High Performance Adaptive Control in the Presence of Time Delays

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Abstract—Many potential applications of adaptive control, such as adaptive flight control systems, require that the controller have high performance, stability guarantees, and robustness to time delays. These requirements typically lead to engineering trade-offs, such as a trade-off between performance and robustness. In this paper, we examine several modifications to the typical direct model reference adaptive control (MRAC) approach which can be used to achieve higher performance as well as higher levels of robustness. A new Time Delay Resistant (TDR) adaptive control framework is proposed using a combination of several modifications to MRAC. The various modifications to MRAC, as well as the TDR adaptive controller, are applied to the control of longitudinal dynamics of an aerial vehicle in simulation.

I. INTRODUCTION

The application of adaptive control to aircraft promises benefits in both safety and robustness, especially for high-performance aircraft. Early attempts at adaptive flight control used controllers with unproven stability properties, sometimes with disastrous consequences; for example the fatal crash of the NASA X–15 in November, 1967. As a result, much of the theoretical work up to the present time has been rightly focused on stability of adaptive architectures. Currently, there exists an assortment of stable adaptive control strategies, as well as techniques for preserving stability in the presence of disturbances, unmodeled dynamics, and time delays [1]-[8].

In addition to these advances in the theory of stable adaptive systems, a number of modifications to the standard model reference adaptive control (MRAC) approach have been presented. One of these involves the combination of direct and indirect MRAC, which is known as Combined or Composite MRAC (CMRAC). By adapting to both estimation and tracking errors, it has been observed that CMRAC systems have smoother transient performance than MRAC systems[9]-[12]. Another such modification is the use of time-varying adaptive gains. In particular, a bounded-gain forgetting (BGF) law for adjusting adaptive gains based on least-squares estimation was developed [13]. This modification has the benefits of faster parameter convergence and smoother parameter estimates. While not specifically designed for systems with time delay, both CMRAC and BGF share a common feature in that they both smoothen the adaptive and estimation parameters. By reducing the high frequency content in the system, the phase lag caused by the time delay will also be reduced. Therefore, we propose that these modifications to MRAC may be useful tools in designing controllers for systems with time delay as well.

In addition to the two modifications described above, the recently developed Adaptive Posicast Controller (APC) [14] is also examined. The APC approach is an adaptive extension of the Smith Predictor, which uses a plant model to predict the future outputs of the plant, and then uses this to cancel the effect of delay on the system.

The paper is organized as follows. In Section II, the problem is defined and the typical MRAC approach is presented. In Section III, the various modifications to the MRAC approach are described. Section IV describes how the various modifications are integrated into one coherent controller design. Simulation results are presented in Section V and conclusion and future research directions are given in Section VI.

II. PROBLEM STATEMENT

Consider a MIMO, state variables accessible system of the form

\[
\dot{x}_p(t) = A_p x_p(t) + B_p \Lambda (u(t - \tau) + d(x_p)),
\]

(1)

where \(B_p \in \mathbb{R}^{m \times n}\) is constant and known, \(A_p \in \mathbb{R}^{m \times n}\) is constant and unknown, \(x_p \in \mathbb{R}^m\), \(u \in \mathbb{R}^m\), \(\Lambda \in \mathbb{R}^{m \times n}\) is an unknown positive definite constant matrix, \(\tau\) is a known time delay, and

\[
d(x_p) = \theta_d^T \phi_d(x_p),
\]

(2)

is a nonlinear state-dependent matched parametric uncertainty where \(\theta_d \in \mathbb{R}^{N \times n}\) is a matrix of unknown constant parameters, and \(\phi_d(x_p) \in \mathbb{R}^N\) is a known \(N\)-dimensional regressor vector, whose components are continuously differentiable functions of \(x_p\). The goal is to track a reference command \(r(\cdot)\) in the presence of the unknown \(A_p, \Lambda, \theta_d\), and the known \(\tau\). This paper deals only with the case of a known input delay. A more complete representation of delays would include various time delays corresponding to sensors, computation, and communication, each with an associated nominal part and uncertain part (e.g. 40 ms ± 10 ms). Note that from this point forward, all time-varying signals are taken to be at time \(t\) unless explicitly noted otherwise.

