a state variable in the final model. Only three points are used to keep the dimension of the state space minimal, thus reducing the computation time of the estimator. The three points identified in Fig. 2 will be used for this purpose. The resulting set of equations is given by

\[
\dot{\theta}_0(t) = \frac{1}{c_{\text{cap}}} i_c(t) + \frac{D}{x_0} \left( \theta_i(t) - \theta_0(t) \right) \tag{11}
\]

\[
\dot{\theta}_i(t) = -\frac{D}{x_0} \left( \theta_i(t) - \theta_0(t) \right) + \frac{D}{x_0} \left( \theta_i(t) - \theta_i(t) \right) \tag{12}
\]

\[
\dot{\theta}_2(t) = \frac{1}{c_{\text{cap}}} i_c(t) - \frac{D}{x_0} \left( \theta_2(t) - \theta_i(t) \right) \tag{13}
\]

This can be further simplified assuming that \( \theta_0 = \theta_2 \) initially. In this case they will remain equal and we can eliminate (13), and replace \( \theta_i \) by \( \theta_0 \) in equation (12). The parameters \( D \) and \( x_0 \) can also be replaced by a single constant \( d_{\text{diff}} \). In addition to these modifications a normalization factor of 2/3 is included so that \( \theta_0 \) and \( \theta_1 \) vary between 0 and 1, yielding the new set of equations

\[
\dot{\theta}_0(t) = \frac{1}{2/3 \cdot c_{\text{cap}}} i_c(t) + d_{\text{diff}} \left( \theta(t) - \theta_0(t) \right) \tag{14}
\]

\[
\dot{\theta}_1(t) = -2d_{\text{diff}} \left( \theta(t) - \theta_0(t) \right) \tag{15}
\]

### B. Further Simplification

The model can be further simplified by assuming negligible oxygen evolution occurs (oxygen evolution is known to be small compared to the main reaction except during large overvoltage conditions [9]) and by lumping the two RC circuits into one. The two voltage responses caused by the double layer capacitance at each electrode have similar time constants. Therefore the two RC circuits in the model can be combined without great loss in accuracy. The variables \( v, \gamma, i_0, \alpha, V_0, i_0, V_{\text{cap}}, A, \) and \( B \) are introduced with meaning analogous to the sub and superscripted versions in (4-7) for the combined RC circuit. Equations (4),(5),(6), and (7) are then replaced by

\[
v(t) = \frac{i_0(t) - \gamma(t)}{c} \tag{16}
\]

\[
\gamma(t) = i_0(t) \left( \text{e}^{\frac{RT}{F} (\theta(t) - \theta_0(t))} - \text{e}^{-\frac{RT}{F} (\theta(t) - \theta_0(t))} \right) \tag{17}
\]

\[
i_0(t) = i_0 \theta_0(t)^{1-a} (1 - \theta_0(t))^a \tag{18}
\]

\[
V_c(t) = V_m + \frac{RT}{n F} \left( A \ln \theta_0(t) - B \ln(1 - \theta_0(t)) \right) \tag{19}
\]

\[
v_f = n_c (v(t) + r_0 \cdot i_c(t)) \tag{20}
\]

This model shares many similarities with other simple models used for EKF based SOC estimation. The model consists of an equilibrium voltage, a resistive component, and an overdamped transient response. The most significant improvement in this model over other simple models is the inclusion of the Butler-Volmer equation (i.e. equation 17). The effect of this equation is that the amplitude and time constant of the transient response is a function of the battery current. This is particularly important in applications where the current frequently switches between low and high charging and discharging. The other is that the equilibrium voltage is dependent on the electrolyte concentration adjacent to the electrode, rather than the SOC.

### III. SELECTION OF MODEL PARAMETERS

The accuracy of the model is dependent on selection of the parameters. The procedure used in this paper to select the model parameters is described in this section.

The parameters \( n, R, \) and \( F \) are known constants (given in Table I). The parameters \( A \) and \( B \) refer to the number of ions used in the discharging and charging reactions respectively. In this case \( A = 8 \) (since the reactants dissociate into \( \text{Pb}, \text{PbO}_2, \text{4H}^+, \) and \( 2\text{SO}_4^\cdot \) and \( B = 4 \) (since the products dissociate into \( 2\text{Pb}^+, \) and \( 2\text{SO}_4^\cdot \)).

