Closed loop control of the sawtooth instability in nuclear fusion

G. Witvoet,1,2 M. Steinbuch,1 E. Westerhof,2 N.J. Doelman,3 M.R. de Baar2 and the TEXTOR team4

Abstract—In nuclear fusion the sawtooth instability is an important plasma phenomenon, having both positive and negative effects on the tokamak plasma. Control of its period is essential in future nuclear fusion reactors. This paper presents a control oriented model of the sawtooth instability, with current drive as input and sawtooth period as output, and validates it with a more extended model and experimental data. This model is numerically implemented and combined with PI-controllers in a feedback loop. Simulations show that closed loop control of the system to a desired sawtooth period is indeed possible, even when time delay is added to the feedback loop.

I. INTRODUCTION

Nuclear fusion research still faces large scientific and technological challenges [1], [2]. Most of this research focuses on the use of a tokamak [3]; a toroidal machine confining extremely hot fusion plasma using large magnetic fields. Unfortunately, tokamak plasmas are subject to various instabilities which can deteriorate their performance [2].

An important instability is the relaxation oscillation in the center of the tokamak plasma called the sawtooth instability [4]–[6]. It is observed as a repetitive rising and crashing of certain plasma variables, like the temperature in the plasma center [6]. The crashes result from a mixing phenomenon in the plasma center, yielding very fast transport of energy and particles from the center to the outside. This can be disadvantageous as fast energetic particles needed for plasma heating may be expelled from the plasma center, while large sawteeth can trigger other (undesired) plasma instabilities [7]. On the other hand the mixing mechanism provides a way to remove helium ash from the center, which can facilitate future control of the helium density, which is necessary to prevent choking of the fusion reaction [8].

Achieving an optimal trade-off between these positive and negative effects of the sawtooth instability requires controlling its period [7]; helium transport from the center is influenced by the period and small periods avoid the triggering of other instabilities. Various studies on different tokamaks, like TCV [8], TEXTOR [9] and ASDEX [10], have shown that the period can be altered using a spatially localized Electron Cyclotron Current Drive (ECCD). Recently, some first experimental feedback results with ECCD have been reported on TCV [11] and Tore Supra [12]. However, these results do not provide a basis for structured controller design, which requires a control-oriented model of the sawtooth behavior. Furthermore, there is currently no simulation environment to test sawtooth controllers offline.

In this paper we present such a control-oriented model, and use it as a case study for applying control techniques to the sawtooth problem. This infinite dimensional impulsive dynamical system of the sawtooth instability, based on Maxwell’s equations and the crash definitions of Porcelli [13] and Kadomtsev [14], is validated with the more extensive model in [15] and experimental data from TEXTOR. The model has been implemented in Simulink®, which yields a testing environment for real-time simulation purposes. After identifying three different regimes from the input-output response of the system, this enables us to apply three different controllers to each of the regimes and assess the corresponding performances. Moreover, we can examine the influence of time delays in the loop, which will arise from the use of sawtooth detection algorithms during real experiments.

This paper is organized as follows. Section II presents the dynamical model of the sawtooth instability, after a short introduction on the TEXTOR control loop and tokamak geometry. This model is validated in Section III, while Section IV shows the simulation feedback loop and presents some closed loop simulation results for different control regions and different time delays. Section V summarizes the conclusions and discusses future research.

II. SAWTOOTH MODEL

In this section we present a control-oriented sawtooth model, based on the model in [15]. This model and its parameters are focussed on the TEXTOR tokamak (Tokamak Experiment for Technology Oriented Research), which is equipped with an Electron Cyclotron Current Drive installation (ECCD) [16] and a steerable mirror to deposit this ECCD very locally in the plasma [17].