The system output is given by

\[
y = C_p x_p,
\]

(3)

and the output tracking error is given by

\[
ey = y - r.
\]

(4)

Augmenting (1) with the integrated output tracking error

\[
\dot{ey}_y = ey = y - r,
\]

(5)

leads to the extended open loop dynamics

\[
\dot{x} = A x + B \Lambda (u(t - \tau) + d(x_p)) + B_c r,
\]

(6)
where \( x = \begin{bmatrix} x^T_p \ c^T_y \end{bmatrix} \) is the extended system state. The extended open-loop system matrices are given by

\[
A = \begin{bmatrix} A_p & 0 \\ C_p & 0 \end{bmatrix}, \quad B = \begin{bmatrix} B_p \\ 0 \end{bmatrix}, \quad B_c = \begin{bmatrix} 0 \\ -I \end{bmatrix},
\]

and the extended system output

\[
y = \begin{bmatrix} C_p & 0 \end{bmatrix} C x.
\]

A nominal controller

\[
u_{nom} = K_x x,
\]

can be designed for the system in (6) in the case where there is no uncertainty or time delay, that is \( \lambda = I^{nxn} \) and \( \tau = 0 \), and \( A \) is taken to be some nominal value. The feedback gains \( K_x \) can be selected using LQR or classical design techniques.

The reference model used by MRAC is the closed loop system given by (6), again in the case of no uncertainty or time delay, along with the control input in (9)

\[
\dot{x}_m(t) = Ax_m(t) + Bu_{nom}(t) = A_m x_m(t) + B_c r(t).
\]

An adaptive control input is added to the baseline controller as

\[
u_{ad} = \dot{\hat{\theta}}^T x - \dot{\hat{d}}^T d(x) = -\dot{\hat{\theta}}^T \phi(x),
\]

where \( \dot{\hat{\theta}}^T = \begin{bmatrix} \dot{\hat{K}}_x^T \ \dot{\hat{\theta}}_d^T \end{bmatrix} \) are time-varying adaptive parameters that will be adjusted in the adaptive law given in (13) below and \( \phi^T = \begin{bmatrix} x^T \ \phi_d^T \end{bmatrix} \) is the regressor vector. The overall control input is thus

\[
u = u_{ad} + u_{nom} = -\dot{\hat{\theta}}^T \phi(x) + K_x x + r.
\]

The canonical adaptive law is given by

\[
\dot{\hat{\theta}} = -\Gamma_\theta \phi e^T P B,
\]

where \( \Gamma_\theta \) is a diagonal positive definite matrix of adaptive gains, \( e = x - x_m \) is the model tracking error, and \( P \) is the unique symmetric positive definite solution of the Lyapunov equation,

\[
A_m^T P + PA_m = -Q,
\]

where \( Q \) is also symmetric positive definite. The stability of this system is a well known result in adaptive control [1], [2].

### III. Modifications to the MRAC Approach

In this section, several modifications to the MRAC approach described above are presented. Each modification is first presented individually using the MRAC as a starting point. Details of the stability proofs will not be included, but can be found in the references.

#### A. Combined / Composite Model Reference Adaptive Control (CMRAC)

It has been observed through numerous simulation studies for various models that better transient characteristics can be obtained by using prediction errors in addition to tracking errors in the design of adaptive controllers [9]-[11], [15]. Thus a Combined or Composite MRAC structure was developed by combining aspects of direct and indirect adaptive control. The particular version of CMRAC used in this paper differs from some previous formulations found in [11], [13], [16] in that: a) it is applicable to a generic class of MIMO dynamical systems, b) online measurements of the system state derivative are not required, and c) the system is designed to augment a baseline linear controller [17].

