Optimal Hedging for Flexible Fuel Energy Conversion Networks

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Abstract—We demonstrate an alternative approach for the hedging of flexible energy conversion networks appropriate for use in campus scale and municipal scale utilities with complex energy requirements, fuel sources, and operational flexibility. This new approach utilizes empirical modeling of price models for alternative commodity fuels coupled with a previously reported class of bilinear models for estimating the efficiency of complex and flexible energy conversion networks. The coupling of financial and operational hedging provides the utility operator with additional mechanisms for mitigating price volatility in the energy markets.

In this paper, we use a steady state bilinear model introduced in an earlier paper to model a power plant that can utilize either coal or natural gas. The model demonstrates the complex trade-off between using a lower cost but less efficient fuel versus a more expensive fuel with higher conversion efficiency. This class of bilinear models incorporates first and second law principles from finite-time thermodynamics to predict energy conversion networks. In the hedging framework, the bilinear models are used to compute deterministic optimal operating conditions which are functions of fuel prices.

Historical fuel price data are collected for coal and natural gas, and a stochastic price model is fit to capture the joint distribution of fuel prices. The fit is used to generate a fuel cost uncertainty model. The model is used to project a joint distribution of fuel prices over a single period planning horizon.

Economic optimization is performed over many fuel price realizations. These results are aggregated to obtain a distribution of prices of fuels and fuel costs of the energy conversion network on the planning horizon. A fuel ‘Cost at Risk’ model is suggested, and it shows how one may choose to take position in coal inventory or in natural gas futures in order to reduce ‘Cost at Risk’.

In summary, this paper presents three main results:

- A stochastic fuel price model is constructed using historical data.
- Economic optimal process conditions are determined for a flexible fuel energy conversion network model and realizations of future prices using the uncertain fuel price model.
- Determination of an optimal hedging strategy for the flexible energy conversion network using Monte Carlo techniques.

I. INTRODUCTION

Financial optimization for flexible fuel utilities is challenging because of uncertainties associated with the energy market and the high degree of operational flexibility available in modern campus-scale utilities. Uncertainties include demand for the electricity, heating, and cooling services provided by the utility, and the market prices of input fuels and purchased electricity. To meet demand, operators generally have at least limited operational flexibility on decisions regarding the generation or purchase of electricity and fuel blend. In addition, utility operators have financial flexibility to use options and futures for the purchase of fuels, to purchase and store some forms of fuel such as coal, and to enter into interruptible contracts for fuel and electricity. Thus the overall financial optimization of these utilities must account for uncertainties in price, demand, and optimal strategies for fuel storage, fuel blending, and the use of commodity options.

Our research focuses on developing models, techniques, and providing insight in characterizing the financial operation of flexible fuel energy systems. Maximizing the value of the energy producing is achieved by integrating the operations, financial instruments, and contractual elements. Our group [1], [2] and others [3]–[5] have shown the value of integrating hedging with operational decisions in energy applications. Hedging provides the means to minimize cost subject to constraints on financial risk. In particular, our group [6] introduced a class of bilinear models for energy conversion networks that incorporates principles from finite-time thermodynamics. In this paper we show how these models can be incorporated into an approach for the economic optimization and risk management of flexible fuel utilities.

This paper investigates different models for price forecasting of commodities, focusing on natural gas and coal. We seek models capturing the essential statistical properties of past performance including the high degree of kurtosis (i.e., ‘fat tails’) observed in empirical data. The empirical model produces joint realizations of future prices over a single period planning horizon. Each joint realization for natural gas and coal is used in conjunction with a model of a plant using the energy conversion network framework determine the optimal operating conditions and fuel costs. Optimization over many realizations yields a distribution of the fuel costs of the energy conversion network. A probability distribution of the fuel costs provides the information needed to evaluate hedging strategies in a single period model.

The following sections of this paper review different price models, demonstrate the development of a simple empirical model for the joint distribution of coal and natural gas futures, the modeling of a simple flexible fuel utility, and the development of a simple one-period hedging strategy to mitigate energy price risk.

II. SINGLE PERIOD HEDGING FOR A FLEXIBLE FUEL POWER PLANT

We consider the hedging of a flexible fuel power plant as a 2-stage stochastic problem with recourse. The flexible
fuel power plant operates with known demand and may use coal and natural gas interchangeably. The decisions today are whether to purchase coal at today’s near future price and store for three months and to purchase natural gas forward at today’s 4-month futures contract value for delivery in three months. The decisions must account for optimal conversion efficiency which is a complex function of energy output, prices, and plant configuration. The financial objective is to minimize expected fuel costs subject to a constraint on financial risk posed by uncertain fuel costs.