A. The feedback loop

The sawtooth control problem for TEXTOR is represented in the general feedback loop of Fig. 1. However, for the remainder of this paper we combine the plasma, temperature diagnostics and sawtooth recognition blocks in a single model, with inputs $\vartheta$ (mirror angle in the vertical plane) and $I_{\text{CD}}$ (total current drive) and output $\tau_s$ (sawtooth period). For simplicity we consider SISO control, where $C$ only computes $\vartheta_{\text{ref}}$ based on $\tau_s$. Therefore it is assumed that $I_{\text{CD}}$ is constant and the mirror is closed loop controlled with a 20 Hz bandwidth [17].
B. Tokamak geometry assumptions

TEXTOR is a circular cross-section tokamak, described by a radius \( r \), poloidal angle \( \theta \) and toroidal angle \( \phi \) (see Fig. 2). We use \( R \) to indicate a distance to the center of the torus, the major radius \( R_0 \) (1.75 m) is the distance from the center of the torus to the plasma center, and the minor radius \( a \) (0.46 m) is the distance from the plasma center to the vessel wall. We assume a large aspect-ratio, i.e. \( R_0 \gg a \).

The sawtooth instability is a magnetic phenomenon \([6]\), hence our model will be based on the time evolution of the magnetic field \( \vec{B} \). A circular tokamak is approximately axisymmetric in both toroidal and poloidal direction, such that \( \vec{B} \) only has a toroidal \( B_\phi \) and poloidal \( B_\theta \) component (see Fig. 2), where \( B_0 = B_0(r,t) \) and \( B_\phi \) is constant (since \( R_0 \gg a \)). Hence, we only need a model for \( B_\phi(r,t) \).

The most relevant variables in our sawtooth model are the safety factor \( q \) and the magnetic shear \( s \), which for large aspect-ratio tokamaks are defined as \([3]\)

\[
q(r) = \frac{r B_\phi}{R_0 B_\theta}, \quad (1)
\]

\[
s(r) = \frac{r}{q} \frac{\partial q}{\partial r} = 1 - \frac{r}{B_\theta} \frac{\partial B_\theta}{\partial r}. \quad (2)
\]

Both \( q \) and \( s \) are uniquely determined by the poloidal field \( B_\theta \), and thus only depend on \( r \) and \( t \). The magnetic shear \( s_1 = s(r_1) \) at the location \( r_1(t) \) where \( q(r_1) = 1 \) is very important in our model, as it is the location where sawtooth oscillations are generally observed \([3]\).

C. Sawtooth instability as impulsive dynamical system

In \([15]\) we have presented an impulsive dynamical model \([18]\) for the sawtooth instability, with partial differential equations (PDEs) to describe the inter-crash evolution and discrete state-jumps to describe the crashes itself. The states in this model were both the magnetic field \( B_\theta \) [T] and the electron temperature \( T_e \) [eV]. The latter state was necessary to simulate temperature measurements on TEXTOR. In this paper we only focus on the sawtooth period itself, which is governed by \( s_1 \) which depends only on \( B_\theta \). This enables a major simplification of our model, i.e. we can remove the temperature measurements and thereby the evolution of \( T_e \) and the corresponding parameter profiles. This simplification yields a similar model as presented in \([19]\).

Hence, in this paper we consider a model described by the magnetic diffusion equation (3a) and crash condition (3b):

\[
\frac{\partial}{\partial t} B_\theta = \frac{\partial}{\partial r} \left( \frac{\eta}{\mu_0 r} \left( B_\theta + r \frac{\partial}{\partial r} B_\theta \right) - \eta J_{CD} \right) \quad \text{if } s_1 \leq s_{crit} \quad (3a)
\]

\[
B_\theta(r,t^+) = \begin{cases} B_\theta(r,t^-) & \text{for } r \geq r_{mix} \\ \frac{1}{\eta} r B_\phi & \text{for } r < r_{mix} \end{cases} \text{ if } s_1 > s_{crit} \quad (3b)
\]

with boundary conditions \( B_\theta(0,t) = 0 \) and \( B_\theta(a,t) = \frac{\mu_0 I_p}{2 \pi a} \), where \( \mu_0 \) is the magnetic permeability in vacuum and \( I_p \) is the total plasma current. The plasma resistivity \( \eta \) depends on the electron temperature \( T_e \) [eV], for which we assume \([3]\)

\[
\eta = 1.65 \times 10^{-9} \ln \Lambda Z_{eff} \left( \frac{T_e}{1000} \right)^{-3/2}. \quad (4)
\]

