Abstract — Control strategies for Hybrid Electric Vehicles (HEVs) are generally aimed at optimally choosing the power distribution between the internal combustion engine and the electric motor in order to minimize the fuel consumption and/or emissions. Using vehicle navigation systems in combination with Global Positioning Systems and Geographical Information Systems allow further optimization of the power distribution by utilizing the route information. In this paper, a new control algorithm based on a combination of dynamic programming and classical optimal control theory is proposed for the Energy Management System in parallel HEVs to improve the fuel economy over a preview route segment. The proposed algorithm optimizes not only the gear position and the engine power yet also the vehicle velocity. The vehicle is controlled to complete this route segment in a predefined time length. Using this method more than 11% fuel saving is computed on an optimized cycle compared to a standard city cycle with equal time length and average speed.

I. INTRODUCTION

Nowadays, the widely usage of vehicle’s onboard navigation system combining with a Global Positioning System (GPS) and a Geographical Information System (GIS) helps driver gaining traffic information over the preview route segment, such as, road characteristics, traffic conditions, speed limits, etc. Therefore, utilizing this information to further reduce the fuel consumption for a vehicular propulsion system, especially for Hybrid Electric Vehicles (HEVs) is an interested topic for researchers in recent years. In [7], a fuzzy-based control algorithm was proposed to utilize GPS information. The route information was used to predict the future vehicle’s speed, which is computed as a weighted average value of look-ahead zone. In [8], the authors specified that the vehicle travels on a specific route as a stochastic process based on collected route data. The route is divided into a number of intervals, and the vehicle’s state at the end of each interval is modeled by a discrete time Markov chain. In [9, 10], the authors use information from GPS and GIS to build a drive cycle corresponding to the preview route segments. The obtained drive cycle is then used to calculate the future power requested for Hybrid Electric Vehicles (HEVs) so that energy management strategy can be derived. However, the drive cycle is built based on the assumption of constant acceleration and deceleration rates, and speed limits for every consecutive route segments. In [11], the authors build the velocity profile along the preview route cycle by dividing it into a series of segments and speed for each segment is chosen from statistical features (mean, variance, etc.). Accordingly, the path forecasting based control algorithm was proposed to optimize charging and discharging of the vehicle’s battery. Apparently, in the studies for HEVs reviewed above, the obtained velocity profile along the preview route cycle or segment is not optimal in terms of fuel consumption and time traveling.

In this paper, for a given route information over a preview route segment, a new control algorithm is proposed for the Energy Management System (EMS) for a parallel HEV to drive the vehicle at the most fuel economy way and complete the preview route segment in a predefined time length. The vehicle speed profile, gear position and engine torque are optimized over the preview route segment. The control algorithm is designed based on a combination of Dynamic Programming (DP) and classical optimal control theory such that the computational efficiency is significantly improved compared to the case of using only DP algorithm for this three decision variables problem. Therefore, optimized velocity profile is expected to be obtained to gain more benefits in terms of fuel economy.

The paper is organized as follows: in Section II the vehicle modeling is outlined. The problem formulation and the control algorithm synthesis are given in Section III. The way of implementing the proposed control algorithm is presented in Section IV. Simulation results are given in Section V. Finally, the conclusions and future study are outlined in Section VI.

II. VEHICLE MODELING

A. Vehicle modeling

The parallel hybrid powertrain topology under investigation in this paper is shown in Figure 1. With aiming at deriving an EMS for such a HEV, the static models [1-3] of the powertrain components are chosen whose dynamics faster than 1 Hz are ignored. Hence, a discretized model with time step of one second for this vehicle is chosen.

Assume that the vehicle rotational speed at a certain time step \( k \) is \( \omega(k) \). The longitudinal motion of hybrid electric vehicle is given by:
\[ \omega_n(k+1) = \omega_{e}(k) + \frac{1}{J_v} (T_w(k) - T_{load}(k)) \cdot \Delta t \] (1)

and the wheel torque demand by:
\[ T_w(k) = (T_e(k) + T_m(k) \cdot r_g(n_g(k))) \] (2)

wherein: \( r_g(n_g(k)) \) denotes the gearbox ratio, which is a function of the gear position \( n_g(k) \); \( T_{load}(k) \) is the load torque acting on the vehicle; \( J_v \) is the vehicle inertia; \( \Delta t \) is the length of time step. For simplicity, the gearbox efficiency is assumed to be 100%.

