Decentralized Communication Range Adjustment Issues in Multi–Agent Mobile Networks

John Stergiopoulos and Anthony Tzes

Abstract—This article addresses the problem of spatial information exchange among the nodes of a mobile network from a communication point of view. The nodes are considered to evolve in discrete time, while one agent moves at each time–step. Given that possible motion of a node is to be performed in a subspace of its Voronoi cell, a lower bound on its communication radius is derived in order to guarantee connectivity with its current, along with all its possible future Delaunay neighbors for any admissible motion. Algorithmic implementations are provided from a decentralized aspect, while results are related to coverage optimization issues in mobile sensor networks.

Index Terms—Voronoi diagrams, Delaunay neighbors, mobile networks, network connectivity, area coverage

I. INTRODUCTION

Swarm coordination has become an issue of major importance for research in the field of networked robotics in the last decade. In the majority of the scenarios considered, mobile agents equipped with sensors and/or RF antennas are spread in areas of interest in order to accomplish a common goal [1–3]. The scope of the robotic group varies from achievement of a desired formation, to optimal spatial configuration for coverage/surveillance purposes, among others [4, 5].

Since centralized coordination algorithms are in most cases impossible to be implemented from a practical point of view, scientific research has turned its interest into finding efficient decentralized algorithms for accomplishing such tasks [6, 7]. In these cases, the corresponding members of the swarm should be able to exchange information with neighboring nodes of the networks, so that they have a local perception of the network’s state and take appropriate decisions as far as concerns their motion [8, 9]. By that way, the mobile group is able to self–organize its action in a manner similar to the behavior observed in most groups in nature.

Considering sensing area–coverage scenarios by a group of mobile robots, the latter should be able to self–organize their actions in an independent manner, so that the area covered by the whole group is maximized [10, 11]. More specifically, independency lays in the way a corresponding agent properly adjusts its communication range to have local perception of the rest of the group, while in the sequel taking appropriate decision considering that its motion will contribute to network coverage.

In this paper, network’s coverage performance is mainly defined via utilization of classical Voronoi tessellation, while the main scope is to provide a lower bound on the communication range of the corresponding node–to–move, so that it can exchange spatial information with neighboring nodes that are needed for evaluating coverage performance of the network.

The article is organized as follows. In section II the main problem is settled up in a formalistic way, while spatial Voronoi tessellation is presented. In section III issues concerning Voronoi diagram alteration are introduced, while relation to coverage optimization scenarios is provided. Main results considering communication range adjustment are presented in section IV, so that the node–to–move can be able to communicate with its current along with its future Delaunay neighbors, while concluding remarks are provided in the last section.

II. PROBLEM FORMULATION

A. Basic Assumptions

Let the region under surveillance $\Omega$ be a convex compact set in $\mathbb{R}^2$. Suppose that $n$ is the number of available mobile agents, laying in its interior, responsible for achievement of a common task, i.e coverage of $\Omega$. Let us define the set $I_n = \{i \in \mathbb{N} : i \leq a\}$ for any $a \in \mathbb{N}$. The agents are considered to move on the plane, while their positions are denoted as $x_i \in \mathbb{R}^2$, $i \in I_n$. The following assumption are made for the network:

Assumption 1: Each agent is supposed to have a uniform circular sensing pattern centered at the agent’s position $x_i$. The sensing radius, $r$, is the same for all agents and the network is considered homogeneous, considering the nodes’ sensing abilities. Let us denote as $C_i$ the sensing region of each agent $i$, i.e.

$$C_i = \{x \in \mathbb{R}^2 : \|x - x_i\| \leq r\}, \quad i \in I_n. \quad (1)$$

Assumption 2: The agents are supposed to evolve in the interior of $\Omega \subset \mathbb{R}^2$ in a discrete manner, i.e. $x_i^k$, $k = 0, 1, 2, \ldots, \in I_n$ (where the superscript index denotes the corresponding time–step), while only one node is supposed to move at each time–instance [11].