For TEXTOR the Coulomb logarithm \( \ln \Lambda \approx 17 \) and the effective ion-charge \( Z_{eff} \approx 1.5 \). In contrast to the model in \([15]\) the temperature \( T_e \) is assumed to be time-invariant, i.e.

\[
T_e(r) = T_0 \left( 1 + q_a \left( \frac{r}{a} \right)^2 \right)^{-4/3}, \quad (5)
\]

where the central temperature \( T_0 \) is typically 2 keV for TEXTOR, and \( q_a = q(a) \) is defined as

\[
q_a = \frac{a B_\phi}{R B_\theta(a)} = \frac{2 \pi a^2 B_\phi}{\mu_0 R_0 I_p}. \quad (6)
\]

The state jump in (3b) is a representation of Porcelli’s sawtooth crash model \([13]\), which states that (under some conditions) a crash occurs as soon as the shear at the \( q = 1 \) location reaches a critical value \( s_{crit} \). At this crash the \( q \)-profile is flattened in our model, as described in Kadomtsev’s model \([14]\), such that \( q = 1 \) everywhere in a center region \( r < r_{mix} \). Note that \( s_{crit} \) is plasma dependent; here a fixed value for \( s_{crit} \) is chosen such that the model yields a realistic sawtooth period \( \tau_s \).

The only input signal in our infinite dimensional impulsive dynamical system (3) is the driven current density \( J_{CD} \), which represents the ECCD input to the plasma. We model this \( J_{CD} \) as a Gaussian distribution over \( r \), hence

\[
J_{CD} = J_0 \exp \left( -\frac{(r - r_{ECCD})^2}{w^2} \right), \quad (7)
\]

where \( w \) is the deposition width (typically 10-20 mm). The deposition location \( r_{ECCD} \) is determined by the ECCD mirror angle in the vertical plane \( \hat{v} \), as illustrated in the

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1. The mixing radius \( r_{mix} \) should be larger than \( r_1 \); here we assume \( r_{mix} \) to be fixed at \( r_{mix} = 1.2 \cdot \frac{a}{q_a} \).
poloidal cross-section in Fig. 3. As explained in [15] this
\( r_{\text{ECCD}} \) can be calculated algebraically by solving
\[
    r_{\text{ECCD}}^2 = z_{\text{ECCD}}^2 + (R_{\text{ECCD}} - R_0 - \Delta(r_{\text{ECCD}}))^2, \tag{8}
\]
using the relations for electron gyration frequency (9a),
mirror vertical angle (9b) and Shafranov shift (9c),
\[
    R_{\text{ECCD}} = \frac{1}{2\pi} \cdot \frac{2eR_0B_0}{f_{\text{ECCD}}m_e}, \tag{9a}
\]
\[
    z_{\text{ECCD}} = (R_{\text{mirror}} - R_{\text{ECCD}}) \cdot \tan(\vartheta), \tag{9b}
\]
\[
    \Delta(r_{\text{ECCD}}) = \Delta_0 \left(1 - \left(\frac{r_{\text{ECCD}}}{a}\right)^2\right), \tag{9c}
\]
where the mirror location \( R_{\text{mirror}} = 2.4 \text{ m} \) and the ECCD
resonance frequency \( f_{\text{ECCD}} = 140 \cdot 10^9 \text{ Hz} \).

The model inputs are now \( \vartheta \) and the total current drive
\( I_{\text{CD}} \) (the surface integral of \( J_{\text{CD}} \) in (7)). The output of the
model is the sawtooth period \( \tau_s \), which is the time between
subsequent crashes or state jumps.

D. Model implementation

The model described above has been discretized over \( r \) in
200 steps, yielding a large 200 states ODE. This ODE, the
algebraic equations defined above and an additional event
function to define the crashes have been implemented in a
Simulink® C-code S-function [20] for real-time simulation
purposes (using the ode15s solver). Note that compared to the
model in [15] the magnetic diffusion equation (3a) is
now linear in \( B_0 \) since \( \eta \) is time-invariant, and the number
of states in the implemented ODE is halved. This makes the
model in Section II-C a factor 20 faster to solve.