![Diagram of Hybrid powertrain prototype.](image)

Figure 1: Hybrid powertrain prototype.

The speed relations in the driveline are:
\[ \omega_e(k) = \omega_m(k); \omega_m(k) = \omega_n(k) \cdot r_g(n_g(k)); \omega_n(k) = \frac{v(k)}{r_w} \] (3)

wherein \( v(k) \) and \( r_w \) are vehicle speed and wheel radius. Furthermore, the constraints on the system are:

- constraints on the angular engine speed \( \omega_e(k) \) and electric machine speed \( \omega_m(k) \):
  \[ \omega_{e\text{ min}} < \omega_e(k) < \omega_{e\text{ max}} \]
  \[ 0 < \omega_m(k) < \omega_{m\text{ max}} \] (4)

- constraints on the engine power \( P_e(k) \) and electric machine power \( P_m(k) \). Assuming that no engine drag power during deceleration phase for this prototype hybrid powertrain.

\[ 0 \leq P_e(k) \leq P_{e\text{ max}}(\omega_e(k)) \]
\[ P_{m\text{ min}}(\omega_m(k)) < P_m(k) < P_{m\text{ max}}(\omega_m(k)) \] (5)

- constraints on the gear position of transmission:
  \[ 1 \leq n_g(k) \leq 6 \] (6)

The next gear position \( n_g(k+1) \) is expressed through the current gear position \( n_g(k) \) and the shift command \( u_g(k) \) as follows:
\[ n_g(k+1) = \begin{cases} 1, & n_g(k) + u_g(k) < 1 \\ 6, & n_g(k) + u_g(k) > 6 \\ n_g(k) + u_g(k), & \text{otherwise} \end{cases} \] (7)

The shift command at time step \( k \) is follows:
\[ u_g(k) = \begin{cases} -1 : & \text{downshift;} \\ 0 : & \text{sustaining;} \\ 1 : & \text{upshift;} \end{cases} \] (8)

For reasons of acceptable driveability, the set of discrete shift command values is chosen as \([-1, 0, 1]\) to avoid a large variation of engine speed for a certain shift at a certain time step \( k \). One gear down or upshift or sustaining for each time step of one second are reasonably, because the average shifting time for an Automated Manual Transmission (AMT) is typically one second. Note that the clutch losses during shifting have been left out of consideration.

The battery is considered as a dynamical system with state \( x(k) \), represented for energy content which is assumed sufficiently large enough, is defined as a function of stored chemical power \( P_e(k) \):
\[ x(k+1) = x(k) + P_e(k) \cdot \Delta t \] (9)

The vehicle speed and traveled distance, denoted by \( v(k) \) and \( s(k) \) respectively, are defined through acceleration rate \( a(k) \):
\[ a_{\text{min}} \leq a(k) \leq a_{\text{max}} \] (10)
\[ v_{\text{min}} \leq v(k+1) = v(k) + a(k) \cdot \Delta t \leq v_{\text{max}} \] (11)
\[ s(k+1) = s(k) + v(k) \cdot \Delta t + \frac{a(k)}{2} \cdot \Delta t^2 \leq L \] (12)

wherein: \( v_{\text{max}} \) and \( v_{\text{min}} \) are the maximum and minimum allowable speed on a certain route segment with a predefined length, denoted as \( L \), of the route segment. \( a_{\text{max}} \) and \( a_{\text{min}} \) are defined by vehicle characteristics, therein \( a_{\text{min}} \) stands for the highest deceleration rate (negative value), and \( a_{\text{max}} \) represents for the highest acceleration rate (positive value).