To construct the CMRAC laws, we first introduce a stable filter

\[
G(s) = \frac{\lambda_f}{s + \lambda_f},
\]

where \( \lambda_f > 0 \) is the filter inverse constant. The filtered version of \( x \) is then given by

\[
\dot{x}_f = \lambda_f (x - x_f).
\]

From this point forward, the subscript \( f \) on a signal denotes a version of that signal that has been filtered by \( G(s) \).

By substituting (12) into (6), filtering both sides and then substituting (16), we can write the filtered dynamics as

\[
(B^T B)^{-1} B^T (\lambda_f (x - x_f) - A_m x_f - B_c r_f) = \Lambda \left( \dot{\hat{\theta}}^T \phi_f + K_x x_f \right),
\]

with the assumption that \( B \) is full rank. We denote the left side of the equation (17) as \( Y \) and note that it can be calculated online at every instant of time. That is,

\[
Y = B^+ \left( \lambda_f (x - x_f) - A_m x_f - B_c r_f \right),
\]

where \( B^+ = (B^T B)^{-1} B^T \). Also note that \( Y \) can be viewed as the known output of the static bilinear-in-unknown parameters truth model,

\[
Y = \Lambda \left( u_f + \hat{\phi}^T \phi_f \right),
\]

We can therefore estimate \( Y \) according to the bilinear predictor model

\[
\hat{Y} = \hat{\Lambda} \left( u_f + \dot{\hat{\phi}}^T \phi_f \right).
\]

The predictor output estimation error is then given by

\[
e_Y = \hat{Y} - Y
\]

It is this error that can be used in addition to the tracking error in (13), leading to the adaptive law

\[
\dot{\hat{\theta}} = -\Gamma_\theta \left( \phi e^T P B - \phi \gamma_c e_Y^T \right),
\]

and the estimation law

\[
\dot{\hat{\Lambda}} = \Gamma_\Lambda \left( u e^T P B - u_f \gamma_c e_Y^T \right).
\]

Comparing the MRAC laws (13) with those of the CMRAC approach (22), it is clear that the former is the special case
of the latter when $\gamma_c = 0$. Also, it can be easily confirmed that the CMRAC laws (22) combine direct and indirect adaptive laws by simply adding the two together. A Lyapunov approach and the application of Barbalat’s Lemma lead to a global asymptotic stability result for the closed-loop tracking dynamics [17].

B. BGF

The bounded-gain forgetting estimator approach replaces the constant adaptive gain matrix in (13) with a time-varying adaptive gain. The update law for the adaptive gain is given by

$$\dot{\Gamma}_\theta^{-1} = -\rho_\theta(t)\Gamma_\theta^{-1} + \phi_f^T \phi_f,$$  \hspace{1cm} (24)

where $\rho_\theta$ is a time-varying forgetting factor adjusted according to

$$\rho_\theta = \rho_\theta_0 \left(1 - \frac{\|\Gamma_\theta\|}{k_{\rho_\theta}} \right)$$  \hspace{1cm} (25)

which ensures that $\Gamma_\theta$ remains bounded. Application of the same Lyapunov function as in the CMRAC case above [17] leads to a similar global asymptotic stability result.

C. Adaptive Posicast Controller (APC)

The Adaptive Posicast Controller (APC) is essentially an adaptive extension of the Smith Predictor, an approach that originated as a method to deal with systems with large delays. The APC method also brings in ideas from finite spectrum assignment [18]. The main idea is to predict the future output of the plant using a plant model, and then to use this prediction to cancel the effect of time delay on the system. It does this by adding an additional term to the control laws (12). The overall control input becomes

$$u = K_\pi x - \tilde{\theta}^T \phi(x) + \int_{t-\tau}^{t} \lambda(t, \eta)^T u(t + \eta) d\eta.$$  \hspace{1cm} (26)