III. MODEL COMPARISON AND VALIDATION

The assumption that \( T_e \) and \( \eta \) are time-invariant will
have an influence on the model output. Therefore we will
compare the solution of our simplified model (3) to the
extended model presented in [15], and validate it with an
actual TEXTOR experiment.

The sawtooth instability is known to have a very charac-
teristic input-output behavior. Additional current drive in
the same direction as \( I_p \) decreases the sawtooth period for
current depositions within \( r_1 \) (close to the plasma center),
and increases it for depositions slightly more outwards [8].
It was shown in [15] that the extended model captures this
behavior successfully. Fig. 4 illustrates that the same holds
for the simplified model (3). This figure shows the response in
\( \tau_s \) of both models with an additional driven current of
\( I_{\text{CD}} = 2.1 \text{ kA} \), when a linear sweep of the vertical mirror angle \( \vartheta \) from \( 2.5^\circ \) to \( 12.5^\circ \) is applied. This continuously
changes the deposition location, to which both models show
a similar dynamical response with similar time-scales.

Fig. 5 points out that this dynamic response is also quite
realistic. This figure shows the sawtooth period as a function
time during an actual TEXTOR measurement, resulting
from a similar linear sweep of the vertical mirror angle. Apart
from a small offset in \( \vartheta \) (during the experiment \( \vartheta \) was swept
from \( 5^\circ \) to \( 15^\circ \) between \( t = 2 \text{ s} \) and \( t = 4 \text{ s} \), there is a large
resemblance between both models and the experiment, in
terms of the influence of \( \vartheta \) on \( \tau_s \).

Based on Fig. 4 and 5 we conclude that both models are
sufficiently accurate from a control point of view, since they
yield correct sawtooth periods and exhibit the characteristic
sawtooth input-output behavior (i.e. the increase and decrease
of \( \tau_s \) depending on \( \vartheta \) [8]) with time-scales in the appropriate
order of magnitude. In our research a sawtooth model must be considered as a case study for this type of system, meant to explore and test suitable control strategies. The input-output behavior and simulation runtime of the simplified model (3) make it very suitable for this purpose.

Remark 1 As can be seen in Fig. 5, especially in its middle region, the observed sawtooth period during experiments can behave rather erratic. This phenomenon is known as compound behavior with aperiodic intermediate crashes, and is considered to be hard (if not impossible) to control. In future controller implementations on tokamaks one needs to take this behavior into consideration, which probably implies avoiding this compound region in practice. Furthermore, the small variations on the measured period for the other regions in Fig. 5 indicate that this signal needs to be lowpass filtered. This will restrict the obtainable bandwidth in practice.

IV. CLOSED LOOP RESULTS

As suggested before, our model does not provide an exact and complete description of the sawtooth instability as physics oriented models (like Porcelli [13]) do, but merely provides a system which can mimic its main behavior in real-time. Consequently, we cannot rely on our exact model definition in a controller synthesis. Changes in plasma settings, parameter mismatches, tokamak imperfections, unmodeled phenomena, etc. will influence plots like Fig. 4, contributing to modeling uncertainties. Currently, there is no classification of these uncertainties available, hence the controller to be used must be as generic and robust as possible. In this paper we therefore restrict ourselves to PI-control, where the controller is given by $C(s) = P + \frac{I}{s}$, where $s$ is the Laplace variable. Any model mismatches or steady-state errors will be dealt with by the integrator.

Such a PI-controller can easily be combined with our implemented model in Simulink®, as is illustrated in the closed loop of Fig. 6. The feedback controlled mirror is represented as a second order low-pass filter with a 20 Hz cut-off frequency and damping of 0.6 [17]. As noted before, only the SISO case will be considered in this closed loop, hence the vertical mirror angle $\vartheta$ is the controllable input, while the current drive is kept constant at $I_{CD} = 2.1$ kA.

