Robust Consensus Tracking of Leader-Based Multi-Agent Systems

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Abstract—In this paper, the use of a robust optimal control as a tool to design a trajectory tracking control for multiple agent systems is discussed. Our approach is based on a leader/follower structure of the multiple robot systems. It is shown that for multiple agent system with cyclic and chain topology of information exchange the result is achieved. A Lyapunov based optimal control design for coordination and trajectory tracking of multiple agent systems with parametric uncertainty is presented. Subsequently, this result is extended to the case of time-delay multi-agent system using the value set characterization to verify the robust stability of the closed loop system. The results are illustrated with several examples and simulations.

I. INTRODUCTION

Coordination control of multiple aerial, ground or underwater vehicles has important applications. They include the transport of heavy or large loads, search and rescue operations, surveillance, space or ocean exploration, etc.

In [1], [2] a leader/follower architecture for multiple agent systems is discussed. In this approach, one agent is designated as leader while the others are designated as followers which should track the leader. Another approach to multiple agents systems is the virtual structure where every agent is considered as an element of a larger structure, e.g. [3], [4]. Finally, the behavioral control in [5] and [6] is based on the decomposition of the main control goal into tasks or behaviors. This approach also deals with collision avoidance, flock centering, obstacle avoidance and barycenter.

Generally, communication between agents is modeled using directed or undirected graphs. Every agent in a multi-agent system is considered as a node in a graph. The information exchange between agents is modeled as directed or undirected edges of the graph. Algebraic graph theory is used to model the information exchange between vehicles. By using this technique several control strategies have been developed, e.g., [4], [7], [8], and [9]. In [9], a coordination control is proposed which is composed of a velocity consensus term and a gradient based term. The gradient term helps the cohesion of the group while the velocity consensus term synchronizes the velocities of the agents. An extension of this approach to include navigational feedback has been also presented in [9]. The navigational term is used to change the orientation of the group or to move the formation to a given reference position. [10] presents a new strategy for consensus in multi-agent systems with a time varying reference. Several cases are presented, such as: all agents have access to the reference, several agents have access to the reference, etc. The analysis presented assumes that each agent evolution is represented by a first order integrator.

The main contribution of this work is to provide a methodology for robust trajectory tracking control for a multiple agent system. In the literature is possible to identify several approaches to robust control, $H_{\infty}$ approach [11], [12], parametric approach [13]-[16], Lyapunov approach [17], [19], among others. We are interested in developing a Lyapunov approach to robust control design. We address the cases of parametric uncertainty in the multi-agent system. An extension of this result for multi-agent systems with input time-delay is also provided. Both cyclic topology and chain topology of information exchange are considered in our analysis and synthesis of control laws for robust forced consensus. The novelty of this approach is to combine optimal control and the value set characterization to robust trajectory tracking control design for multi-agent systems with time delay.

This work is organized as follows: section II gives a general overview on graph theory and multi-agent systems. In section III, the robust control design of multi-agent systems with uncertainty and time-delay is developed. The combination of optimal control and value set characterization will be considered in order to ensure robust stability of the system. In Section IV, simulation results are presented. Finally in section V, conclusions are presented.

II. PRELIMINARIES

A graph $G$ is a pair $G(N,E)$ consisting of a set of nodes $N = \{n_i; n_i \in N, \forall i = 1, \ldots, n\}$ together with their interconnections $E$ on $N$ [20]. Each pair $(n_1,n_2)$ is called an edge $e \in E$. Therefore, a multi-agent dynamic system can be modeled as a group of dynamical systems which has an information exchange topology represented by a directed or undirected graph. An undirected graph is one where nodes $i$ and $j$ can get information from each other. In a directed graph or simply digraph, the $i^{th}$ node can get information from the $j^{th}$ node but not necessarily vice versa. Thus, information exchange between agents can modeled as a digraph graph but also as an undirected which implies a more complicated problem. A graph is connected if for every pair $\{n_1,n_2\}$ of distinct vertices there is a path from $n_1$ to $n_2$. A connected graph allows the communication between all agents through the network. A graph is said to be balanced if its in-degree (number of communication links arriving at the node) is equal to its out-degree (number of communication links leaving the node).
Now, we present the chain topology and the cyclic topology of information exchange for multi-agent systems.