In discrete time, the integral is approximated as

$$u = K_\pi x - \tilde{\theta}^T \phi(x) + \lambda_{t}^T u_{\bar{\eta}},$$  \hspace{1cm} (27)

where $u_{\bar{\eta}} \in \mathbb{R}_{mT/dT}$ and $\lambda_{t} \in \mathbb{R}_{mT/dT \times m}$ are a vector of control inputs at times $t+\eta$ and $\lambda(t, \eta)$ evaluated at $\eta = \bar{\eta}dT$. The indices $\bar{\eta}$ are given by the integer values of $\eta/dT$ over $\eta \in [-\tau/dT, 0]$. It is assumed that the time delay $\tau$ is a multiple of the time step $dT$.

The parameters $\lambda_{t}$ are adjusted according to

$$\dot{\lambda}_{t} = -\Gamma_{\lambda} u_{\bar{\eta}}(t-\tau)e^T PB.$$  \hspace{1cm} (28)

A semi-global proof of signal boundedness for all $\tau$ less than some critical $\tau^*$ is achieved using a unique Lyapunov Krasovskii functional [14].

IV. TIME DELAY RESISTANT ADAPTIVE CONTROLLER

We now combine the modifications of MRAC suggested in Section III into one control framework. To accomplish this, we go through each of the modifications presented in the previous section, reconciling them with one another. The equations for $Y$ and $e_Y$ are unchanged, however, the output of the bilinear predictor model $\tilde{Y}$ must be adjusted to include the new adaptive parameters as

$$\tilde{Y} = \hat{\Lambda} \left( u_f(t-\tau) - \hat{\Theta}^T \omega_f \right)$$  \hspace{1cm} (29)

where $\hat{\Theta}^T = [\hat{\theta}^T \lambda_f^T]$ and $\omega_f^T = [\phi_f^T u_{\bar{\eta}}^T]$. Note that the MRAC adaptive parameters have been augmented with the posicast parameters and the filtered regressor vector now includes the filtered, delayed control signals $u_{\bar{\eta}}$.

The adaptive laws can then be written as

$$\dot{\hat{\Theta}} = -\Gamma_{\Theta} (\omega(t-\tau)e^T PB - \omega_f(t-\tau)\gamma_c e_Y^T),$$  \hspace{1cm} (30)

where $\omega^T = [\phi_f^T u_{\bar{\eta}}^T]$ and $\Gamma_{\Theta} = \text{diag}(\Gamma_\theta, \Gamma_\lambda)$.

The adaptive gains in (30) can be made time-varying according to using the BGF least-squares estimator approach as

$$\dot{\Gamma}_{\Theta}^{-1} = -\rho_\Theta \Gamma_{\Theta}^{-1} + \omega(t-\tau)\omega(t-\tau)^T,$$  \hspace{1cm} (31)

with the forgetting factor given by

$$\rho_\Theta = \rho_\Theta_0 \left(1 - \frac{\|\Gamma_{\Theta}\|}{k_{\rho_\Theta}} \right).$$  \hspace{1cm} (32)

The parameter estimation laws are given by

$$\dot{\hat{\Lambda}}^T = \Gamma_{\Lambda} \left( u(t-\tau)e^T PB - u_f(t-\tau)\gamma_c e_Y^T \right).$$  \hspace{1cm} (33)

Similarly, the adaptive gains $\Gamma_{\Lambda}$ in (33) can be made time-varying according to the BGF approach as

$$\dot{\Gamma}_{\Lambda}^{-1} = -\rho_{\Lambda} \Gamma_{\Lambda}^{-1} + u_f(t-\tau)u_f(t-\tau)^T,$$  \hspace{1cm} (34)

with the forgetting factor

$$\rho_{\Lambda} = \rho_{\Lambda_0} \left(1 - \frac{\|\Gamma_{\Lambda}\|}{k_{\rho_{\Lambda}}} \right).$$  \hspace{1cm} (35)

Finally, the overall control input can be written as

$$u = K_\pi x - \tilde{\Theta}^T \omega + r.$$  \hspace{1cm} (36)

Equations (30)-(36) fully describe the time delay resistant adaptive controller.