Assumption 3: Each agent is supposed to be equipped with radio transceivers in order to be able to exchange spatial information with other members of the network. The radiation pattern $S_i$ of the antennas is considered as a uniform circular one, centered at $x_i$, i.e.

$$S_i = \{x \in \mathbb{R}^2 : \|x - x_i\| \leq R_i\}, \quad i \in I_n, \quad (2)$$
while the communication radii of the nodes, \( R_i \) are considered to be adjustable.

As far as concerns Assumption 2, the selection of the node–to–move must be performed by the nodes themselves, and not by a global supervisor, considering decentralized applications. This can be achieved by determining the corresponding node either in a cyclic or in a random manner. In the first case, an arbitrary node \( i \) moves only at time–steps \( k = i + p \ n, \ p \in \mathbb{N} \), while in the intermediate time–intervals it can be set in standby mode in order to preserve power. Alternatively, the node that is to perform possible motion can be chosen randomly by the group itself, where the random generators that run on each processor have the same seed value, so that at each step the node–to–move is unique and same for all members of the network.

### B. Spatial Voronoi Tessellation

For a compact polygonal set \( P \subset \mathbb{R}^2 \) let \( \partial P \) be its boundary. Then \( P \) is fully defined by the vertices of \( \partial P \) denoted as \( P_j, \ j \in N(P) \), where \( N(P) \) is the number of the latter’s vertices. Considering the region of interest \( \Omega \), a responsibility region is assigned at each agent based on its spatial coordinates on the plane. The set of these regions is well–known as a Voronoi diagram [12]. For the convex compact set \( \Omega \) and the \( n \) agents, the region of interest is partitioned into \( n \) compact convex subsets \( V_i, \ i \in I_n \), which are defined as

\[
V_i = \{ x \in \Omega : \| x - x_i \| \leq \| x - x_j \|, \ \forall j \in I_n \}, \ i \in I_n. \tag{3}
\]

The above definition is appropriate for homogeneous networks (Assumption 1), while modified definitions for heterogeneous ones (considering coverage capabilities) can be found in [13–15]. It should be noted that a Voronoi diagram is a full tessellation of \( \Omega \). A Voronoi cell \( V_i \) is uniquely characterized by the set of its vertices \( v_{i,j}, \ j \in I_{N(V_i)} \).

Two nodes that share an edge of their Voronoi cells (i.e. their Voronoi cells are adjacent) are considered as Delaunay neighbors [16]. The Delaunay neighbors of node \( i \), denoted as \( \mathcal{N}_i \), are then defined by the set

\[
\mathcal{N}_i = \{ j \in I_n : V_i \cap V_j \neq \emptyset \ or \ \{ a \}, \ j \neq i \}, \ i \in I_n, \tag{4}
\]

where \( a \) is an arbitrary point of \( \mathbb{R}^2 \). It should be noted that when \( V_i \cap V_j \) is a singleton, then the two nodes share a common Voronoi vertex (instead of a Voronoi edge) and are not considered as Delaunay neighbors. The edges of the Voronoi cell of an arbitrary node \( i \) that do not lay on the boundary of \( \Omega \) are then defined as

\[
\Delta_{ij} = V_i \cap V_j, \ i \in I_n, \ j \in \mathcal{N}_i. \tag{5}
\]

By taking into account the sensing range of the nodes, the \( r \)–limited Voronoi cells are then defined as

\[
V_i' = V_i \cap C_r, \ i \in I_n, \tag{6}
\]

which apart a collection of all sensed parts in the interior of the Voronoi cell of a node. An important property of these sets is that the total area sensed by the network can be expressed as the summation of the independent areas of the Voronoi cells, i.e.

\[
\mathcal{I} = \sum_{i \in I_n} \mathcal{A}(V_i'), \tag{7}
\]

where \( \mathcal{A}(\cdot) \) denotes the area-function of the argument-set.