III. CONTROL ALGORITHMS

A. Problem formulation

Based on the preview route information obtained from GPS, GIS and traffic information systems, it is assumed that that the length of route segment \( L \), the minimum speed \( v_{\text{min}} \), the maximum speed \( v_{\text{max}} \) are known a-priori. The objectives are to design an EMS for the prototype HEV such that it finishes this preview route segment at a given time \( N \) in the cost fuel efficient way. Driving the vehicle along a given route segment length \( L \) at a given time \( N \) is equivalent to finishing this given distance at a given average speed \( \frac{v_{\text{avg}}}{L/N} \). The vehicle is expected to keep its speed inside an interval \([v_{\text{min}}, v_{\text{max}}]\).

It is clear that the operating points of this hybrid powertrain system are determined by three main state variables \( v(k), n_g(k) \) and \( T_e(k) \) which are controlled by \( a(k), u_g(k) \) and \( P_e(k) \) respectively. Let \( m_f(T_e(k), \omega_e(k)) \) denote the instantaneous fuel consumption of engine to produce torque \( T_e(k) \) at speed \( \omega_e(k) \), then the problem of minimizing fuel consumption of the parallel hybrid electric vehicle under such conditions can be formally formulated as follows:

Find the optimal control law \( u^*(k) = [a^*(k), u^*_g(k), P^*_e(k)] \) that minimizes the fuel cost function:
\[ J = \sum_{k=0}^{N-1} m_f(u(k), k) \cdot \Delta t \] (13)

subject to constraints:
(1) – (12)
\[ s(N) = L \]  
(14)
\[ x(N) - x(0) = 0 \]  
(15)

B. Proposed control algorithm

The optimization problem for the EMS of a HEV can be solved using different method, e.g., based on the results from a DP algorithm, or using gradient-based control optimization algorithms herein after called “classical optimal control”. Applying a DP algorithm for this problem will yield globally optimal results. We need to design search spaces for the three state variables \( s(k), n_g(k) \) and \( x(k) \) and compute for the optimal values along the preview route segment with length \( L \) by using Bellman’s optimality principle [12]. Apparently, this method due to the ‘curse of dimensionality’ becomes computation time consuming.

1) Control algorithm

In an effort to seek for an alternative method that overcomes the drawbacks of DP (large computation time) to solve for the optimal solutions to the formulated problem (13)-(15), a new control optimization algorithm is proposed for the EMS based on a combination both numerical DP and classical optimal control, which reduces the computation time. The numerical DP algorithm is applied for the outer loop of optimization process (indicated by DP algorithm in Fig. 3) to find the optimal acceleration profile \( a^*(k) \) resulting in the optimal velocity profile \( v^*(k) \) along the preview route segment. This optimization problem is reformulated as follows:

**Problem 1:** Find the admissible optimal control law for \( u^*_1(k) = a^*(k) \) that minimizes the cost function:

\[
J_1 = \sum_{k=0}^{N-1} C^{i,j}_k(u_i(k), k)
\]

subject to constraints:

\[(1) - (12) \text{ and } (14)\]

In (16) \( C^{i,j}_k \) denotes the instantaneous optimal equivalent fuel consumption cost for every possible movement from vehicle speed \( v_i(k) \) at time step \( k \) to vehicle speed \( v_i(k+1) \) at time step \( k+1 \) given a control variable \( u_i(k) = a(k) \). This cost function is found by solving a sub-problem as follows:

**Sub-problem 2:** Find the instantaneous optimal control variables \( u^*_2(k) = [u^*_g(k), P^*_e(k)] \) at a given \( u^*_1(k) \) that minimize the instantaneous equivalent fuel consumption cost as:

\[
J_2 = \tilde{m}_{f-eq}(u^*_2(k), k) | u^*_1(k) | \Delta t
\]

Denoting: \( C^{i,j}_k(u_i(k), k) = \tilde{m}_{f-eq}(u^*_2(k), k) | u^*_1(k) | \Delta t \)

A gradient-based optimization method is used in an inner loop (indicated inside the *For* loop in Fig. 3) to compute the instantaneous optimal value for \( u^*_2(k) \) which depends on all admissible values for \( u_i(k) \) to reduce the computation time. The inequality constraint (15) is incorporated in \( J_2 \) by a Lagrange multiplier. The detail of this gradient-based optimization method will be presented in sub-subsection III.B.2.