The remaining parameters are found using data obtained from the experimental setup described in [12]. The voltage data is divided by \( n_c \) to find the voltage per cell, shown in Fig. 3. The parameters are extracted through a procedure combining curve fitting and manipulation of the data. To find \( c_{\text{cap}} \) and \( V_{\text{cap}} \), a technique developed in [13] is used to find the open circuit voltage (OCV) vs. SOC curve. The extracted charge at points \( V_1, V_5, \) and \( P6 \) to \( P25 \) in Fig. 3 is first found based on the current profile used. Then voltages at points \( P6 \) to \( P15 \) are averaged with corresponding values at points \( P16 \) to \( P25 \). These averages are assumed to be equal to the equilibrium voltage. The resulting curve is fit to (19) with \( \theta_0 \) replaced by

\[
\theta_0 = \theta(0) = A_{\text{bd}} / c_{\text{cap}}, \tag{21}
\]

where \( \theta(0) \) is the value of \( \theta_0 \) at point \( V1 \), and \( A_{\text{bd}} \) is the number of amp-hours discharged at each point. The Trust-Region algorithm provided by Matlab’s curve fitting toolbox is used to find \( \theta(0), c_{\text{cap}}, \) and \( V_{\text{cap}} \).

Next \( r_0 \) is found by identifying voltage drops and rises (V3-V2 in Fig. 3) caused by current charge/discharge and dividing by the change in current (in this case 4A). This is repeated at points \( P1 \) to \( P25 \) and the average value is used for \( r_0 \).

During the rest period prior to \( V5 \) in Fig. 3, (14-15) will give the exponential response

\[
\theta_0 = \theta(0) + \theta_m e^{-3d_{\text{eff}}}, \tag{22}
\]
Fig. 3. Recorded current waveform and voltage waveform divided by number of cells, along with expanded view of voltage waveform.

where \( \theta(0) \) is now the value of \( \theta_0 \) at the beginning of the transient and \( \theta_m \) is the magnitude of the transient response. By substituting (22) into (19), the parameters \( \theta(0) \), \( \theta_m \), and \( d_{ag} \) can be found using the Trust-Region algorithm provided by Matlab's curve fitting toolbox and the data prior to \( V5 \).

Next, we use the exponential response at points P6 to P15 and P16 to P25 to find the remaining parameters. We use the curve fitting toolbox again to fit the exponential portion of each curve (i.e. corresponding to that between points V3 and V4 in Fig. 3). The transients at P6 to P15 are fit to the equation

\[
V(t) = V_\infty - V_\gamma e^{-\frac{t}{\tau}}, \tag{23}
\]

where \( V_\infty \) is the voltage that \( V(t) \) asymptotically approaches, \( V_\gamma \) is the magnitude of the response and \( \tau \) is the time constant of the response. Similarly the transients at P16 to P25 are fit to

\[
V(t) = V_\infty + V_\gamma e^{-\frac{t}{\tau}}, \tag{24}
\]

where \( V_\gamma \) is the magnitude of the response.

To find \( \alpha \), we assume that prior to the rest period where the transient is measured, (16) has reached an equilibrium, therefore \( i_T = \gamma \). The polarization \( \nu-V_0 \) in (17) is then given by \( V_\gamma \) or \(-V_\gamma\) depending on the polarity of the current. During discharge (points P6 to P15) the first exponential term in (17) is assumed to be negligible. This yields

\[
i_T = -i_0 e^{-\frac{(1-n)\nu}{RT}}. \tag{25}
\]

Similarly, the second exponential term is assumed negligible for P16 to P25:

\[
i_T = i_0 e^{-\frac{\nu}{RT}}. \tag{26}
\]

By pairing the polarization at points with equal values of \( A_{hd} \), (25-26) can be combined at these points to yield

\[
\alpha = \frac{V_\gamma}{V_\gamma + V_\gamma}. \tag{27}
\]

The exchange current \( i_0 \) can then be found be found by

\[
i_0 = i_T \left( \frac{\alpha F \nu}{RT} \right). \tag{28}
\]

The values of \( i_0 \) and \( \alpha \) found at each point are averaged to yield the corresponding model parameters.

Finally the value of \( c \) is found by linearizing (17) around \( \nu = V_0 \). This gives

\[
\gamma = \frac{nF}{RT} \nu. \tag{29}
\]

Substituting (29) in (16), and using the fact that \( i_T = 0 \) during the transient, the value of \( c \) should then be given by

\[
c = \pi i_0 nF (RT)^{-1}. \tag{30}
\]

This calculation is performed at points P6 to P25 and the average is taken to find the model parameter. At this point all the required model parameters have been found. The resulting values are given in Table I.