### III. LOCAL VORONOI DIAGRAM ALTERATION

#### A. Motivation

Considering sensing coverage applications, the nodes should one–by–one (Assumption 2) move to such spatial locations in a way that the total covered area \( \mathcal{I} \) of \( \Omega \) is non–decreasing as time evolves [11]. Taking into account that, in most practical scenarios, global knowledge of the network’s state by a node is impossible (since that would lead to extremely large communication ranges), each node should have sufficient spatial information of the nodes in its neighborhood, in order to determine its position at the next step in a way that network coverage will increase via its motion.

However, considering (7), in order for the node–to–move to evaluate coverage performance of the network at an arbitrary time–step, it should have appropriate knowledge of the state of the nodes whose Voronoi cells are to be affected via its motion. In fact, spatial information from these nodes is adequate in order for the node–to–move to take decision about the spot to move at, in a decentralized concept. This article deals with issues concerning communication range adjustment so that the corresponding node can obtain adequate information in order to evaluate network coverage performance and coordinate its action. In the rest of the article, the index \( i \) will stand for the node–to–move at step \( k \) and not for an arbitrary node.

#### B. Delaunay Triangulation Alteration

Considering now the Voronoi space–partitioning defined in (3), the corresponding agent should first define the region of responsibility (own Voronoi cell) that is assigned to it. Taking into account the fact that no other node (apart from \( i \)) moves at that time–step, the only part of the network that alters (considering region assignment and thus coverage performance, as concluded by (7)), are the Voronoi cells of the union of Delaunay neighbors of node \( i \) before and after its motion, along with the Voronoi cell of the moving node itself.

Let us denote as \( \mathcal{N}_i^k \) and \( \mathcal{N}_i^{k+1} \) the Delaunay neighbors of node \( i \) at steps \( k \) and \( k+1 \), which correspond to the time–instances before and after the motion of the latter, respectively. Figure 1 shows such a scenario, where a node moves in the interior of its Voronoi cell, resulting in alteration of its Delaunay neighbors. The red dot represents the node–to–move to the spot denoted with the \( \times \) sign in Fig. 1(a). Consider that Fig. 1(a) and 1(b) correspond to time–steps \( k \) and \( k+1 \), respectively. The green dots correspond to the Delaunay neighbors of the red node in each case, i.e. \( \mathcal{N}_i^k \) (Fig. 1(a)) and \( \mathcal{N}_i^{k+1} \) (Fig. 1(b)). The grey node at Fig. 1(b)
corresponds to the position of the moving (red) node before its motion, i.e. $x^k_i$.

Important is the fact that the rest of the network’s state (considering Voronoi cell alteration), apart from the set $N^k \cup N^{k+1} \cup \{i\}$, does not alter at all. Indeed, if we suppose that the Voronoi cell of a node that does not belong in $N^k \cup N^{k+1} \cup \{i\}$ alters, this means that a Delaunay neighbor of that node has moved and thus has perturbed a Voronoi edge. But, since the only node that moves is node $i$, then the aforementioned node should belong to the above set.

C. Relation to Coverage-oriented Coordination Schemes

Suppose now that the possible motion of node $i$ at step $k$ is restricted in a compact convex subset of $V^k_i$, denoted as $W^k \subset V^k_i$, containing $x^k_i$, i.e. $x^k_i, x^{k+1}_i \in W^k$. Let the notation $q^{k+1} \mid I$ stand for the estimated value of the arbitrary variable $q$ at step $k + 1$, given the motion of node $i$ at $x^{k+1}_i$, where the evaluation is performed at step $k$ based on information from nodes in the set $I$. Consequently, the Delaunay neighbors of node $i$ at step $k + 1$, given $x^{k+1}_i \in W^k$, as estimated at step $k$ via knowledge of the current Delaunay neighbors is denoted as $N_{i}^{k+1} \mid x^k_i$.