The optimal solutions for control variables \( u^*_1(k) \) and \( u^*_2(k) \) are dependent control design variables. Applying the DP algorithm in the outer loop to find only the optimal control variable for \( u^*_1(k) \) (or \( a(k) \)) instead of state variable \( s(k) \) will reduce the computation time. Yet, it requires us to derive a method in order to decouple the optimal solution of \( u^*_1(k) \) from \( u^*_2(k) \). Moreover, the constraints (10), (11) and (13) must be respected during the DP optimization process. A method of changing the boundaries on the admissible values of \( a(k) \) will be proposed in order to create this decoupling (see Section IV). Certainly, this leads to a suboptimal solution for \( u^*_1(k) \) and \( u^*_2(k) \).

2) Classical optimal control for HEV

Assuming at current time step \( k \), the vehicle speed is \( v_i(k) \) and the possible vehicle acceleration is \( a(k) \). The vehicle speed is \( v_i(k+1) \) at next time step \( k+1 \). Then we can define the wheel speed \( \omega_e(k) \) and required power at the wheel \( P_e(k) \). It is quite clear that the operating points for drivetrain system at current time step \( k \) are defined by two control variables: the shift command \( u_g(k) \) and the engine power \( P_e(k) \). In this section, a new control algorithm based on classical optimal control is developed to derive those instantaneous optimal values of \( u_g(k) \) and \( P_e(k) \) that such that computational time is significantly reduced. The readers can find this approach is somewhat similar with that of presented in [4]. Actually, the new of this method is on: deriving the optimal shift command which is a constraint control variable (see eq. (8)); problem is formulated and solved in form of power flow based model for consistency. This approach is presented as follows.

We need to define the shift command \( u_g(k) \) such that the next gear position \( n_g(k+1) \) belongs to the admissible gear range at time step \( k+1 \) to fulfill the constraints (4) on engine speed and electric machine speed. In the same way, the required power at the wheel \( P_e(k) \) can be produced by the powertrain, i.e. by both engine and electric machine. It means that the constraints on engine power and electric machine power can be incorporated into one constraint as:

\[
P_{e \text{ max}}(u_g(k), k) \leq P_e(k) \leq P_{e \text{ min}}(u_g(k), k)
\]

(17)

The lower and upper limits on engine power in the above equation are expressed as:

\[
P_{e \text{ min}}(u_g(k), k) = \max \left\{ 0, P_u(k) - P_{m \text{ max}}(u_g(k), k) \right\}
\]

(18)

\[
P_{e \text{ max}}(u_g(k), k) = \min \left\{ P_{e \text{ max}}(u_g(k), k), P_u(k) - P_{m \text{ min}}(u_g(k), k) \right\}
\]

(19)

Electric power consumption \( P_u(T_u(k), \omega_u(k)) \) and fuel consumption \( \tilde{m}_f(T_e(k), P_e(k)) \) are converted into \( P(u_g(k), P_e(k)) \) and \( D(u_g(k), P_e(k)) \) respectively as follows:

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\[ P(u_g(k), P_e(k)) = P_w \left[ u_g(k), \left( P_e(k) - P_e(k) \right) \right] \]  
(20)

\[ D(u_g(k), P_e(k)) = m_f \left[ u_g(k), P_e(k) \right] \]  
(21)

Then, the optimization is reformulated at time step \( k \) as:

**Problem-3:** Find the control values \( u_g^*(k), P_e^*(k) \) that minimize the instantaneous cost function:

\[ J_1 = D(u_g(k), P_e(k)) \cdot \Delta t \]  
(22)

subject to the constraints:

\[ x(k + 1) = x(k) + P(u_g(k), P_e(k)) \cdot \Delta t \]  
(23)

\[ P_e^*_{\text{min}}(u_g(k), k) \leq P_e(k) \leq P_e^*_{\text{max}}(u_g(k), k) \]  
(24)

