Robust Nonlinear Control of Fuel Cell Ultra-Capacitor Hybrid System

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Abstract—In this paper we design robust control strategies for a hybrid solid oxide fuel cell ultra-capacitor system. Fuel cell control is established by using an invariant property of fuel utilization within an input-shaping framework. Two control strategies are developed. The first design uses a nonlinear control approach for which we prove the stability of the closed-loop system in presence of system uncertainties. The second uses a standard $H_{\infty}$ robust control approach. Both strategies enforce the control of State of Charge (SOC) of the ultra-capacitor to a desired level. A hardware-in-the-loop test-stand is developed and experimental results are provided.

I. INTRODUCTION

Among different fuel cell technologies, the Solid Oxide Fuel Cell (SOFC) technology has attracted significant interest in recent years. They are fuel flexible solid state devices that operate at high temperatures of (800 to 1000°C). SOFCs are fuel flexible, tolerant to impurities and hence often carry simple on-board fuel reformers. They fit well in combined heat and power (CHP) applications. An important performance variable of SOFCs is fuel utilization $U$, defined as the ratio of hydrogen consumption to the net available hydrogen in the anode. While high utilization implies high efficiency, very high utilization leads to reduced partial pressure of hydrogen in the fuel cell anode, which can cause irreversible damages due to anode oxidation [13]. Typically, a range of 80 – 90% is optimum over a wide range of operating conditions [12]. In applications with significant power transients, fluctuations in $U$ due to slow dynamic response of the fuel supply system and reformer dynamics can adversely affect stack life through fuel starvation and drastic voltage drop, [5], [17].

A majority of existing work addressing transient control of fuel cells consider the Polymer Electrolyte Membrane (PEM) fuel cell technology. Control of hybrid PEMFC systems are proposed in [1], [9], [14] and [18]. Explicit constraint handling is addressed in fewer works such as [5], [16], [17] and [19]. In comparison to PEMFC, few works on power management and control of hybrid SOFC systems appear in the literature, [8], [7]. There is increased complexity of sensing and estimation in SOFCs which is supplied with a gas mixture consisting of several species, in comparison to pure hydrogen in PEMFC. Hence measuring $U$ requires sensing several species concentrations and flow rates leading to cost and reliability considerations, or designing observers, [2], increasing computational burden and reliance on an accurate system model.

In this paper, we address the aforementioned issue through the development of a control strategy that performs feedback based regulation of the fuel cell current, in conjunction with a steady-state invariance property of $U$. The resulting deficit or surplus power delivered by the fuel cell during transients is compensated by an ultra-capacitor. The proposed fuel utilization control strategy fits directly within the overall power splitting control for the hybrid system that additionally controls the ultra-capacitor’s SOC at a target value regardless of system uncertainties. Two robust control strategies are proposed in this work, a nonlinear and an $H_{\infty}$ based control.

A detailed control-oriented mathematical model of an SOFC system is developed, that captures the thermodynamics, chemical kinetics, heat transfer and pressure dynamics phenomena of the fuel cell. The model has been validated against published results in [10], [13]. A system description and the uncontrolled response of $U$ to transient current are presented in section II. Steady-state relation for this system, derived in [4], is used for open-loop control of $U$ in section III. Feedback based input shaping of fuel cell current is presented in section IV. The hybrid fuel cell configuration and the comprehensive control designs are detailed in sections V and VI. The hardware-in-the-loop experimental test-stand is explained in VII, followed by experimental results in VIII that confirm the efficacy of our control designs. Finally concluding remarks are stated in section IX.

II. FUEL CELL SYSTEM

In this work we consider a steam reformer based tubular SOFC system. The system consists of three primary components, namely, the steam reformer, the solid oxide fuel cell and the combustor. Methane is chosen as the fuel for the system, with a molar flow rate of $N_f$. It is noted that the control development approach in this paper can be extended to other SOFC configurations and fuels as well. The SOFC system is described in Fig.1.