Special attention should be given to the difference between the $N_{i}^{k+1} \mid x^k_i$ and $\hat{N}_i^{k+1} \mid x^k_i$ notations. The set $N_{i}^{k+1} \mid x^k_i$ corresponds to the Delaunay neighbors of node $i$ at step $k + 1$ as evaluated after the motion of the latter. Thus, it represents the value of a variable at $k + 1$. On the other hand, $\hat{N}_i^{k+1} \mid x^k_i$ corresponds to the estimated Delaunay neighbors of node $i$ at step $k + 1$, as estimated at step $k$ (before motion is performed) given $x^{k+1}_i$, supposing knowledge of $N_{i}^{k}$. Consequently, it corresponds to an evaluation performed at step $k$.

Let us now define the set $\mathcal{W}^k_i \{k, k+1\}$ as

$$\mathcal{W}^k_i \{k, k+1\} = \mathcal{A}_i^k \cup \{i\} \bigcup_{x^{k+1}_i \in W^k} \mathcal{A}_i^{k+1},$$

which corresponds to the union of the current (step $k$) Delaunay neighbors of node $i$, the moving node itself, along with the union of all possible Delaunay neighbors at the next step for all possible node’s motions in $W^k$.

Lemma 1: The set defined in (8) contains the nodes of the network whose Voronoi cell is possibly affected by the motion of node $i$, given the restriction of $x^{k+1}_i \in W^k$.

Proof: The proof is omitted for brevity.

The main scope of this article is the derivation of sufficient conditions as far as concerns communication range, such that the node-to-move, $i$, will be able to exchange information at each time-step $k$ with the nodes of the set $\mathcal{W}^k_i \{k, k+1\}$. The main innovation lays in the fact that the corresponding node-to-move attains connectivity with both its current and future Delaunay neighbors, which can be related to coverage optimization coordination algorithms.

In fact, considering Assumption 1 and the corresponding Voronoi tessellation of the space, according to Lemma 1, the only nodes in the network whose coverage contribution is affected by motion of node $i$ are the nodes of the set $\mathcal{W}^k_i \{k, k+1\}$ defined in (8). Thus, in a coverage optimization scenario, node $i$ should move to such a location in the interior of its Voronoi cell so that $\mathcal{A}^{k+1} - \mathcal{A}^k$ is maximized [11]. However, taking into account that

$$\mathcal{A}^{k+1} - \mathcal{A}^k = \sum_{j \in \mathcal{W}^k_i \{k, k+1\}} \left( \mathcal{A}(V^r_j \{k+1\}) - \mathcal{A}(V^r_j \{k\}) \right),$$

it is clear that in order for node $i$ to be able to evaluate coverage performance of the above set it must be ensured that it has spatial information from these nodes before motion is performed. Although this is the main motivation, coverage optimization issues are beyond the scope of this paper.

IV. SPATIAL INFORMATION EXCHANGE—COMMUNICATION ISSUES

A. Connectivity with Current Delaunay Neighbors

Considering communication issues, node $i$ should be able to acquire information from the nodes of the set (8) at step $k$, given the subset $W^k \subset V^k_i$, in order to be able to evaluate their current and future coverage performance. Ignoring the trivial case of receiving information from the node itself, as far as concerns the first part of the aforementioned union set, the node’s transceiver should first have adequate communication range so that it can obtain information from its current Delaunay neighbors $\mathcal{A}_i^k$.