A detailed proof of stability for the TDR adaptive controller can be found in [19], but a brief description is as follows. A Lyapunov candidate function for the TDR adaptive controller can be constructed by a sum of component Lyapunov functions. The full Lyapunov function candidate consists of the Lyapunov function used in the global asymptotic stability proof for CMRAC and the Lyapunov functional used in the semi-global signal boundedness proof for APC. The resulting Lyapunov function derivative contains negative semi-definite terms from the CMRAC control laws and the BGF adaptive gain laws and a term which is negative outside a compact set from the APC laws. Therefore, the TDR adaptive controller shares a similar semi-global proof of signal boundedness with the APC.
V. Simulation Results

In this section, we will compare the performance of the developed TDR architecture with the performance of the individual modifications to MRAC as well as MRAC alone through simulation. The simulation model used is an F-16 short period longitudinal dynamics model. Neglecting the effects of gravity and thrust, the short period dynamics are given by [20]

\[
\begin{bmatrix}
\dot{\alpha} \\
\dot{q}
\end{bmatrix} = \begin{bmatrix}
\frac{Z_\alpha}{M_{\alpha}} & 1 + \frac{Z_q}{M_q} \\
\frac{Z_\delta_e}{M_{\delta_e}} & \frac{Z_q}{M_q}
\end{bmatrix} \begin{bmatrix}
\alpha \\
q
\end{bmatrix} + \begin{bmatrix}
\frac{Z_\delta_e}{M_{\delta_e}} \\
\frac{Z_q}{M_q}
\end{bmatrix} \delta_e
\] (37)

where \(\alpha\) is the aircraft angle of attack, \(q\) is the pitch rate, \(\delta_e\) is the elevator deflection, and \(V\) is the trimmed (constant) air speed. The partial derivatives of the aerodynamic vertical force \(Z\) with respect to \(\alpha\), \(q\), and \(\delta_e\), are given by \(Z_\alpha\), \(Z_q\), and \(Z_{\delta_e}\) respectively. Similarly, the partial derivatives of the pitching moment \(M\) with respect to \(\alpha\), \(q\), and \(\delta_e\), are given by \(M_{\alpha}\), \(M_q\), and \(M_{\delta_e}\) respectively. The numerical values for these aerodynamic derivatives can be found in [20], Example 5.5.3, Table 3.4.3. These particular values correspond to an F-16 aircraft trimmed at sea level, with \(V_T = 502\) ft/sec, \(\bar{Q} = 300\) lb/ft², and \(CG = 0.35\bar{c}\).

With \(\alpha\) in units of radians, \(q\) in radians/second, and \(\delta_e\) in degrees, the resulting open-loop system matrices are given by

\[
A = \begin{bmatrix}
-1.0189 & 0.9051 \\
0.8223 & -1.0774
\end{bmatrix}, \quad B = \begin{bmatrix}
-0.0022 \\
-0.1756
\end{bmatrix}.
\]

The following matched uncertainties are introduced into the system: a) linear-in-state uncertainty \(K_{x_p,\text{pert}}^{T}\), b) constant loss of control effectiveness \(\Lambda > 0\), and c) nonlinear-in-state uncertainty in the form of (2). Numerical values for these uncertainties are given by

\[
K_{x_p,\text{pert}}^{T} = \begin{bmatrix}
-4.6839 \\
-9.8197
\end{bmatrix}, \quad \Lambda = 0.5, \quad d(\alpha) = e^{-\frac{(\alpha - \alpha_0)^2}{2\sigma^2}}
\] (38)