The reformer produces a hydrogen-rich gas which is supplied to the anode of the fuel cell. Electrochemical reactions occurring at the anode due to current draw results in a steam-rich gas mixture at the anode exit. A fixed known fraction $k$ of the anode exhaust is recirculated through the reformer into a mixing chamber where fuel is added. The mixing of the two fluid streams and pressurization is achieved in the gas mixer using an ejector or a recirculating fuel pump, [6]. The steam reforming process occurring in the reformer catalyst bed is an endothermic process. The energy required to sustain the process is supplied from two sources, namely, the combustor exhaust that is passed through the reformer, and the aforementioned recirculated anode flow, as shown...
in Fig.1. The remaining anode exhaust is mixed with the cathode efflux in the combustion chamber. The combustor also serves to preheat the cathode air which has a molar flow rate of $N_{air}$. We assume our system to be comprised of $N_{cell}$ tubular Solid Oxide Fuel Cells, connected in series.

Fuel utilization $U$ is a critical indicator of the steady-state and transient performance of an SOFC system. Fuel utilization, defined as the ratio of hydrogen consumption to the hydrogen availability in the fuel cell, can be expressed mathematically as follows:

$$U = 1 - \frac{N_a (4X_{1,a} + X_{2,a} + X_{4,a})}{N_{in} (4X_{1,r} + X_{2,r} + X_{4,r})}$$

where, $X_{1,a}$, $X_{2,a}$, $X_{4,a}$, and $X_{1,r}$, $X_{2,r}$, $X_{4,r}$, represent the molar concentrations of $CH_4$, $CO$ and $H_2$ in the anode and the reformer respectively, [12], [3]. $U$ is sensitive to fuel cell current $i_{fc}$. This is illustrated in the simulation results presented in Fig.2. The fuel cell model is run in open loop for steady-state utilization,

$$U_{ss} = \frac{1 - k}{(4nFN_f/i_{fc}/N_{cell}) - k}$$

For detailed analysis, see [4]. Eq.(2) is independent of internal variables such as internal flow rates, temperatures and reaction rates, etc. and is therefore an invariant property of $U$. Furthermore, since $k$, $i_{fc}$ and $N_f$ are measurable and known inputs, Eq.(2) can serve as an open-loop control to achieve a target $U_{ss}$. Consider the demanded fuel demand $N_{f,d}$, that satisfies a target $U_{ss}$ is,

$$N_{f,d} = \frac{i_{fc}N_{cell}}{4nFU_{ss}} \left[ 1 - (1 - U_{ss})k \right]$$

Eq.(3) only addresses steady-state behavior. Hence we must assess its effectiveness in the presence of transient current demand. Control of $U$ using Eq.(3) is shown in Fig.3. The fuel cell system simulated is same as in Fig.2 with $i_{fc} = 10A$ for $t < 150s$ and target $U_{ss} = 85\%$. Four simulations are presented with $i_{fc} = 11, 14, 18$ and $22A$ for $t \geq 150s$, Fig.3(a). The actual fuel injected, $N_f$, is shown in Fig.3(b). Note that while $N_{f,d}$ changes instantaneously according to Eq.(3), $N_f$ experiences a lag due to the dynamics of the fuel supply system that typically comprises of valves and mass flow controllers [13]. At steady-state, $N_f = N_{f,d}$. In this simulation, although we have assumed a first order dynamics with a time-constant of 2s, similar response is obtained with other types of fuel supply dynamics, such as ramped or rate-limited behavior. In Fig.3(c), $U$ is plotted for individual simulations. $U$ reaches the target $U_{ss} = 85\%$ at steady-state.

III. OPEN-LOOP CONTROL OF U

From mass balance equations of individual species in the reformer and the anode, we obtain the following expression

![Schematic Diagram of the SOFC System](image)

**Fig. 1.** Schematic Diagram of the SOFC System

![Open-loop Response to Transient Current Demand](image)

**Fig. 2.** Open-loop Response to Transient Current Demand

![Open-loop Control of U Using Eq.(3)](image)

**Fig. 3.** Open-loop Control of $U$ Using Eq.(3)
> 10A. This is due to hydrogen starvation manifested by $U \to 100\%$, as seen in Fig.3(c) for step change of 12A. Note that hydrogen starvation occurred at a step increase of only 1A in the uncontrolled case, Fig.2. We make the following observations:

- In the comparison to the uncontrolled case, under open-loop control the fuel cell is capable of handling a significantly greater amount of current fluctuation.
- Under the open-loop control, $U$ deviates from target $U_{ss}$ during transient phase.
- Transient $U$ is a result of the delay between a change in the fuel command $\dot{N}_{f,d}$ and its corresponding effect in the fuel cell.