Let us denote by $R^k_i(I)$ and $R^k_i(I)_{wes}$ the minimum communication radius of node $i$ at step $k$ required in order to exchange information with a set of nodes $I \subset I_n$, from a centralized and decentralized point of view, respectively. It is easy to see that, from a centralized aspect, the minimum range required for guaranteeing connectivity of node $i$ with $\mathcal{A}_i^k$ at step $k$ is equal to

$$R^k_i(\mathcal{A}_i^k) = 2 \max \left\{ d(x^k_i, \Delta_{ij}) : j \in \mathcal{A}_i^k \right\} = \max \left\{ \left\| x^k_i - x^k_j \right\| : j \in \mathcal{A}_i^k \right\},$$

where $\Delta_{ij}$ are defined in (5) and $d(x, M)$ is the distance of point $x$ from the set $M$. Although, expression (10) provides indeed the minimum range required, it cannot be used as a bound, from an independent point of view. In fact, node $i$ needs to increase even more its communication range until it is ensured that its Voronoi cell will not be affected further, even if another node falls in range.
Considering algorithmic implementations of communication range adjustment, the main concept lays in gradually increase of the latter until sufficient information from neighbor nodes is obtained. In fact, node $i$ gradually increases its range and updates its Voronoi cell according to the nodes that fall in its range. The procedure ends when the node’s range becomes twice the distance between the node and its farthest Voronoi cell vertex, i.e.

$$R^k_i \left( \mathcal{N}^{k+1}_i \right)_{\text{wcs}} = 2 \max \left\{ \| x_i - v_{i,j} \| : j \in I_N(v^+_i) \right\}, \quad (11)$$

since from that time on, any other node identified does not alter its Voronoi cell. The above are summarized in the algorithm shown in Table I.

**TABLE I**

<table>
<thead>
<tr>
<th>ALGORITHM I: DECENTRALIZED COMMUNICATION RANGE ADJUSTMENT</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Goal:</strong> Identify current Delaunay neighbors and Voronoi cell</td>
</tr>
<tr>
<td>$R^0_i \leftarrow 0$</td>
</tr>
<tr>
<td>$\mathcal{N}^0_i \leftarrow \emptyset$</td>
</tr>
<tr>
<td>$V^0_i \leftarrow \Omega$</td>
</tr>
<tr>
<td><strong>while</strong> $R^k_i \leq 2 \max \left{ | x_i - v_{i,j} | : j \in I_N(v^+_i) \right}$</td>
</tr>
<tr>
<td><strong>increase</strong> $R^k_i$</td>
</tr>
<tr>
<td><strong>if</strong> node $j$ detected</td>
</tr>
<tr>
<td>$\mathcal{N}^k_i \leftarrow \mathcal{N}^k_i \cup j$</td>
</tr>
<tr>
<td><strong>end if</strong></td>
</tr>
<tr>
<td><strong>end while</strong></td>
</tr>
</tbody>
</table>

It should be noted that $R^k_i \left( \mathcal{N}^{k+1}_i \right)_{\text{wcs}}$ is larger than the centralized bound, due to the decentralized nature of the scheme. During the rest of the analysis that follows in this section, node $i$ will be considered to obtain information from its current Delaunay neighbors $\mathcal{N}^k_i$ via decentralized proper adjustment of its communication radius at $R^k_i \left( \mathcal{N}^{k+1}_i \right)_{\text{wcs}}$.

**B. Connectivity with future Delaunay neighbors**

Considering (8), what needs to be further ensured is connectivity of node $i$ with the set $\bigcup_{k=1}^{k+1} W_k$, given a subset $W_k \subset V^k_i$. Suppose an arbitrary point $x_i^{k+1} \in W_k \subset V^k_i$. The goal is to find the minimum communication radius of node $i$ at step $k$ in order to guarantee connectivity at that step with $\mathcal{N}^{k+1}_i$. At step $k$, node $i$, positioned at $x_i^k$ with communication range $R^k_i \left( \mathcal{N}^{k+1}_i \right)_{\text{wcs}}$ (Algorithm I), has information about the coordinates of the nodes $\mathcal{N}^k_i$ of the network. Thus, at that time, the first can evaluate its future Voronoi cell, supposing that its motion is to be performed at $x_i^{k+1}$, by taking into account only the nodes in $\mathcal{N}^k_i$ and itself (i.e. ignoring the rest of the network). Let us denote as $\hat{V}^k_i \left( \mathcal{N}^{k+1}_i \right)_{\text{wcs}}$ the aforementioned evaluated Voronoi cell, according to the notations introduced in section III-C. At this point, node $i$ can be aware if a node $j \in \mathcal{N}^k_i$ is about to leave the set of its Delaunay neighbors at the next step, supposing that its motion will be performed at $x_i^{k+1}$, via simple evaluation of $\hat{V}^k_i \left( \mathcal{N}^{k+1}_i \right)_{\text{wcs}}$. What is unknown to the node–to–move yet are possible nodes of the network that may enter $\mathcal{N}^{k+1}_i$.