where the center of the Gaussian was set to \(\alpha_0 = 2\frac{\pi}{180}\) radians, and its width was \(\sigma = 0.0233\) radians. The values of the uncertainties given in (38) are equivalent to a 50% increase in \(M_{\alpha}\), an 80% decrease in \(M_q\), and a 50% decrease in the elevator effectiveness \(M_{\delta_e}\). Thus, the vehicle has become 50% more statically unstable, lost 80% of its pitch damping ability, and the aircraft controllability has decreased by 50%. Time delays tested ranged from 0-250 ms. Obviously these uncertainties are quite drastic and perhaps even unrealistic. Nonetheless, the choice of such a “harsh” uncertainty was motivated by our intent to demonstrate effectiveness of the proposed TDR design, and to highlight the differences between the various modifications to the MRAC approach.

The full aircraft dynamics, along with the uncertainties described above now take the form of (1),

\[
\dot{x}_p = (A_p + B_p\Lambda K_{x_p,\text{pert}}) x_p + B_p\Lambda (u(t - \tau) + d(x_p))
\] (39)

where \(x_p = [\alpha \quad q]^T\) is the state vector, \(\Lambda > 0\) is the uncertainty in the elevator effectiveness, and the state-dependent nonlinearities are captured by the function \(d(x_p) = d(\alpha)\), which represents the unknown nonlinear increments in the vehicle aerodynamic \(Z\)-force and pitching moment \(M\). The controlled output state \(y\) was chosen to be the angle of attack \(\alpha\). Thus, the control goal becomes tracking of a bounded, time-varying angle of attack command \(r = \alpha_{\text{cmd}}\) in the presence of the various uncertainties and the time delay \(\tau\). To ensure steady state tracking performance, integrated \(\alpha\) tracking error dynamics, \(\dot{\bar{e}}_\alpha = \alpha - \alpha_{\text{cmd}}\), are included as well. The resulting extended open-loop dynamics takes the form of (6).
of the RBFs were placed within $\alpha \in [-10, 10]$ and $q \in [-10, 10]$ with a spacing of 2° and 2°/sec respectively. The widths of the RBFs were all set to $\sigma = 0.0233$. The reference matrix $A_m$ was defined as in (9)-(10) with the baseline feedback gains selected using the LQR method [20], with $Q_{lqr} = \text{diag} \left( \begin{bmatrix} 0 & 100 \end{bmatrix} \right)$ and $R_{lqr} = 1$. This resulted in LQR feedback gains $K_x = [10.8786 \ 6.0589 \ 10]$. A direct MRAC controller was then designed as described in Section 2. The algebraic Lyapunov equation (14) was solved with $Q = \text{diag} \left( \begin{bmatrix} 1 & 800 & 0.1 \end{bmatrix} \right)$ and rates of adaptation were set to $\Gamma_\theta = \text{diag} \left( \begin{bmatrix} 400 & 400 & 1 & 20 \end{bmatrix} \right)$. For the later tests using CMRAC $\Gamma_\Lambda = 8$, $\lambda_f = 10$, and $\gamma_c = 4$ were used.

<table>
<thead>
<tr>
<th>$\gamma_k$</th>
<th>$\rho_\sigma$</th>
<th>$k_{\rho_\sigma}$</th>
<th>$\rho_\Lambda$</th>
<th>$k_{\rho_\Lambda}$</th>
<th>$\Gamma_\Lambda$</th>
</tr>
</thead>
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<tr>
<td>MRAC</td>
<td>0.01</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>CMRAC</td>
<td>0.1</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
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<tr>
<td>BGF</td>
<td>0.02</td>
<td>0.0432</td>
<td>10</td>
<td>0.16</td>
<td>0.5</td>
</tr>
<tr>
<td>APC</td>
<td>0.05</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>0.001</td>
</tr>
<tr>
<td>TDR</td>
<td>0.05</td>
<td>200</td>
<td>5</td>
<td>0.4</td>
<td>0.5</td>
</tr>
</tbody>
</table>