Next, we propose a feedback approach that builds on the open-loop control to alleviate the transient deviation in $U$.

### IV. FEEDBACK BASED CURRENT SHAPING

An advantage of the above approach is that $U_{ss}$ is achieved without the knowledge of internal flow rates, temperatures, intermediate species, species concentrations or chemical reactions. However, it results in transient deviations in $U$. We address this issue by dynamically shaping $i_{fc}$ using feedback. Noting that $\dot{N}_f \neq \dot{N}_{f,d}$ during transients, the actual fuel cell current $i_{fc}$ is shaped using Eq.(2), as follows:

$$i_{fc} = \frac{4nF U_{ss} \dot{N}_f}{\eta_{cell} \left[ 1 - (1 - U_{ss}) k \right]}$$

Implementing Eq.(4) requires the measurement of the actual fuel flow $\dot{N}_f$, which is assumed to be available. The feedback based input shaping scheme and the open loop approach are represented in Fig.4 by the switch positions CL and OL respectively. Simulations are presented in Fig.5 to demonstrate the effect of regulating $i_{fc}$ on transient utilization. The system simulated is the same as that in Figs.2 and

In Fig.5, $i_{fc} = i_{fc,d} = 10A$ for $t < 150s$, and $i_{fc,d} = 18,30,50A$ for $t \geq 150s$, and $U_{ss} = 85\%$. In response to the step changes in $i_{fc,d}$, the target fuel $\dot{N}_{f,d}$ also undergoes step changes, Figs.5(a) and (b). $\dot{N}_f$ changes according to the fuel supply dynamics which is assumed to be first order with a time constant of 2s Fig.5(b), as in the previous simulations. The regulated $i_{fc}$ is computed using Eq.(4) and is shown in Fig.5(a). The resulting transient $U$ is shown in Fig.5(c).

It is evident from the results above that the feedback based shaping of $i_{fc}$ drastically reduces transient $U$. It is also clear that this regulation increases transient current handling capability of the fuel cell by a considerable margin. Although in sections III and IV we assumed a first order fuel-supply dynamics, the observations made are applicable to a wide variety of dynamic responses [4].

### V. HYBRID FUEL CELL CONFIGURATION

Dynamic regulation of $i_{fc}$ will lead to a mismatch between the demanded power and fuel cell delivered power during transients. This mismatch can be compensated by a storage device that discharges and charges when compensating for deficit and excess power respectively. A schematic diagram of a hybrid fuel cell system is shown in Fig.6. The hybrid system consists of a fuel cell and an ultra-capacitor connected to an electrical bus through DC/DC converters. In the following sections we develop strategies for combined control of the SOFC ultra-capacitor hybrid system. The objectives of the control strategy are first, to control fuel utilization and second, to control SOC of the ultra-capacitor, at their respective target values. Within these constraints, we wish to achieve load following in the presence of rapid load transients.

In the control development, we make the assumption that the DC/DC converters can be treated as static power conversion devices. This leads to the following instantaneous power balance equation, developed using Fig.6

$$V_{L}i_{L} = \eta_1 V_{fc,i_{fc}} + \eta_2 V_{uc,i_{uc}}$$

where, $\eta_1$ and $\eta_2$ are the efficiencies of the power converters $C_1$ and $C_2$ respectively. The following aspects are also considered in the control development:
Among the two DC/DC converters shown in Fig. 6, \( C_1 \) operates in voltage control mode and \( C_2 \) in the current control mode. In particular, \( C_1 \) maintains a constant bus voltage and \( C_2 \) follows the commanded ultra-capacitor current \( i_{uc,c} \). (we treat \( i_{uc} = i_{uc,c} \)).

The ultra-capacitor current \( i_{uc} \) and the fuel demand \( N_{f,d} \) are treated as control inputs.

Measurements of \( V_{fc}, V_{ac}, i_c, \) and \( i_{fc} \), are available.

The DC/DC converter efficiencies \( \eta_1 \) and \( \eta_2 \) vary with operating conditions and are treated as unknowns with constant estimates \( \bar{\eta}_1 \) and \( \bar{\eta}_2 \).