Considering decentralized aspects, let us consider the worst case scenario (similarly to section IV-A). Suppose that $m \in I_m$ is an extra node to possibly enter the set $\mathcal{N}^{k+1}_i$, where existence of $m$ is unknown to node $i$ yet. Considering (3), let $h_{ij}$ stand for the line that equally divides the space into two halfplanes between two arbitrary nodes of the network, i.e.

$$h_{ij} = \{ x \in \mathbb{R}^2 : \| x - x_i \| = \| x - x_j \| \}, \quad i, j \in I_m, \; i \neq j. \quad (12)$$

The critical case for node $m$ to enter the set $\mathcal{N}^{k+1}_i$ is when $h_{im}$ marginally crosses the farthest vertex of $\hat{V}^k_i \left( \mathcal{N}^{k+1}_i \right)_{\text{wcs}}$. Let $x_i^{k+1} \left| x_{j}^{k+1}$ denote the farthest vertex of $\hat{V}^k_i \left( \mathcal{N}^{k+1}_i \right)_{\text{wcs}}$, where the index $j^*$ is given as

$$j^* = \arg \max \left\{ \| x_i^{k+1} - x_{j,j^*}^{k+1} \| : j \in I_N(v^+_i) \right\}. \quad (13)$$

It is well–known that, given two points $a, b \in \mathbb{R}^2$ and a family of straight lines $\mathcal{L}$, where $b \in \ell$, $\forall \ell \in \mathcal{L}$, the farthest line from $a$ is the one that is perpendicular to the line that connects $a$ and $b$. Considering the above, one can conclude that the worst case scenario for the position of node $m$ is when it lays along the line that connects $x_i^{k+1}$ and $x_{j^*}^{k+1}$, at a distance from $x_i^{k+1}$ equal to twice that of the aforementioned points. This case is depicted graphically in Fig. 2. The red dot represents $x_i^{k+1}$, while the blue line connects that node with the farthest vertex of its estimated Voronoi cell $\hat{V}^{k+1}_i \left| x_{j^*}^{k+1}$. The blue dot represents the worst case scenario (comparing to the other possible cases denoted by grey color) for the existence of node $m$. The rest of the nodes in the network are omitted for visualization purposes.

Thus, according to Fig. 2, the worst–case for the position of node $m$ is

$$x_m^{k+1} = x_i^{k+1} + 2 \left( x_{j^*}^{k+1} - x_i^{k+1} \right).$$

Consequently, node $i$ at $x_i^k$ should have adequate communication range $R^k_i \left( \mathcal{N}^{k+1}_i \right)_{\text{wcs}}$ so that it can at least exchange
information with that node, \(m\), provided as
\[
R^k_i \left( \mathcal{N}^{k+1}_i \right)_{\text{wcs}} = \left\| x^k_i - (x^k_i + \left( \frac{1}{2} \sum_{j \neq i \in \mathcal{N}^{k+1}_i} |x^k_i - x^k_j| \right) \right\|,
\]
where the index \(j^*\) is defined in (13). It should be noted that \(x^k_i\) appears in the norm–argument of the upper part of (14), and not \(x^k_j\), since we are interested in finding the appropriate communication range of node \(i\) in order to communicate with node \(m\) at step \(k\). Furthermore, special attention should be given to the fact that the range defined in (14) is not the minimum required radius (from a centralized point of view) for guaranteeing connectivity with \(\mathcal{N}^{k+1}_i\), since the first depicts the worst case scenario (wcs). However, it is considered as the optimum range from a decentralized point of view, considering that, at time–step \(k\), node \(i\) has knowledge of existence for the nodes \(\mathcal{N}^{k+1}_i\).