**TABLE I**
GAINS USED FOR THE 250MS TEST CASE

The first simulation study consists of simulation of the closed-loop system with the three types of uncertainties mentioned previously, but with no time delay, that is $\tau \equiv 0$. Figure 1 shows that the MRAC approach solves the tracking problem, while the LQR approach is unable to accommodate the uncertainties and has very poor tracking performance. This result is unsurprising given that adaptive control has been shown to be very effective in dealing with parametric uncertainty and MRAC has a global asymptotic proof of stability for the simulated system.

However, when even a small amount of time delay (40 ms, in this case) is added to the system, the performance of our well-tuned adaptive controller deteriorates dramatically. In Figure 2 (a), the MRAC approach displays undesirable oscillations. Figure 2 (b) shows that the elevator deflection is unreasonably large.

<table>
<thead>
<tr>
<th>$f[\varepsilon]$</th>
<th>1.00</th>
<th>75.0</th>
<th>0.323</th>
<th>0.770</th>
<th>0.398</th>
<th>0.232</th>
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<tr>
<td>$f[\delta_e]$</td>
<td>1.00</td>
<td>24.2</td>
<td>0.867</td>
<td>0.945</td>
<td>0.877</td>
<td>0.868</td>
</tr>
<tr>
<td>$f[\delta_e]$</td>
<td>1.00</td>
<td>19.3</td>
<td>0.657</td>
<td>1.02</td>
<td>0.822</td>
<td>0.782</td>
</tr>
</tbody>
</table>

**TABLE II**
COMPARISON OF TRACKING ERROR AND CONTROL EFFORT

One possible solution to this problem is to reduce the adaptive gains. A single scaling factor, $\gamma_k$ across both $\Gamma_\sigma$ and $\Gamma_\Lambda$ allows for simple adjustment. This reduction in gains eliminates the undesirable oscillations at the cost of transient performance and parameter convergence. As a control designer, one has the choice of either using the MRAC approach with reduced gains, or using a modification to MRAC to improve performance. The command tracking performance of the various MRAC modifications discussed previously for a more severe 250ms time delay is shown in Figure 3. The various gains and other design parameters were tuned to achieve an optimal balance of tracking error and control effort. The values used for simulation can be found in Table I.

All three modifications display increased tracking performance over the MRAC-with-reduced-gains approach. A comparison of the tracking performance of the various approaches can be found in Table II. For comparison purposes, in this table each of the L1 norms of tracking error $e$, control position $\delta_e$, and control rate $\dot{\delta}_e$ over the duration of the simulation are normalized by the corresponding value for the MRAC approach. Both the CMRAC and APC approaches show more than 60% reduction in tracking error. Typically in control systems there is a design trade-off between tracking performance and control power used. However, by utilizing these modifications to MRAC, it is possible to both increase tracking performance and decrease control effort as compared with MRAC. Table II also contains comparisons of...
the control position effort and control rate effort.

The TDR adaptive controller is designed as described in Section 4, with the design parameters given in Table I, and is tested using the same uncertainties and 250ms time delay. Figure 4 shows that the TDR adaptive controller exhibits an over 75% reduction in tracking error over the MRAC approach. The control effort used is also 15%-20% less than that of MRAC.

VI. CONCLUSIONS

Several modifications to the typical MRAC approach were examined with specific application to systems with time delay. The modifications presented were either designed specifically to counter time delays, or had the effect of smoothing the adaptive or estimated parameters. All of the modifications were combined into one coherent control architecture, the time delay resistant adaptive controller. Simulation results show that the proposed modifications to the MRAC approach dramatically increase performance in the presence of time delays. Future work in this area includes examining additional methods for dealing with time delays such as wave variables, as well as application of the proposed approach on mini-UAVs.

REFERENCES