VI. CONTROL DESIGN

A. Nonlinear Robust Control

A schematic diagram of the control strategy incorporating the current regulation algorithm of Fig. 4 is provided in Fig. 7. The fuel cell serves as the primary power source and the ultra-capacitor supplies transient demands. This, along with the objective of controlling the ultra-capacitor SOC prompts the following design of fuel cell current demand

\[
\begin{align*}
\dot{i}_{fc,d} &= \frac{V_{Li}L_i + g(E_s) + \delta_1}{\bar{\eta}_1 V_{fc}} \quad E_s = S - S_t, \quad S = \frac{V_{ac}}{V_{max}} (6)
\end{align*}
\]

where \( S \) is the instantaneous SOC, \( S_t \) is the target SOC and \( V_{max} \) is the maximum ultra-capacitor voltage. The function \( g(E_s) \) and the robustness term \( \delta_1 \) will be designed in the ensuing analysis. The fuel demand, \( N_{f,d} \), is an algebraic function of \( i_{fc,d} \), given by Eq.(3), that satisfies the target utilization criterion. The target fuel cell current, \( i_{fc,t} \), is based on \( N_f \) as shown in Fig.7, and is computed using Eq.(4) as

\[
\dot{i}_{fc,t} = \frac{4nF U_{st}N_f}{\gamma_{cell}} \frac{1}{[1 - (1 - U_{ac}) k]} (7)
\]

The fuel cell target current \( i_{fc,t} \) is indirectly achieved through the ultra-capacitor current command as follows

\[
\dot{i}_{uc} = \dot{i}_{uc,c} = \frac{V_{Li}L_i - \bar{\eta}_1 V_{fc}i_{fc,t}}{\bar{\eta}_2 V_{uc}} + h(E_{fc}) + \delta_2, \quad E_{fc} = i_{fc} - i_{fc,d} (8)
\]

where the function \( h(E_{fc}) \) and the robustness term \( \delta_2 \) will be designed in the ensuing analysis.

Note that the current regulation approach detailed in section IV does not require any knowledge of the dynamics of the fuel supply system. However, in developing a comprehensive control strategy we assume a stability characteristics of the fuel supply system. Specifically, we consider the error variable \( E_{fl} = N_f - N_{f,d} \) to show an exponentially stable property, i.e.,

\[
|E_{fl}(t)| \leq \gamma|E_{fl}(0)| e^{-\alpha t}, \quad \gamma, \alpha > 0 (9)
\]

where, \( \gamma \) and \( \alpha \) are unknown but their upper and lower bounds respectively exist and can be estimated. This is not a restrictive condition, since any stable linear \( n \)th order behavior or nonlinear behaviors such as ramped response or rate limited response can be bounded by an exponential decay within a limit of accuracy. From Eqs.(3) and (7), it is evident that Eq.(9) implies

\[
|\dot{E}_{fc,t}(t)| \leq \gamma|E_{fc,t}(0)| e^{-\alpha t}, \quad E_{fc,t} = i_{fc,t} - i_{fc,d} (10)
\]

Our control objective is to stabilize the origin \( E_s = E_{fc} = 0 \) in the presence of the exogenous input \( i_{fc} \) with the aforementioned design of \( i_{fc,d} \) and \( i_{uc,c} \) in Eqs.(6) and (8) respectively. In this section, we adopt a Lyapunov-based approach to develop a robust nonlinear control strategy. We first observe from Converse Lyapunov Theorems, [11], that \( E_{fc,t} \) has an exponentially stable behavior, there exists a positive definite function \( \bar{V}_{fc} \) such that,

\[
\alpha_1 E_{fc,t}^2 \leq \bar{V}_{fc}(E_{fc,t}) \leq \alpha_2 E_{fc,t}^2, \quad and \quad \bar{V}_{fc} \leq -\alpha_3 E_{fc,t}, \quad \alpha_3 > 0 (11)
\]

where \( 0 < \alpha_1 < \alpha_2 \). We use the Lyapunov function candidate

\[
\bar{V} = \frac{1}{2} E_{fc}^2 + \bar{V}_{fc} + \frac{1}{2} E_{fc}^2 (12)
\]

Next, we consider the following dynamical equation of the ultra-capacitor,

\[
\dot{V}_{uc} = \frac{i_{uc}}{C} \Rightarrow \dot{E}_s = -\frac{i_{uc}}{V_{max}} (13)
\]