C. Guaranteeing connectivity with all possible future Delaunay neighbors

Considering the above, the minimum communication radius of node \(i\) at step \(k\) required in order to exchange information with the nodes in the set \(\bigcup_{n=1}^{k+1} W^k \cdot \mathcal{N}^{k+1}_i\) is
\[
R^k_i \left( \bigcup_{n=1}^{k+1} W^k \cdot \mathcal{N}^{k+1}_i \right)_{\text{wcs}} = \sup \left\{ R^k_i \left( \mathcal{N}^{k+1}_i \right)_{\text{wcs}} : x^k_i \in W^k \right\},
\]
where \(R^k_i \left( \mathcal{N}^{k+1}_i \right)_{\text{wcs}}\) is defined in (14). It is clear that the above expression is dependent on the admissible region \(W^k \subset V^k\), while its evaluation is impossible due to the lack of distinctiveness of \(W^k\).

The goal is to find a closed–form expression for \(R^k_i \left( \bigcup_{n=1}^{k+1} W^k \cdot \mathcal{N}^{k+1}_i \right)_{\text{wcs}}\) evaluated via distinct computations. Before proceeding, let us first introduce some preliminaries. Considering duality between a Voronoi diagram and the corresponding Delaunay triangulation, each vertex of a node’s Voronoi cell is the center of a circle that circumscribes a triangle, consisted of the aforementioned node \(i\) and two adjacent Delaunay neighbors of the latter [12].

Considering node \(i\) and its current Delaunay neighbors \(\mathcal{N}^{k}_i\), let us denote as \(\triangle (x^k_i, x^k_p, x^k_q)\) the triangle (including its interior), whose vertices are defined by the positions of the nodes \(i, p, q\) at step \(k\). It is clear that \(p, q \in \mathcal{N}^k_i\) and \(p \in \mathcal{N}^{k+1}_i\) (since \(p, q\) are adjacent nodes). By denoting as \(G (\triangle (x^k_i, x^k_p, x^k_q))\) the center of the circle circumscribing the corresponding triangle \(\triangle (x^k_i, x^k_p, x^k_q)\), it holds by definition that the latter is a vertex of \(V^k_i\) provided as
\[
G (\triangle (x^k_i, x^k_p, x^k_q)) = \Delta p^k \cap \Delta q^k,
\]
where the Voronoi edges are defined in (5). The reason that the aforementioned Voronoi vertex is given as the section of two Voronoi edges is just to avoid complex notations in the subscript indices.

Thus, each vertex of \(V^k_i\) corresponds to the center of the circle that circumscribes a Delaunay triangle. Considering (15) along with (14), what needs to be determined is the farthest future Voronoi vertex from \(x^k_i\), for all possible motions of node \(i\) in \(W^k\). At this point, a gradient–based approach is to be followed in order to study how the current Voronoi vertices alter by the node’s motion. The procedure described in the sequel will concern an arbitrary triangle of the Delaunay triangulation; accordingly, the same one can be followed for the rest of the triangles.