![Diagram showing chain topology]

Fig. 1. Information flow configuration: a) Ring topology (left), b) Chain topology (right).

A. The cyclic topology in the general case

Controllability and observability of cyclic topology in the general case is shown. The state space representation of the cyclic topology is the following

$$\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2 \\
\vdots \\
\dot{x}_n
\end{bmatrix} =
\begin{bmatrix}
-1 & 1 & & \\
1 & -2 & 1 & \\
& \ddots & \ddots & \\
& & 1 & -2 & 1 \\
& & & \ddots & \ddots \\
1 & & & & -1
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
\vdots \\
x_n
\end{bmatrix} +
\begin{bmatrix}
1 \\
0 \\
\vdots \\
0
\end{bmatrix}u$$

which for simplicity will be rewritten as

$$\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2 \\
\vdots \\
\dot{x}_n
\end{bmatrix} = -L_c
\begin{bmatrix}
x_1 \\
x_2 \\
\vdots \\
x_n
\end{bmatrix} + bu_1$$

C. General case

As described in previous sections the interconnection of $u_1$ agents using the coordinating control strategy lead to systems represented as

$$\begin{cases}
\dot{x} = -L\dot{x} + bu \\
y = c^T\dot{x}
\end{cases}$$

where $L$ is the laplacian matrix of the information exchange graph. We assume that the information exchange graph is balanced. Let us assume also that in the coordinating controller the gains multiplying the signals in between agents are all equal to 1. For the $i$-th row of $L$, the entries $l_{ij} = -1$ for $i \neq j$ correspond to the gains multiplying the signals from other agents coming to agent $i$. For the $i$-th column of $L$, the entries $l_{ji} = -1$ for $i \neq j$ correspond to the gains multiplying the signals going out of agent $i$ towards the other agents.

System (5) uses a velocity coordination control strategy given by the laplacian matrix $L$. It should be noticed that position coordination control strategy can be achieved by using a change of variable of the form

$$\xi \triangleq \dot{x} + \lambda x$$

where

$$\begin{cases}
\dot{\xi} = -L\xi + bu \\
y = c^T\xi
\end{cases}$$

Let us first review the controllability and observability properties of multi-agent system discussed in [18].

Proposition 1: Consider the multiple agent system whose evolution is described by (7). This system is not observable if there exists a right eigenvector $\omega_i$ of $L$ such that $c^T\omega_i = 0$.

Proposition 2: Consider the multiple agent system whose evolution is described by (7). This system is uncontrollable if there exist an eigenvector $v_i$ of $L^T$ such that $v_i^T b = 0$.

Recall that for $i = 1$ we have $v_1 = \omega_1$ and therefore $c^T\omega_1 \neq 0$ and $v_1^T b \neq 0$. Thus the mode corresponding to $(\lambda_1, v_1)$ is controllable and the mode corresponding to $(\lambda_1, \omega_1)$. If for $i = 2, \ldots, n$ there exists a $c^T\omega_i = 0$ or $v_i^T b = 0$, such mode is not observable or not controllable.
respectively. Nevertheless, such modes are asymptotically stable and converges to zero.

A leader-follower approach for multi-agent systems (5) will be adopted, then a similarity transformation can be applied such that the multi-agent system can be represented as follows

$$
\dot{y}(t) = \begin{bmatrix} 0 & 1 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ -q_0 & -q_1 & \cdots & -q_{n-1} \end{bmatrix} y(t) + \begin{bmatrix} 0 \\ 0 \\ \vdots \\ b \end{bmatrix} u(t)
$$

$$
y_1 = e^T \begin{bmatrix} \eta_1 \\ \eta_2 \\ \vdots \\ \eta_n \end{bmatrix}
$$

where $\eta \in \mathbb{R}^n$ and $u \in \mathbb{R}^m$ are the state variables and control inputs respectively. It should be noticed that under a similarity transformation the system’s characteristic equation, transfer function, eigenvalues, eigenvectors are all preserved.