From Eqs.(5), (6), (8) and (10) noting that \( E_{fc} = i_{fc} - i_{fc,t}, \quad E_{fc,t} = i_{fc,t} - i_{fc,d} \Rightarrow i_{fc} = E_{fc} + E_{fc,t} + i_{fc,d} \) we have,

\[
\dot{E}_s = -\left(\frac{1}{C V_{max}}\right) [\frac{V_{Li}/\eta_1 V_{ac}}{\eta_1 V_{fc}} (E_{fc} + E_{fc,t} + i_{fc,d})]
\]

\[
\dot{E}_s = -\left(\frac{1}{C V_{max}}\right) [\frac{V_{Li}/\eta_2 V_{ac}}{\eta_1 V_{fc}} (E_{fc} + E_{fc,t} + (V_{Li}/\bar{\eta}_1) V_{fc} + g(E_s) + \delta_1)]
\]

We now design the compensator \( g(E_s) \) as

\[
g(E_s) = -k_s E_s, \quad k_s > 0 (14)
\]

Thus,

\[
\dot{E}_s = -\left(\frac{1}{C V_{max}}\right) [\frac{V_{Li}/\eta_1 V_{ac}}{\eta_1 V_{fc}} (E_{fc} + E_{fc,t} + (V_{Li}/\bar{\eta}_1) V_{fc} + k_s E_s + \delta_1)]
\]

Next, note from Eqs.(5) and (8) that

\[
V_{Li}L_i = \eta_1 V_{fc}I_{fc} + \eta_2 V_{ac} \left[\frac{V_{Li}L_i - \bar{\eta}_1 V_{fc}i_{fc,t}}{\bar{\eta}_2 V_{uc}} + h(E_{fc}) + \delta_2\right] (15)
\]

We now design the function \( h(E_{fc}) \) as follows

\[
h(E_{fc}) = k_p E_{fc} + k_d \dot{E}_{fc}, \quad k_p, k_d > 0 (17)
\]

Combining Eqs.(16) and (17), we have

\[
\dot{E}_{fc} = -\alpha E_{fc} + \frac{\beta - \delta_2}{k_d} (18)
\]

where,

\[
\begin{align*}
\alpha &= \left[ k_p + (V_{fc}k_d V_{ac})(\bar{\eta}_1/\eta_2) \right]
\beta &= \left(1/\eta_2 V_{ac}\right) [V_{Li}(1 - \eta_2/\eta_1) - \eta_1 V_{fc} \left\{ 1 - (i_{fc,t}/i_{fc})(\bar{\eta}_1/\eta_1)(\eta_2/\eta_2) \right\}]
\end{align*}
\]
From Eqs.(11), (12), (15) and (18), we have,
\[ \dot{V} \leq -\delta^T Q \delta + \frac{E_s}{CV_{max} \eta_2 V_{uc}} \left[ \eta_1 V_f c \delta_1 - V_L i_L (1 - \frac{\eta_1}{\eta_1}) \right] + \frac{E_{fc}}{k_d} (\beta - \delta_2) \] (19)

where,
\[ \delta = \begin{bmatrix} E_s \\ E_{fc} \\ E_{fc} \end{bmatrix}, \quad Q = m \begin{bmatrix} k_s & -0.5 & -0.5 \\ -0.5 & \alpha_3/m & 0 \\ -0.5 & 0 & \alpha/m \end{bmatrix} \] (20)

and \( m = \eta_1 V_{fc}/CV_{max} \eta_2 V_{uc} \). Note in Eq.(20), that \( Q \) is symmetric. Furthermore, \( m > 0 \) and has finite positive upper and lower bounds over the range of operation. Among the positive constants \( k_s, \alpha \) and \( \alpha_3 \), note that \( \alpha_3 \) is not a tunable parameter since it is determined by the characteristics of the fuel supply system. However, by choosing \( k_s \) and \( \alpha \) appropriately, we can ensure \( Q > 0 \) for all feasible values of \( m \). Thus, from the Rayleigh-Ritz Inequality, we have
\[ \delta^T Q \delta \geq \inf(\lambda_{\min,Q}) ||\delta||^2 > 0 \quad \forall \delta \neq 0 \] (21)