To apply the classical optimal control theory, the constraint on engine power must be written as an equality constraint by introducing a parameter \( p(k) \):

\[ P_e(k)^2 + A(u_g(k), k) \cdot P_e(k) + B(u_g(k), k) + p(k)^2 = 0 \]  
(25)

where:

\[ A(u_g(k), k) = -\left( P_e^*_{\text{min}}(u_g(k), k) + P_e^*_{\text{max}}(u_g(k), k) \right) \]

\[ B(u_g(k), k) = P_e^*_{\text{min}}(u_g(k), k) \cdot P_e^*_{\text{max}}(u_g(k), k) \]

The new cost function is built as:

\[ J_{SN} = D(u_g(k), P_e(k)) \cdot \Delta t \]

\[ -\lambda(k)(x(k + 1) - x(k) - P(u_g(k), P_e(k)) \cdot \Delta t) \]

\[ -\gamma(k)(P_e(k)^2 + A(u_g(k), k) \cdot P_e(k) + B(u_g(k), k) + p(k)^2) \]  
(26)

Optimality conditions:

\[ \frac{\partial J_{SN}}{\partial x(k)} = 0 \iff \lambda(k + 1) = \lambda(k) \]

\[ \Rightarrow \lambda(k) = \lambda(0) \forall k \in [0, ..., N - 1] \]  
(27)

\[ \frac{\partial J_{SN}}{\partial P_e(k)} = 0 \iff \frac{\partial D(u_g(k), P_e(k))}{\partial P_e(k)} + \lambda(k) \left( \frac{\partial P(u_g(k), P_e(k))}{\partial P_e(k)} - \gamma(k) \left( 2P_e(k) + A(u_g(k), k) \right) \right) = 0 \]  
(28)

\[ \frac{\partial J_{SN}}{\partial p(k)} = 0 \iff 2\gamma(k) \cdot p(k) = 0 \]

\[ \Rightarrow \begin{cases} \gamma(k) = 0 \\ p(k) = 0 \end{cases} \]  
(29)

\[ \frac{\partial^2 J_{SN}}{\partial P_e(k)^2} \geq 0 \iff \frac{\partial^2 D(u_g(k), P_e(k))}{\partial P_e(k)^2} + \lambda(k) \left( \frac{\partial^2 P(u_g(k), P_e(k))}{\partial P_e(k)^2} - 2\gamma(k) \right) \geq 0 \]  
(30)

According to (29) we have two cases needed to be considered as follows:

- **Case 1:** if \( p(k) = 0 \), then

\[ P_e(k) \in \left[ P_e^*_{\text{min}}(u_g(k), k), P_e^*_{\text{max}}(u_g(k), k) \right] \]  
(31)

- **Case 2:** if \( \gamma(k) = 0 \), then (26) becomes:

\[ J_{SN} = D(u_g(k), P_e(k)) \cdot \Delta t \]

\[ -\lambda(0)(x(k + 1) - x(k) - P(u_g(k), P_e(k)) \cdot \Delta t) \]  
(32)

So, the constraint (28) becomes:

\[ \frac{\partial D(u_g(k), P_e(k))}{\partial P_e(k)} + \lambda(0) \left( \frac{\partial P(u_g(k), P_e(k))}{\partial P_e(k)} \right) = 0 \]  
(33)

To solve (33), \( \partial D(u_g(k), P_e(k))/\partial P_e(k) \) and \( \partial P(u_g(k), P_e(k))/\partial P_e(k) \) are needed to express analytically. Basically, the fuel consumption of engine, electric motor power consumption and battery power consumption are given by static maps. Therefore, it is a convenient way to choose an approximation of \( D(u_g(k), P_e(k)) \) and \( P(u_g(k), P_e(k)) \) as a second-order piecewise function [3-4]. So, their first derivatives will yield out first-order piecewise linear functions:

\[ \frac{\partial D(u_g(k), P_e(k))}{\partial P_e(k)} \approx a_e \left( u_g(k) \right) P_e(k) + b_e \left( u_g(k) \right) \]  
(34)

\[ \frac{\partial P(u_g(k), P_e(k))}{\partial P_e(k)} \approx a_m \left( u_g(k) \right) P_e(k) + b_m \left( u_g(k) \right) \]  
(35)

with \( i \in \{0, 1, ..., n - 1\} \) and \( n \) is the number of piece-wise functions.