### IV. EKF IMPLEMENTATION

While a detailed description of the implementation of the EKF can be found in [14], this section provides a brief outline:

The Kalman filter is a least squares optimal estimator for discrete time linear dynamic systems subject to process and measurement noise modeled as stochastic processes in the state equation and the output equation. The algorithm iteratively estimates the optimal (i.e. lowest variance) state vector at each time step by first estimating the state vector at the previous time step and its covariance based on the state-space model, and then correcting this estimate in an optimal fashion using the latest measurement of the output of the actual system. The EKF extends this algorithm to nonlinear dynamic systems. The state is still estimated using the system model, but to estimate the covariance and to correct the state estimate, the system is linearized around the latest estimate of the state vector [14].

To implement the EKF, the discretized version of (16-20) is used for the system model. The current \( i_T \) is treated as the system input, while \( v_T \) is considered to be the output. In addition to discretization we must also define the sources of noise in our system. We assume current and voltage to be corrupted by white Gaussian noise source \( w \) and \( \xi \) according to

\[
i_m[k] = i_T[k] + w[k], \tag{31}
\]

\[
v_m[k] = v_T[k] + \xi[k], \tag{32}
\]

and the state vector 

<table>
<thead>
<tr>
<th>Param</th>
<th>Value</th>
<th>Param</th>
<th>Value</th>
<th>Param</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( c )</td>
<td>5128 F</td>
<td>( c_{exp} )</td>
<td>1.97 \times 10^7 As</td>
<td>( i_0 )</td>
<td>2.5061 A</td>
</tr>
<tr>
<td>( V_{00} )</td>
<td>2.151 V</td>
<td>( \alpha )</td>
<td>0.2608 A</td>
<td>( n )</td>
<td>8</td>
</tr>
<tr>
<td>B</td>
<td>4</td>
<td>( d_{ag} )</td>
<td>6.267 \times 10^{-4} s^{-1}</td>
<td>( \tau )</td>
<td>8.315 / K mol</td>
</tr>
<tr>
<td>( n )</td>
<td>2</td>
<td>R</td>
<td>5.6 \times 10^{-5} \Omega</td>
<td>T</td>
<td>298 K</td>
</tr>
<tr>
<td>F</td>
<td>96485 C/mol</td>
<td>( n_c )</td>
<td>6</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
where $i_T[k]$ and $v_T[k]$ are the discretized true current and voltage and $i_m[k]$ and $v_m[k]$ are the measured values.

Using these sources of noise and using Euler's method to discretize the battery model, the model equations become

$$\theta_0[k+1] = \theta_0[k] + h \cdot f_1(\theta_0[k], \theta_1[k], i_T[k])$$
$$\theta_1[k+1] = \theta_1[k] + h \cdot f_2(\theta_0[k], \theta_1[k], i_T[k])$$
$$v[k+1] = v[k] + h \cdot f_3(\theta_0[k], \theta_1[k], v[k], i_T[k])$$
$$v_T[k] = g_v(v[k], i_T[k]).$$

Fig. 4. Parallel RC circuit model used for comparison

where $\theta_0[k], \theta_1[k]$, and $v[k]$ are state variables sampled at time $kh$, $h$ is the sampling period, $f_1, f_2$, and $g_v$ are the right hand sides of (14), (15), and (20) respectively, and $f_3$ is found by substituting (17-19) into (16), and taking the right hand side. Note that although $v[k]$ occurs due to noise in the current measurement, it is considered process noise rather than measurement noise because it enters into the state update equation.

For the EKF implementation we need to determine the following parameters: the variance of the process noise, the variance of the measurement noise, the expected value of the state vector at $k = 0$ and the covariance of the state vector at $k = 0$. The variance of the process noise and measurement noise are estimated based on voltage and current measurements of the battery in steady state taken over a period of time.

To estimate the mean and covariance of the state vector at $k = 0$, we assume the state variables to be independent, so the covariance matrix will be diagonal with the individual variances of each variable along the diagonal. Based on prior knowledge of the system, we estimate an upper and lower bound for each state variable that the variable will exceed with very small probability. The midpoint of this range yields the mean of the state variable at $k = 0$, and half the length of the interval is assumed to be three times the standard deviations from the mean. Since the EKF is based on the assumption of Gaussian random processes the variance can be found by dividing this length by 3 and squaring the resulting value.