III. PRICE MODELS FOR ENERGY COMMODITIES

Price models for energy commodities have been an intense area of research in the financial literature. Here we review a few aspects of these models preliminary to developing an empirical model for the joint distribution of coal and natural gas futures.

A. Models Using Theory of Storage and Convenience Yield

Fama and French [7] tested two models in their landmark paper on the prices of commodity futures. The first model is given by Eq. 1, where $F(t,T)$ is the futures price at time $t$ for delivery of a commodity at time $T$, $S(t)$ is the spot price at time $t$, $R(t,T)$ is the risk-free interest rate at time $t$ maturing at time $T$, $W(t,T)$ is the marginal storage cost, and $C(t,T)$ is the convenience yield:

$$ F(t,T) - S(t) = S(t) R(t,T) + W(t,T) - C(t,T) $$

(1)

The second model (Eq. 2) uses forecast power and premiums where $E_t[P(t,T)]$ is the expected premium and $E_t[S(T) - S(t)]$ is the expected difference in the spot price at times $T$ and $t$:

$$ F(t,T) - S(t) = E_t[P(t,T)] + E_t[S(T) - S(t)] $$

(2)

These models assume differences in the futures prices and spot price are based convenience yield and the cost of carry including storage and opportunity costs. The stochastic process is implicit in the convenience yield or expected premium.

B. Joint Diffusion Models

Gibson and Schwartz [8] studied the price of financial contracts contingent on oil prices, and proposed using the model in Eq. 3 and 4:

$$ \frac{dS_t}{S_t} = (r_t - \delta_t)dt + \sigma_1 dz_1 $$

(3)

$$ d\delta_t = \kappa (\theta - \delta_t) dt + \sigma_2 dz_2 $$

(4)

The spot price equation is coupled with a stochastic convenience yield equation, which assumes the convenience yield is a mean-reverting process. The stochastic terms $dz_1$ and $dz_2$ are modeled as Brownian motion with a correlation coefficient of $\rho$.

Deng [9] proposed several mean-reverting jump-diffusion models in his modeling of natural gas and electricity prices. The first model (Eq. 5) assumes constant volatility:

$$ d \begin{pmatrix} X_t \\ Y_t \end{pmatrix} = \begin{pmatrix} \kappa_1(t) (\theta_1(t) - X_t) \\ \kappa_2(t) (\theta_2(t) - Y_t) \end{pmatrix} dt + \begin{pmatrix} \sigma_1(t) \sigma_1(t) \\ \rho(t) \sigma_2(t) \sqrt{1 - \rho(t)^2} \sigma_2(t) \end{pmatrix} \Delta W_t + \sum_{i=1}^{2} \Delta Z^i_t $$

(5)

where $X_t$ and $Y_t$ are the log electricity price and natural gas price, respectively, $\kappa_1(t)$ and $\kappa_2(t)$ are the mean-reverting coefficients, $\theta_1(t)$ and $\theta_2(t)$ are the long term means, $\sigma_1(t)$ and $\sigma_2(t)$ are the instantaneous volatility rates of $X_t$ and $Y_t$, $W_t$ is Brownian motion, and $Z_t^i$ is a vector Poisson process.

The second model adds a Markov regime switching model to the original model. The third model includes a third state $V_t$, which is a stochastic volatility factor, to the original model.

C. Autoregressive Models

An autoregressive model assumes that the current value is dependent on a number of the most recent values plus an error term, as given in Eq. 6:

$$ X_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i X_{t-i} + \epsilon_t $$

(6)

This approach seeks to find an independent, identically distributed (i.i.d.) innovation term $\epsilon_t$ by identifying the dependence of $X_t$ on its previous $q$ values (AR($q$) model). Akaike [10] has an early paper on fitting these models.

The innovations or residuals in these models are also fitted using an autoregressive model. Typical models are autoregressive conditional heteroskedasticity (ARCH) introduced by Engle [11] and an extension of this model called generalized ARCH (GARCH). The variance of an ARCH($q$) model is equal to a constant plus the weighted sum of the past $q$ residuals and is given by Eq. 7:

$$ \sigma^2_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i \epsilon^2_{t-i} $$

(7)

A GARCH($p$, $q$) model includes a term for the $p$ most recent variances of the past residuals as well as the $q$ past residuals and is given by Eq. 8:

$$ \sigma^2_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i \epsilon^2_{t-i} + \sum_{j=1}^{p} \beta_j \sigma^2_{t-j} $$

(8)

D. Other Models

Escobar-Hernández-Seco [12] proposed an additive, quadratic fitting model for the term structure of futures prices (Eq. 9):

$$ F_{t,T} = \chi_{0,t} + \xi_{1,t} (T - t + 1) + \eta_{2,t} (T - t + 1)^2 $$

(9)
where $\chi_{0,t}$ is the short term deviation in price, $\xi_{1,t}(t)$ is the equilibrium price level, and $\eta_{2,t}(t)$ is the quadratic term.