### III. ROBUST OPTIMAL CONSENSUS

In this section, a robust state feedback control design to stabilize a system with uncertain parameter values is presented. A combination of the Lyapunov method and the LQR approach are used to find the robust control law which guarantee the stability property and the system robustness. Subsequently the result is extended to time delay system using the value set characterization to proof the robust stability of the close loop system. To do this, we consider systems as in (8) where $q_i \in Q$ are the parametric uncertain values and $b \in R$. $Q$ is a set that represents the parametric uncertainty defined as:

$$
Q \triangleq \left\{ \bar{q} = [q_0 \cdots q_{n-1}]^T : q^-_i \leq q_i \leq q^+_i \right\}
$$

These type of sets are known as boxes.

**Remark 1:** It is worth to recall that in engineering problems there is always a noise present in every signal coming from sensors. Our approach considers that noise as uncertainty in parameters of the multi-agent system, i.e., every non-zero entry in the Laplacian matrix is uncertain with a nominal value (typically $l(i,j) \in I$ depending on the information exchange structure of the multi-agent system) and bounded by inferior and superior limits, e.g. let the nominal value of the signal $x_i = 1$ for some $i$ with minimum and maximum values between 0.9 and 1.1, then the signal $x_i$ can be considered as $x_i(q)$ where $q \in [0.9,1.1]$. Then, the Laplacian matrix represents the exchange of information between agents and we are interested in modeling the noise of those signals as uncertainty in the entries of the Laplacian matrix.

Let us consider the following representation of the multi-agent system (8) as in [19]

$$
\Sigma_{\text{nom}} \triangleq \left\{ \dot{\eta} = A(q^-)\eta + Bu + B\Gamma(r)\eta, \ y = C\eta \right\}
$$

where:

$$
\Gamma(r) = \begin{bmatrix} r_0 & \cdots & r_{n-1} \end{bmatrix}
$$

$$
0 \leq r_i \leq r_i^+ = \frac{q^+_i - q^-_i}{b}
$$

$\Sigma_{\text{nom}}$ represents the nominal system and $\Sigma_{\text{un}}$ represents the uncertain system. Consider the system (11), and the following control law

$$
u = -B^TS\eta
$$

where:

$$
SA(q^-) + A(q^-)^T S + F + I - SBB^T S = 0, S > 0
$$

Now, the $F$ matrix is defined in such a way that the following condition is satisfied:

$$
\Gamma(r)^T \Gamma(r) \leq F \quad \forall r_i \in [0, r_i^+]
$$

To find the solution of the LQR optimal control problem for the $\Sigma_{\text{nom}}$ system, the following cost functional is considered:

$$
V(\eta) = \min_{u \in \mathbb{R}} \int_0^\infty (\eta^TF\eta + \eta^T S \eta + u^Tu)dt
$$

It is possible to verify that the proposed control law (12) corresponds to the solution of the LQR optimal control problem for the $\Sigma_{\text{nom}}$ system (10), considering the cost functional $V(\eta)$, and the relative weights matrices $Q = F + I$ and $R = 1$. Obviously, the above control law stabilizes the nominal system $\Sigma_{\text{nom}}$. Next, a proof that the same control law also stabilizes the uncertain system $\Sigma_{\text{un}}$ will be presented. Using the results of the LQR optimal control problem, it is possible to obtain the following solution to the problem in (15):

$$
V^*(\eta) = \int_0^\infty (\eta^TF\eta + \eta^T S \eta + \eta^T SBB^T S\eta)dt
$$

Since (12) is the solution of the LQR optimal control problem, then the following conditions should hold

$$
\eta^TF\eta + \eta^T S \eta + \eta^T SBB^T S\eta + \left[ \frac{\partial V(\eta)}{\partial \eta} \right]^T (A(q^-)\eta + BB^T S\eta) = 0
$$

$$
2\eta^T S B + \left[ \frac{\partial V(\eta)}{\partial \eta} \right]^T B = 0
$$