For the 3 states described above, we assume a range of 0 to 1 for $\theta_0$ and $\theta_1$ and between 1.75 and 2.4 for $v$. This gives mean values of 0.5, 0.5 and 2.075, and variances of 0.0278, 0.0278, and 0.0117. The EKF can then be implemented following the description in [14]. The output of the EKF yields an estimate of each state variable ($\theta_0, \theta_1,$ and $v$). The state of charge $q$ is proportional to the average acid concentration of the battery. Since $\theta_0$ equals $\theta_2$, this is given by

$$q = \frac{2\theta_0 + \theta_2}{3}.$$

V. COMPARISON

In an effort to demonstrate the accuracy of the proposed model, it is compared to a frequently encountered model used for both EKF based and sliding mode observer based SOC estimation [7, 15, 16]. The model used for comparison consists of two parallel RC branches and is shown in Fig. 4. This model uses the capacitance of one branch to model SOC, while the other branch models the transient behavior of the battery.

The parameters of this model are estimated as described in [7] with some modification due to the differences in the tests used for parameter estimation. Voltages $V_1$ to $V_5$ and times $T_1$ and $T_2$ labeled in Fig. 3 are used to estimate the model parameters. Note that since these voltages are per cell, the values used in the following are scaled to the full battery voltage. The parameter $C_{bulk}$ is determined by integrating the discharge current $i_T$ between points $V_1$ and $V_5$ in Fig. 3, giving

$$C_{bulk} = \frac{1}{n_c (V_1 - V_5)} \int_{t_{V1}}^{t_{V5}} i_T dt = 62,400 \text{ F},$$

where $t_{V1}$ is the time at point $V1$ and $t_{V2}$ is the time at point $V5$. We assume the current through the $C_{surf}$ branch to be negligible since the time constant of this branch is assumed to be much smaller than that of $C_{bulk}$.

The equation used to estimate the time constant of the $C_{surf}$ branch is taken from [7]. However, in [7], the discharge pulses used are short enough to assume the battery voltage returns to the equilibrium voltage prior to the discharge. The equation then must be modified using the estimated equilibrium voltage $V_{Eq}$ that the battery should reach after the discharge. The equation for $\tau$ is

$$\tau = T_2 \ln \left(1 - \frac{V_4 - V_3}{V_{Eq} - V_5}\right) = 49.9s.$$  

The equilibrium voltage is estimated by dividing the voltage drop between $V1$ and $V5$ by the number of discharge pulses $n_d$. This is given by

$$V_{Eq} = n_d \left(\frac{V_1 - V_5}{R_d}\right) = 12.43 \text{ V}.$$

We now assume $R_E = R_S$, and find the value of these resistances by assuming that at $V2$ the $C_{surf}$ branch is in equilibrium, so that the entire discharge current is flowing through $R_E$. The value of $R_E$ is then determined by dividing

![Fig. 4. Parallel RC circuit model used for comparison](image-url)
the voltage drop across this resistance by the current $I$ used in the discharge, therefore,

$$R_E = R_S = \frac{n_c (V_{eq} - V_3)}{I} = 0.0525 \Omega.$$  \hfill (41)

The value of $C_{Surf}$ is calculated using the equation

$$C_{Surf} = \frac{\tau}{R_E + R_S} = 470 \text{ F},$$  \hfill (42)

taken directly from [7]. Finally, the resistance $R_F$ is calculated using the drop in voltage between $V_2$, and $V_3$.

$$R_F = \frac{n_c (V_3 - V_2)}{I} - \frac{R_E}{2} = -0.0089 \Omega.$$  \hfill (43)

A test on the battery using an independent current profile is used to compare the two models. The recorded current and voltage waveforms along with the simulated voltage waveforms from the two models are presented in Fig. 5. Both the recorded data and the simulation have a sampling rate of 10Hz. The accuracy of each model is analyzed using the RMS error defined by

$$\text{err} = \sqrt{\frac{1}{T} \sum_{k=0}^{T} (v_{act}^k - v_{sim}^k)^2}.$$  \hfill (44)

Here $v_{act}$ is the recorded voltage and $v_{sim}$ is the simulated voltage. The parallel RC model yields an RMS error of 0.3615 V (0.0301 p.u.), while the newly proposed model yields an RMS error of 0.3160 V (0.0263 p.u.).

VI. CONCLUSION

This paper presents a new battery model for use with an EKF based SOC estimator. The model is simple, is based on underlying physical principles, and takes into account nonlinearities that have not been accounted for in other simple models. The model is compared with another popular model used in EKF based SOC estimation demonstrating the improved accuracy of the new model.

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VIII. REFERENCES