As a first system identification step, we can derive an estimation of the system’s DC-gain as a function of the mirror angle $\vartheta$, by estimating the derivative $\frac{d\vartheta}{dt}$ of the static input-output map of our system (see [15]; closely related to the sweep results in Fig. 4). The result in dB is shown in Fig. 7, from which we can roughly identify three different control regions:

A. center region, approximately $\vartheta < 5.6^\circ$, with a small DC-gain and negative sign;

B. middle region, approximately $5.6^\circ < \vartheta < 8.3^\circ$, with a large DC-gain and positive sign;

C. outer region, approximately $\vartheta > 8.3^\circ$, with a relatively large DC-gain and negative sign.

In each region the P-action will be kept small to prevent sudden steps in the vertical mirror angle, which can bring the system to a different region, possibly causing instability. Here we take $P = 0$, and choose $I$ such that its product with the system’s DC-gain around the considered setpoint is approx. 30 dB for each region. The open loop $L(s) \approx \frac{L_{DC}}{s}$ then has a 5 Hz bandwidth, resulting in a 0.2 s closed loop time scale (neglecting time scales of the system itself). Generally both the initial condition and the setpoint should lie in the region we are controlling at that time, since the DC-gains and their signs are significantly different in each of the regions.

A. The center region

To illustrate center region control, we choose an initial condition $\vartheta_0 = 0^\circ$ and a setpoint of $\tau_{a,\text{ref}} = 10$ ms. The DC-gain around this sawtooth period is relatively small and negative, hence we choose $I = -38$ to obtain our 30dB target. The corresponding closed loop response is shown in blue in Fig. 8. Between $t = 0$ s and $t = 0.5$ s control is switched off, allowing the system to settle around $\tau_a = 11.8$ ms, corresponding to a 2.1 kA current drive in the plasma center. At $t = 0.5$ s the controller is switched on, resulting in an immediate change of $\vartheta$ in an outward direction, and an almost immediate change of the period $\tau_a$. The blue curves in Fig. 8 show that the desired period of 10 ms is reached after 0.2 s, corresponding to a vertical mirror angle of $3.9^\circ$, which is in accordance with Fig. 4. This proves the success of our closed loop PI-controller.
B. The middle region

For the middle region we take $\tau_{s,\text{ref}} = 16$ ms and $\vartheta_0 = 11^\circ$, i.e. an initial vertical mirror angle in the outer region (for demonstrational reasons). The DC-gain around this sawtooth period is very large, so that we now need to choose $I = +2$ to meet the 30dB target.

As can be seen in green in Fig. 8, this choice of parameters initially results in an exponential growth of the sawtooth period once the controller is switched on at $t = 0.5$ s. The mirror angle is decreased, yielding an increase of $\tau_s$ from 16.5 ms to 30.5 ms. At this point the period is lifted across the maximum in Fig. 4 into the middle region where the sawtooth period is stabilized. The desired period is reached around 0.2 s after this entrance in the middle region.

Note that the total response is stable because the initial period (16.5 ms) is larger than the setpoint (16 ms), so that the controller is steering the mirror in the right direction, i.e. towards the plasma center. A setpoint of e.g. 20 ms would yield an unstable solution, since $\vartheta$ would then increase towards the plasma edge, decreasing the sawtooth period. However, the suggested PI-controller is stabilizing for all $\tau_{s,\text{ref}}$ and $\vartheta_0$ inside the middle region.

C. The outer region

The red curves in Fig. 8 show the closed loop response when the outer region is controlled, with $\tau_{s,\text{ref}} = 24$ ms and $\vartheta_0 = 24^\circ$, hence the initial angle is far from the plasma center. Using the same reasoning as for the other regions, $I = -4.5$ is chosen for this region.

D. Influence of time delay

In the numerical model used above the times at which a crash occurs are exactly and immediately known. Hence, there is no time delay in the determination of the sawtooth period; the new period is known as soon as a new crash occurs. In practice the sawtooth period should be detected

When control is switched on at $t = 0.5$ s the I-action starts to decrease $\vartheta$ immediately, depositing the 2.1 kA current drive closer to the plasma center. Initially this hardly changes $\tau_s$, since $\vartheta$ is still too large. However, after 0.3 s the period rapidly increases, and 0.3 s later settles after a small oscillation at the desired 24 ms.