If \( \lambda(0) \) is known, the set of solutions \( Q(k) \) with respect to (30) can be obtained. A way to choose the best value is to rewrite \( J_2 \) as follows:

\[ J_{SN} = -\lambda(0)(x(N) - x(0)) + D(u_g(k), P_e(k)) + \lambda(0).P(u_g(k), P_e(k)) \cdot \Delta t \]  
(36)

For the whole preview route segment, the cost function (36) becomes:

\[ J_{SN} = -\lambda(0)(x(N) - x(0)) + \sum_{k=0}^{N-1} \left[ D(u_g(k), P_e(k)) + \lambda(0).P(u_g(k), P_e(k)) \right] \cdot \Delta t \]  
(37)

Therefore, at each time step \( k \), the control value is obtained as:

\[ u_g^*(k), P_e^*(k) = \arg \min \left[ D(u_g(k), P_e(k)) + \lambda(0).P(u_g(k), P_e(k)) \right] \cdot \Delta t \]  
(38)

The problem (22)-(38) is reduced to the choice of a unique value \( \lambda(0) \). This value can be determined by using a search method or using a PI controller to adapt its value in time to ensure \( |\Delta SOE| = \left| x(N) - x(0) \right| < \xi \) with \( \xi \) small enough.

IV. IMPLEMENTING THE PROPOSED CONTROL ALGORITHM

1) Gridding the controlled variable for DP loop

To apply DP for optimal vehicle acceleration along the route segment, we need to grid this control variable \( a(k) \) and its state variable \( v(k) \). The vehicle acceleration is gridded with a predefined grid step \( \Delta a \) as:

\[ A_{\text{grid}} = \{ A_{\text{dec}}, 0, A_{\text{acc}} \} \]  
(39)

wherein:

\[ A_{\text{acc}} = \{ a_{\text{max}}, a_{\text{max}} + \Delta a, ..., a_{\text{max}} \} \]

\[ A_{\text{dec}} = \{ -a_{\text{max}}, -a_{\text{max}}, -\Delta a, ..., -a_{\text{max}} \} \]
If we define $\delta t = \Delta t$, the vehicle speed at any time step $k$ should belong to the grid $V_{\text{grid}}$ as:

$$ V_{\text{grid}} = \{v_{\min}, v_{\min} + \delta t, ..., v_{\max}, v_{\max} \} $$ (40)

Therefore at time step $k$, the vehicle speed grid $V_{\text{grid}}(k)$ is reduced further by reconstructing the minimum speed and maximum speed as follows:

$$ v_{\min}(k) = \max(v_{\max}(k-1) + a_{\min} \cdot \Delta t, 0) $$ (41)

$$ v_{\max}(k) = \min(v_{\min}(k-1) + a_{\max} \cdot \Delta t, v_{\max}) $$ (42)

2) Analyzing the cost function

Solving the optimization Problem 1 by using the proposed method does not guarantee global optimal solutions because of the constraints imposed on traveled distance, velocity profile and the way of constructing the cost function therein.

From (14), this end constraint requires the solver must know the rate of change of instantaneous equivalent fuel consumption cost $C_{k}^{ij}$ with respect to vehicle speed $v(k)$ and acceleration $a(k)$ to obtain global optimal solution. Moreover, from (16), the DP algorithm just only looks for optimal acceleration rate in term of the total of equivalent fuel consumption cost. It does not consider the rate of change of acceleration with respect traveled distance to satisfy the constraint (11), (12), and (14).

To overcome this difficulty, the solver must vary the rate of change of acceleration with respect to the traveled distance. In other words, the solver changes the lower and upper bounds on $A_{\text{grid}}$ (see eq. (38)) and pre-constructs the boundary of feasible traveled distance profiles along the preview route segment such that constraint (11), (12) and (14) are respected. This boundary region is a search space where DP algorithm in the outer loop can find corresponding optimal solutions under the mentioned constraints. The whole sequence is repeated with another pair of lower and upper bounds on $A_{\text{grid}}$. The final optimal solutions are corresponded to the case of giving the smallest value of the cost function (16).

Structurally explaining the way of implementing the proposed control algorithm is given in Fig.3.