Along the trajectories of the system $\Sigma_{\text{un}}$ (11) we have

$$
\dot{V}(\eta) = \left[ \frac{\partial V(\eta)}{\partial \eta} \right]^T \dot{\eta} = \left[ \frac{\partial V(\eta)}{\partial \eta} \right]^T [A(q^-)\eta + Bu + B\Gamma(r)\eta]
$$

$$
= \left[ \frac{\partial V(\eta)}{\partial \eta} \right]^T [A(q^-)\eta + BB^T S\eta] + \left[ \frac{\partial V(\eta)}{\partial \eta} \right]^T B\Gamma(r)\eta
$$
Then (17) and (18), leads to

\[
\dot{V}(\eta) = -\eta^T F \eta - \eta^T SBB^T S \eta \\
- 2\eta^T SBB^T \eta \eta \\
- \eta^T \left[ F - \Gamma \right] \eta
\]

from condition (14) it follows

\[
\dot{V}(\eta) \leq -\eta^T \eta \quad \text{(19)}
\]

Then, $\dot{V}(\eta) < 0$ for all $\eta \neq 0$ and $\dot{V}(\eta) = 0$ if and only if $\eta = 0$.

Now, we consider the case of time delays due to sensor information process, actuator time delay, etc. Considering a time delay $\tau$ in the input, system (8) can be rewritten as

\[
\dot{\eta}(t) = 0 1 \ldots 0 \\
0 0 \ldots 0 \\
\vdots \\
- q_0 - q_1 \ldots - q_{n-1}
\]

\[
\eta(t) + \begin{bmatrix} 0 \\ 0 \\ 0 \\ r_0 \end{bmatrix} u(t-\tau)
\]

(20)

Consider the multi-agent system with time delay (20), then this system is robustly stable if the same optimal control law (12) is used considering a maximum time-delay $\tau_{\text{max}}$.

The uncertain time-delay system (20) has the following characteristic equation:

\[
p(s, q, r, e^{-s\tau}) = s^n + \sum_{k=n-1}^{n} (q_k + s q_{k-1}) s^{k-1} + \ldots + \left(q_1 + q_0 s + \sum_{k=0}^{n} k s + k_0 e^{-[0,\tau_{\text{max}}]}\right)
\]

(21)

where $K = \begin{bmatrix} k_0 & k_1 & \cdots & k_{n-1} \end{bmatrix}$ is the optimal LQR gain obtained using the above methodology. These kind of functions are known as quasipolynomials. It is clear that the above characteristic equation (21) represents an infinite number of quasipolynomials that have to be considered to verify the robust stability property. This family is defined as follows:

\[
P_\tau = \left\{ p(s, q, r, e^{-s\tau}) : q \in Q; \quad r \in R; \quad \tau \in [0, \tau_{\text{max}}] \right\}
\]

(22)

Now we define the value set as follows:

**Definition 1:** (Value Set): The value set of $P_\tau$ denoted by $V_\tau(\omega)$ is the graph in the complex plane of $p(s, q, r, e^{-s\tau})$ when $s = j\omega$:

\[
V_\tau(\omega) = \left\{ p(s, q, r, e^{-j\omega \tau}) : q \in Q; \quad r \in R; \quad \tau \in [0, \tau_{\text{max}}]; \quad \omega \in \mathbb{R} \right\}
\]

(23)

It is clear that the value set of $P_\tau$ is a set of complex numbers plotted on the complex plane for values of $q$, $r$, $\omega$ and $\tau$ inside the defined boundaries. Next, the zero exclusion principle is presented in order to verify the robust condition [13].

**Lemma 3:** Consider the characteristic equation (21), also called quasipolynomials. Suppose that (21) has at least one stable member. Then the robust stability property of the control system is guaranteed if and only if:

\[
0 \notin V_\tau(\omega) \quad \forall \omega \geq 0
\]

(24)

IV. EXAMPLES

In this section we present some examples describing the proposed methodology developed in previous sections. Simple configurations of information exchange are described.

A. Cyclic Topology

A 3-agents system with cyclic topology of information exchange is given by

\[
\dot{x}(t) = \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 0 \\ 1 & 0 & -1 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u(t)
\]

(25)

The multi-agent system in terms of this new variables is given by

\[
\dot{\eta}(t) = \begin{bmatrix} 0 & 1 \\ 0 & 0 \\ 0 & -3 \end{bmatrix} \eta(t) + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} u(t)
\]

(26)

We will consider a $[10\%, 10\%, 10\%]$ uncertainty in the last row coefficients of the state matrix. As a result of this assumption, the following matrix is obtained

\[
A(q) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -q_1 & -q_2 \end{bmatrix}
\]

(27)

where $q_1 \in [2.7, 3.3]$ and $q_2 \in [2.7, 3.3]$.