There are certainly other models available as well. This is an overview of some of the more popular model and isn’t meant to be an exhaustive review of the literature.

IV. EMPIRICAL MODEL FOR COAL AND NATURAL GAS FUTURES

A. Prices and Return Innovations

The daily price history of natural gas near futures and coal near futures from the New York Mercantile Exchange (NYMEX) is available at http://www.eia.doe.gov. The available history for coal starts in July, 2001, while the available history for the four nearest futures (Henry Hub) for natural gas begins in January 1994. The price data was imported into Matlab and matched by date from the earliest available coal data point. Missing data that were present in the coal data was interpolated using the closest dates. Fig. 1 shows the combined history of the near futures of these commodities since July 2001.

![Fig. 1. Natural Gas and Coal Near Futures Settlement Price](image)

Statistical properties of the error from models of pricing are not independent and identically distributed (i.i.d.). Empirically, and consistent with models for the equity and spot commodity markets, we choose to model the joint evolution of log returns ($r_t$) by taking the difference between the log of price at time $t$ and the log of price at time $t-1$, or (Eq. 10):

$$r^C_t = \ln \left( \frac{r^C_t}{r^C_{t-1}} \right) = \ln(r^C_t) - \ln(r^C_{t-1}) \quad (10)$$

where $r^C_t$ is the nearest futures price for coal. The log returns of daily nearest future for both coal and natural gas are shown in Fig. 2.

B. Estimate a Model

Log return for these commodity data sets were tested for long-term memory effects using the Hurst coefficient. The Hurst coefficients were estimated to be 0.605 and 0.550 for natural gas and coal, respectively. Compared to a null hypothesis of 0.5 for the case of no long-term memory effects, these were judged to be statistically insignificant.

We then chose to model the log returns data using an autoregressive approach. The $\text{AR}(1)$ model follows the form of the $\text{AR}(q)$ model given above (Eq. 6).

$$X_t = c + B X_{t-1} + \epsilon_t$$

where

$$X_t = \begin{bmatrix} r^NG_t \\ r^C_t \end{bmatrix}$$

Least squares regression produces estimates (after conversion to annualized basis for return):

$$c = \begin{bmatrix} -0.224 \\ 0.010 \end{bmatrix} \quad B = \begin{bmatrix} -0.0345 & 0.0164 \\ 0.0130 & -0.0030 \end{bmatrix}$$

The estimated residuals $\epsilon_t$ of the log returns are distinctly non-Gaussian. Fig. 3 shows the empirical cumulative distribution function of the joint residuals. The measured kurtosis is 8.6 for natural gas residuals and 46.9 for coal residuals, indicating that both residuals have “fat tails.”

Expressing residuals in terms of quantiles of the cdf, and plotting the joint residuals produces the empirical copula shown in Fig. 4. We first fitted the residuals data to a t-distribution copula, which also estimates $\hat{\upsilon}$, or the degrees of freedom. We found the estimate for $\hat{\upsilon}$ to be large (218), indicating the error data are close to a Gaussian distribution.
A new estimate for $\hat{\rho}$ was fit with a Gaussian copula with an estimated correlation coefficient of 0.15. For this purpose we used the maximum likelihood estimator for copulas provided in the Matlab Statistics Toolbox available from the Mathworks, Inc.

C. Price Projections

The linear correlation matrix $\hat{\rho}$ is used to generate random copula distributions. These random distributions are simulated for a particular time horizon. We chose 3 months out, which corresponds to roughly 63 trading days. The simulated data are interpolated empirically to obtain the projected error for the AR(1) model. The AR(1) parameters $c$ and $B$ are used with the return for the last recorded trading day to construct the projected price trajectories for both fuels.

We ran 10,000 simulations and recorded the prices on the final trading day. Fig. 5 shows 100 realizations of the projected prices for natural gas and coal. Fig. 6 shows the final natural gas and fuel near futures settlement price distributions from the simulation. The mean price of natural gas and coal is identified by the red dot.

V. MAPPING FUEL COSTS

Given a price model and fitted parameters, that model may be used to produce realizations of commodities prices over a relevant time horizon. The statistical properties of these realizations maintain the same statistical properties as the original data within the framework of the price model. These realizations can be utilized in a model of a power plant based on the energy conversion network (ECN) framework we introduced in an earlier paper [6].