Let us consider an arbitrary triangle, one vertex of which is node \(i\), i.e. \(\triangle (x_i, x_p \cdot x_q)\), where the coordinates of its vertices are \(x_i, x_p, x_q\), at an arbitrary time–step. Then the center of the circle that circumscribes it, considering vector format, is given by
\[
\lambda (\triangle (x_i, x_p, x_q)) = x_i + \lambda p x_p + \lambda q x_q,
\]
\[
\lambda_i = \frac{1}{2D} \| x_i - x_q \|^2 \left( x_i - x_p \cdot (x_i - x_q) \right),
\]
\[
\lambda_p = \frac{1}{2D} \| x_i - x_q \|^2 \left( x_p - x_i \cdot (x_p - x_q) \right),
\]
\[
\lambda_q = \frac{1}{2D} \| x_i - x_p \|^2 \left( x_q - x_i \cdot (x_q - x_p) \right),
\]
\[
D = \| (x_i - x_p) \times (x_p - x_q) \|^2,
\]
where \(\cdot, \times\) denote the dot and cross products, respectively. Taking into account that only node \(i\) moves at time–step \(k\), the circumcenter of an arbitrary triangle of the aforementioned Delaunay triangulation at time–step \(k + 1\) can be considered as a function of \(x^k_i\).

Consequently, node \(i\) should solve \(N(V^k_i)\) optimization problems, in order to identify the coordinates of the farthest future Voronoi vertices, defined as
\[
x^k_{i, (j)^*} = \arg \max \left\{ \left\| x^k_i - G (\triangle (x^k_i, x^k_p, x^k_q)) \right\| : x^k_i \in W^k \right\},
\]
\[
p.q \in \mathcal{N}^k_{i}, q \in \mathcal{N}^{k+1}_{i}, j \in I_N(V^k_i),
\]
where the bracketed \(j\) subscript in the optimal arguments denotes optimization of the corresponding vertex of \(V^k_i\). The aforementioned optimizations can easily be solved numerically at each step via nonlinear optimization techniques [17]. Having computed the “worst” cases for the Voronoi vertices of node \(i\), considering the motion of node \(i\) at step \(k + 1\) in \(W^k\), the minimum communication radius of the latter at step \(k\) required in order to exchange information with the nodes in the set \(\bigcup_{n=1}^{k+1} W^k \cdot \mathcal{N}^{k+1}_i\) is given as
\[
R^k_i \left( \bigcup_{n=1}^{k+1} W^k \cdot \mathcal{N}^{k+1}_i \right)_{\text{wcs}} = \max \left\{ R^k_i \left( \mathcal{N}^{k+1}_i \right)_{\text{wcs}} : x^k_{i, (j)^*} = x^k_{i, (j)^*}, j \in I_N(V^k_i) \right\},
\]
where \(R^k_i \left( \mathcal{N}^{k+1}_i \right)_{\text{wcs}}\) are given by (14).

Comparing (23) to (15), it is clear that the infinite–dimensional problem of defining the minimum communication radius for a node in order to guarantee connectivity with all possible future Delaunay neighbors has been reduced into \(N(V^k_i)\) distinct computations. The above are summarized in the following proposition.

Proposition 1: The communication radius of node \(i\) at
step \( k \) in order to guarantee connectivity with both current and future Delaunay neighbors, should be at least
\[
R_i^k \left( \omega_{i,k}^{\{k, k+1\}} \right)_{\text{wcs}} =
\]
\[
= \max \left\{ R_i^k \left( \mathcal{N}_i^{\{k\}} \right)_{\text{wcs}}, R_i^k \left( \bigcup_{i' \in \mathcal{W}_k} \mathcal{N}_{i'}^{k+1} \right)_{\text{wcs}} \right\},
\]
where the corresponding radii are given by (11) and (23).

V. CONCLUSIONS

In this paper a lower bound on the communication range of a node in a sensor network is derived, in order for the latter to be able to exchange information with its current along with all its possible future Delaunay neighbors, given that possible motion of the node is to be performed in a subspace of its Voronoi cell. Emphasis was given in the decentralized approach of the schemes, while algorithmic implementations were provided, where needed. Results were related to coverage optimization issues in autonomous mobile sensor networks.

REFERENCES