Then,

\[
A(q^\tau) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -3.3 & -3.3 \end{bmatrix}
\]

(28)

\[
\Gamma(r) = \begin{bmatrix} r_0 & r_1 & r_2 \end{bmatrix}
\]

where $r_0 = 0$; $r_1 \in [0, 0.6]$; $r_2 \in [0, 0.6]$.

The F matrix is given by:

\[
F = \begin{bmatrix} 0.00 & 0.00 & 0.00 \\ 0.00 & 0.36 & 0.36 \\ 0.00 & 0.36 & 0.36 \end{bmatrix}
\]

(29)

Thus, the optimal control law is given by

\[
K = \begin{bmatrix} 1.0000 & 1.1591 & 0.5168 \end{bmatrix}
\]

The uncertain time-delay system (25) has the following characteristic equation:

\[
p(s, q, r, e^{-s\tau}) = s^3 + [2.7, 3.3] s^2 + [2.7, 3.3] s + (0.5168 s^2 + 1.1591 s + 1) e^{-[0.4,3]}
\]

(30)

After a simple inverse transformation of the optimal control law (29), the resulting optimal control law is applied to the multi-agent system (25). The performance of the robust optimal control is shown in Figure 2. The results presented in [21] permit building the value set $V_\tau(\omega)$ for the characteristic equation (30) is presented in Figure 3.
Fig. 2. Multi-agent system

Fig. 3. Value set of the uncertain time-delay system (25)

It can be noted that the zero is not included in the value set \( V_\tau (\omega) \). Multi-agent System with cyclic topology of information exchange (25) and \( \tau < 4.3 \) is then robustly stable.

B. Chain Topology

A 3-agents system with cyclic topology of information exchange is given by

\[
\dot{x}(t) = \begin{bmatrix}
-1 & 1 & 0 \\
1 & -2 & 1 \\
0 & 1 & -1
\end{bmatrix} x(t) + \begin{bmatrix}
0 \\
0 \\
1
\end{bmatrix} u(t) \tag{31}
\]

The multi-agent system in terms of the canonical form is given by

\[
\dot{\eta}(t) = \begin{bmatrix}
0 & 1 & 0 \\
0 & 0 & 1 \\
0 & -3 & -4
\end{bmatrix} \eta(t) + \begin{bmatrix}
0 \\
0 \\
1
\end{bmatrix} u(t) \tag{32}
\]

Considering a [10\%, 10\%, 10\%] uncertainty in the last row coefficients of the state matrix. As a result of this assumption, the following matrix is obtained

\[
A(q^-) = \begin{bmatrix}
0 & 1 & 0 \\
0 & 0 & 1 \\
0 & -q_1 & -q_2
\end{bmatrix} \tag{33}
\]

where \( q_1 \in [2.7, 3.3] \) and \( q_2 \in [3.6, 4.4] \).

Then,

\[
A(q^-) = \begin{bmatrix}
0 & 1 & 0 \\
0 & 0 & 1 \\
0 & -3.3 & -4.4
\end{bmatrix}
\]

\[
\Gamma(r) = \begin{bmatrix}
r_0 & r_1 & r_2
\end{bmatrix}
\]

where \( r_0 = 0; r_1 \in [0, 0.6]; r_2 \in [0, 0.8] \).

The F matrix is given by:

\[
F = \begin{bmatrix}
0.00 & 0.00 & 0.00 \\
0.00 & 0.36 & 0.48 \\
0.00 & 0.48 & 0.64
\end{bmatrix} \tag{34}
\]

Thus, the optimal control law is given by

\[
K = \begin{bmatrix}
1 & 1.3908 & 0.4766
\end{bmatrix}
\]

The uncertain time-delay system (31) has the following characteristic equation:

\[
p(s, q, r, e^{-\tau s}) = s^3 + [3.6, 4.4]s^2 + [2.7, 3.3]s + (0.4766s^2 + 1.3908s + 1)e^{-[0.4, 2]} \tag{36}
\]

The value set \( V_\tau (\omega) \) for the characteristic equation (36) is presented in Figure 5.