V. SIMULATIOM RESULTS AND DISCUSSIONS

A mild hybrid powertrain system for a passenger car is chosen for this study, in which the diesel engine is of 1.3lits, maximum torque of 200Nm, maximum power of 68kW; the electric machine is of maximum power of 6kW, see [2]; battery system is of 6Ah, 110V.

An arbitrary route segment, a final part of ECE drive cycle, is chosen for simulation. This route has a length of 626m and required time of traveling of 71s.

![Figure 3: Proposed control algorithm structure.](image)

Figure 2: Traveled distance region

Fig.2 illustrates an example of the boundary of feasible traveled distance profiles along the preview route segment. The upper bound is constructed based on assumption of fastest travelling situation such that vehicle can fulfill the route characteristics. Vice verse, the case of lower bound is based on assumption of slowest travelling situation. Therefore, these bounds are proper when the upper bound is greater or equivalent to the lower bound and both of them satisfy (12) and (14).

![Figure 4: Optimal fuel consumptions when changing the lower bound and the upper bound on $A_{\text{grid}}$.](image)

The results in Fig. 4 show that when choosing the bounds on $A_{\text{grid}}$ with $a_{\text{acc}}$ around 1.5m/s² and $a_{\text{dec}}$ in the set $A_{\text{dec}}$, the corresponding optimal fuel consumptions will be the smallest ones. The DP algorithm navigates for the optimal acceleration profiles whose values at any certain time will be in this $A_{\text{grid}}$. 
Fig. 5 shows the optimal traveled distance $s_{opt}$ and optimal velocity profile $v_{opt}$ compared with other three driving profiles. The traveling distance requirement of this route is satisfied by four driving profiles. The optimal velocity profile possesses a moderate acceleration and a small deceleration. The phase of decelerating lasts at a longer time to take the advantage of regenerative braking.

![Distance vs Time Graph](image1)

![Velocity vs Time Graph](image2)

Figure 5: Optimal profile along the route segment; comparison with other three driving profiles.

Simulation results in table 1 show that if driving profile $v_1$ (final part of ECE cycle) is performed with optimizing shift command and engine power, the fuel economy is reduced up to 11.5% compared with that of optimal profile $v_{opt}$. Meanwhile, the other two arbitrary driving profiles $v_2$ and $v_3$ yield out the severe situations in term of fuel economy despite the fact that these driving missions were performed with both the shift command and the engine power optimized. The state of energy at the end of route segment is guaranteed by $\Delta$SOE$<1\%$. Although, the problem is partly solved using a surface response (surrogate) model to find the optimal setting for the boundaries on $A_{grid}$ and creates a suboptimal solution, it shows a significantly fuel economy improvement.

<table>
<thead>
<tr>
<th>Velocity profile</th>
<th>$\Delta$SOE (%)</th>
<th>Fuel (gr)</th>
<th>Economy (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$v_{opt}$</td>
<td>-0.5</td>
<td>14.14</td>
<td>baseline</td>
</tr>
<tr>
<td>$v_1$ (ECE)</td>
<td>-0.6</td>
<td>15.76</td>
<td>-11.5</td>
</tr>
<tr>
<td>$v_2$</td>
<td>-0.6</td>
<td>16.28</td>
<td>-15.1</td>
</tr>
<tr>
<td>$v_3$</td>
<td>-0.8</td>
<td>16.48</td>
<td>-16.6</td>
</tr>
</tbody>
</table>

Table 1: Simulation results

VI. CONCLUSION

In this paper, a control algorithm was proposed based on a combination of Dynamic Programming and classical control theory for the Energy Management System of the parallel hybrid electric vehicle using preview route information. The method is able to optimize the vehicle velocity, gear position and engine power over a preview route segment such that the fuel economy is improved and vehicle completes this route segment within a predefined time length. It is recommended that for gaining fuel economy when driving in urban region, the vehicle should be moderately accelerated to a certain speed, then it should be kept constant and decelerated as long as possible before finishing the route segment.

It’s necessary to improve the control algorithm to further reduce the computational burden such that this method can be applied to a longer route segment. In future, a full analytical method for this problem will be studied to find a global optimal solution.

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