The ECN modeling approach models a complex energy system at a high level to capture the essential characteristics and trade-offs that must be made without going into complex equipment details. Larger systems are broken down into basic components based on first and second law principles from finite-time thermodynamics. The resulting model presents a bilinear optimization problem which is solved using linearization techniques.

Utilizing realizations from a commodities price model, the prices at the final time point are used as inputs to the flexible fuel ECN model. For computational efficiency, we choose to simulate the flexible fuel ECN model using a grid of prices covering the range of prices produced by the realizations. We divide each range into equally spaced points and simulate the flexible fuel ECN model at every combination of points, recording the amount of each fuel used and the fuel costs. Given the commodity prices of a particular realization, the amounts of each commodity used and fuel costs can be determined by linear interpolation.

A. Payoffs

The set of projected settlement prices for 3 months out is used as inputs to a flexible fuel ECN model. Fig. 7 shows the energy conversion network model used. This model consists of 2 engines and 7 heat nodes. Engine 1 and engine 2 represent the operation of the equipment using natural gas and coal, respectively. The isentropic efficiencies of the natural gas and coal engines are 0.85 and 0.9, respectively.

The parameters used for this energy conversion network model are as follows:
Fig. 7. 2-Engine Natural Gas and Coal Model

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$K_{12} = K_{34} = 25$ MW/K  
$K_{56} = 1$ MW/K  
$K_{45} = K_{67} = 25$ MW/K  
$W = 350$ MW

For computational efficiency, instead of running all 10,000 final prices through the model, a grid of prices was created based on the range of prices from the simulated projected final prices, recording the price and amounts of each fuel used at each point (shown in Fig. 8). Linear interpolation is used to estimate the cost and amounts of each fuel on all 10,000 simulation points.

The fuel costs is lowest using coal over most of the range of natural gas and coal price. Natural gas is favored when coal prices are high and natural gas prices remain low. There is also a narrow region where a combination of both natural gas and coal provides the lowest cost.

Fig. 9 show the histogram of the cost projections. The mean of all cost projections is $19.96$/MWh. The skewness and kurtosis of this distribution were estimated at 0.28 and 4.00, respectively.

The cost projections are used to construct a cumulative distribution function (shown in Fig. 10). The mean is identified on the graph with the green line. The gold line is the cost of running the plant 3 months from the last recorded fuel prices purchasing the fuel or fuels at those prices. The red line represents the 'Fuel Cost at Risk' (FCaR).

For a confidence level $\alpha$ and time horizon, the FCaR is the threshold value such that the probability of higher fuel cost is $1 - \alpha$. We choose $95\%$ as the probability level. Our notion of 'Fuel Cost at Risk' (FCaR) is directly analogous to the familiar Value At Risk (VaR) used for investment portfolios. The FCaR identifies a threshold value at a probability level and time horizon such that the value of the portfolio has a probability of being less than the threshold value at the probability level. Both are measures of the downside risk.

We calculate the cost of running the plant 3 months from today purchasing the fuel or fuels at the last recorded fuel prices as follows. If coal is needed, it is purchased at the near futures settlement price today and stored for 3 months.
before use. Therefore, the price today must be adjusted by the cost of storage for 3 months and lost opportunity cost on the money. The cost of storage is assumed to be 2% of the purchase cost for coal. The risk-free interest rate is 0.18% annually, which translates to 0.05% of the cost of the coal purchase over the 3-month period. If natural gas is needed, it is purchased as a forward for delivery 3 months from the purchase. We use the 4-month futures price to estimate the price of the forward. Since no cash is exchanged until delivery, no adjustment of the price is necessary. For this example, purchasing and storing coal provides the lowest cost given the most recently recorded prices.

B. Hedging

Given a level of risk tolerance, an optimal hedge can be constructed using the cumulative distribution graph in Fig. 10. If one is highly risk averse, the fuel costs by purchasing coal at the last recorded price and storing the coal reduces the FCaR to zero. However, choosing to operate in this manner does not allow one to achieve the lower fuel costs, including the mean cost. The opposite strategy is to take no hedge on the FCaR. The cost will result in the mean on average and allows one to realize the downside risk but also exposes one to the upside risk as well. Other hedges can be identified but is dependent on one’s desired risk tolerance.

VI. Conclusions

We use a flexible fuel example of an energy conversion network model, a class of bilinear models introduced in an earlier paper, to construct an analysis of cost distribution to utilize for hedging. We demonstrate the flexibility of the energy conversion network framework by illustrating that any pricing model may be used to project the future fuel prices. For large number of simulated projections, computational speed can be increased by interpolating on a grid of prices covering the range produced by the projections. These projections were used to construct a cumulative distribution function that can be used with a desired risk tolerance to determine the optimal hedging strategy.

REFERENCES