Fig. 4. Multi-agent System with Chain Topology

Fig. 5. Value set of the uncertain time-delay system (31)

It can be noted that the zero is not included in the value set \( V_\tau (\omega) \). Multi-agent System with chain information exchange topology (31) and \( \tau < 4.2 \) is then robustly stable.
C. Balanced Graph Topology

A 4-agents system with balanced topology of information exchange is given by
\[
\dot{x}(t) = \begin{bmatrix} -2 & 1 & 0 & 1 \\ 1 & -3 & 1 & 1 \\ 0 & 1 & -1 & 0 \\ 1 & 1 & 0 & -2 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} u(t) \quad (37)
\]

The multi-agent system in terms of the canonical form is given by
\[
\dot{\eta}(t) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \eta(t) + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} u(t) \quad (38)
\]

Considering a [10%, 10%, 10%, 10%] uncertainty in the last row coefficients of the state matrix. As a result of this assumption, the following matrix is obtained
\[
A(q) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -q_1 & -q_2 & -q_3 \end{bmatrix} \quad (39)
\]

where \(q_0 = 0, q_1 \in [10.8, 13.2], q_2 \in [17.1, 20.9] \) and \(q_3 \in [7.2, 8.8]\).

Then,
\[
A(q^-) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -13.2 & -20.9 & -8.8 \end{bmatrix}
\]
\[
\Gamma(r) = \begin{bmatrix} r_0 & r_1 & r_2 & r_3 \end{bmatrix}
\]

where \(r_0 = 0; r_1 \in [0, 2.4]; r_2 \in [0, 3.8]; r_3 \in [0, 1.6]\).

the F matrix is given by
\[
F = \begin{bmatrix} 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 5.76 & 9.12 & 3.84 \\ 0.00 & 9.12 & 14.44 & 6.08 \\ 0.00 & 3.84 & 6.08 & 2.56 \end{bmatrix} \quad (40)
\]

Thus, the optimal control law is given by
\[
K = \begin{bmatrix} 0.3162 & 0.5221 & 0.2538 & 0.0489 \end{bmatrix} \quad (41)
\]

The uncertain time-delay system (37) has the following characteristic equation:
\[
p(s, q, r, e^{-\tau s}) = s^4 + [7.2, 8.8]s^3 + [17.1, 20.9]s^2 + [10.8, 13.2]s + [0.0489s^3 + 0.2538s^2 + 0.5221s + 0.3162e^{-[0.4, 7]}] \quad (42)
\]

The value set \(V_r(\omega)\) for the characteristic equation (42) is presented in Figure 7.

It can be noted that the zero is not included in the value set \(V_r(\omega)\). Multi-agent System with chain information exchange topology (37) and \(\tau < 4.7\) is then robustly stable.

D. Simulation

In order to validate the results described in previous sections, numerical simulations of several multi-agent dynamic models have been run using Matlab Simulink\textsuperscript{TM}. Coupling gain between agents have been set to unit gain, as noted in previous sections, 10% uncertainty was consider in every signal. A representative gaussian noise is consider in measured signals. In general, a consensus agreement is not useful for real multi-robot systems. Instead, synchronization is a very nice tool for cooperative behavior and autonomous vehicles formation. The proposed algorithm finds a constant output feedback that effectively synchronizes every agent (signal). Also, it allows the multi-agent system to follow a reference signal or trajectory. In this case, we propose a constant reference and a sinusoidal signal. The behavior of the cyclic topology is shown in Figures 8 and 9. Similarly, chain topology behavior is shown in Figures 10 and 11.

V. CONCLUSIONS AND FUTURE WORKS

We have presented a simple methodology for robust trajectory tracking control design for multi-agent systems with uncertainty and time delay. A parametric uncertainty was considered in the transformed system to simulate noise in the measured signals. We have presented sufficient conditions on maximum time delay for robust stability of the multi-agent consensus and regulation system. Several cases have been considered such as cyclic topology, chain topology, as
well as other balanced configurations. It has been shown that the robust optimal control is enough to achieve multi-agent consensus and trajectory tracking to a smooth time varying reference.

REFERENCES