The above discussion shows that the sawtooth instability can be successfully controlled with PI-controllers. We needed to divide the problem in three different regions, and define separate controllers for each one. By scaling the controller gains with the estimated system DC-gain, we could achieve the desired periods without any steady state error and with comparable settling times (0.2 - 0.3 s). However, this strategy only achieved local stability for each separate region. To obtain global stability we need to design e.g. some higher-level supervisory control loop or hybrid control strategy. This hybrid approach is part of current research.

Remark 2 Notice that in this paper none of the controllers needs information on the inversion radius $r_{\text{inv}}$ [15] (a measure for $r_1$, the location of $q = 1$). Instead, they all solely rely on the sawtooth period measurement. However, when combining the separate controllers this $r_{\text{inv}}$ measurement will be useful, since it indicates in which region the system is operating at a certain time.

D. Influence of time delay

In the numerical model used above the times at which a crash occurs are exactly and immediately known. Hence, there is no time delay in the determination of the sawtooth period; the new period is known as soon as a new crash occurs. In practice the sawtooth period should be detected
and determined from e.g. temperature measurements in the plasma center region. Consequently, any sawtooth detection algorithm will add time delay to the feedback loop, which will alter the closed loop performance.

To illustrate this we add a time delay to the model output in Fig. 6, controlling the center region with \( P = 0, I = -30, \vartheta_0 = 0^\circ \) and \( \tau_{s, \text{ref}} = 9.5 \text{ ms} \). The closed loop responses with varying delays are shown in Fig. 9. The response changes from very fast without overshoot (no delay), to relatively fast with some overshoot (20 ms delay), to very slow with large oscillations (34 ms delay), to even instability (38 ms delay). Hence, sawtooth detection algorithms which need even a few crashes to determine the period can be very problematic.

The critical amount of delay before the closed loop becomes unstable (34 ms in the above example) depends on many factors. A smaller controller gain allows larger delays, e.g. when \( I = -20 \) the critical delay is 60 ms. Setpoints closer to the middle region (i.e. smaller \( \tau_{s, \text{ref}} \)) allow less delay, e.g. when \( I = -30 \) and \( \tau_{s, \text{ref}} = 9 \text{ ms} \) the critical delay is 14 ms, which is less than two sawtooth periods.

Consequently, to get fast and robust closed loop responses a sawtooth detection algorithm should be as fast as possible, adding a time delay of preferably less than a sawtooth period. This will be taken into account in the design of such algorithms for future use in tokamak experiments.

V. CONCLUSIONS AND FUTURE WORK

In this paper we have presented a simplification of the sawtooth instability model presented in [15], and shown that its output in terms of sawtooth period and input-output behavior is very similar. It was shown that this simplified model still mimics the experimental observations fairly well, while its numerical implementation is 20 times faster than the extended model in [15].

The model was then embedded in a Simulink® feedback control loop, and several simulations with different PI-controllers were performed. We have shown that the sawtooth instability can be divided into three different regions, each having unique controller demands. Each region could be controlled with 0.3 s settling time to any desired sawtooth period inside that region. This illustrates the feasibility of the sawtooth control problem. Furthermore, we have shown that time delays induced by detection algorithms can be dealt with successfully, as long as the amount of time delay is reasonably small.

Unfortunately, global stability cannot be proven yet. The separate PI-controllers should in some way interact with each other to guarantee global stability, e.g. by means of a higher-level supervisory loop or hybrid switching. This is part of current research. Also non-linear control strategies instead of the standard PI-controllers will be investigated.

Finally, the model in this paper enables further control oriented analysis of the sawtooth instability, e.g. linearization around operating points and system identification techniques. Its numerical implementation also provides a very useful testing environment for future sawtooth controller designs, before implementation on real tokamaks.

VI. ACKNOWLEDGMENTS

The authors would like to thank Prof. N.J. Lopes Cardozo for his critical remarks and suggestions, and I.D. Maassen van den Brink for her contribution to the results in Fig. 5.

The work in this paper has been performed in the framework of the NWO-RFBR Centre of Excellence (grant 047.018.002) on Fusion Physics and Technology. This work, supported by NWO, ITER-NL and the European Communities under the contract of the Association EURATOM/FOM, was carried out within the framework of the European Fusion Programme. The views and opinions expressed herein do not necessarily reflect those of the European Commission.

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