where \( \lambda_{\min,Q} \) represents the smallest eigenvalue of \( Q \) at any instant. Furthermore, note that by choosing \( \delta_1 \) and \( \delta_2 \) as
\[ \delta_1 \begin{cases} \leq \left( V_L i_L / \eta_1 V_{fc} \right) (1 - \eta_1 / \eta_1) & \text{for } E_s > 0 \\ \geq \left( V_L i_L / \eta_1 V_{fc} \right) (1 - \eta_1 / \eta_1) & \text{for } E_s < 0 \end{cases} \] (22)

and
\[ \delta_2 \begin{cases} \geq \beta & \text{for } E_{fc} > 0 \\ \leq \beta & \text{for } E_{fc} \leq 0 \end{cases} \] (23)

we have from Eqs.(19) and (21)
\[ \dot{V} \leq -\delta^T Q \delta \leq \inf(\lambda_{\min,Q}) ||\delta||^2 < 0, \quad \forall \delta \neq 0 \] (24)

Note that the conditions in Eqs.(22) and (23) are implementable since \( E_s, E_{fc} \), and all voltage and current variables are measurable. Also, upper and lower bounds of \( \eta_1 \) and \( \eta_2 \) can be estimated. From Eqs.(11), (12) and (24), we conclude that the control design in Eqs.(6) and (8), with \( g(E_s), h(E_{fc}) \) designed as in Eqs.(14) and (17) respectively, and \( \delta_1, \delta_2 \) chosen as in Eqs.(22) and (23) respectively, guarantee exponential stability of the origin \( \delta = 0 \). It is noted that the exponential stability condition in Eq.(9) can be relaxed. For instance, if the fuel supply dynamics is simply

**Input-to-State Stable, ISS [11]**, we can show that the origin \( \delta = 0 \) will also satisfy the ISS property. We omit this proof for conciseness.

### B. \( H_{\infty} \) Control

In this section, we design an alternate control scheme incorporating an \( H_{\infty} \) based approach [15]. Referring to Fig.7 and Eq.(6), the only difference introduced by the \( H_{\infty} \) approach compared to the nonlinear control in section VI-A is that \( i_{fc,d} \) is computed as
\[ i_{fc,d} = \frac{V_L i_L}{\eta_1 V_{fc}} + \delta_{fc} \] (25)

where, \( \delta_{fc} \) is obtained through an \( H_{\infty} \) synthesis. A state space representation of the system is needed to perform the \( H_{\infty} \) control synthesis where all uncertainties and disturbances are included. From Eqs.(5) and (13), we have
\[ \dot{V}_{uc} = \frac{1}{CV_{uc} \eta_2} \left[ V_L i_L (1 - \eta_1 / \eta_1) - V_f c \eta_1 (\delta_{fc} + \sigma) \right] \] (26)

Expanding \( i_{fc} \) as
\[ i_{fc} = i_{fc,d} + \sigma \] (27)

where \( \sigma \) represents a bounded unknown error term, we get from Eqs.(25), (26) and (27),
\[ \dot{V}_{uc} = \frac{1}{CV_{uc} \eta_2} \left[ V_L i_L (1 - \eta_1 / \eta_1) - V_f c \eta_1 (\delta_{fc} + \sigma) \right] \] (28)

Eq.(28) can be expressed as
\[ V_{uc} = f - g(\delta_{fc} + \sigma), \quad f = \frac{V_L i_L}{CV_{uc} \eta_2} \left( 1 - \frac{\eta_1}{\eta_1} \right), \quad g = \frac{V_f c \eta_1}{CV_{uc} \eta_2} \] (29)

We design \( \delta_{fc} \) as follows,
\[ \delta_{fc} = \frac{1}{g} (\hat{f} - v) \Rightarrow i_{fc,d} = \frac{V_L i_L}{\eta_1 V_{fc}} + \frac{1}{g} (\hat{f} - v) \] (30)

where \( \hat{f} \) and \( \hat{g} \) are the estimates of the functions \( f \) and \( g \) respectively, evaluated with nominal values. In the \( H_{\infty} \) control design, we evaluate the maximum bounds of deviation by evaluating \( f \) and \( g \) with maximum upper and minimum lower
bounds possible for all uncertain and variable quantities. Substituting for $\delta_{fc}$ from Eq.(30) into Eq.(29), we get

$$V_{uc} = \frac{g}{\delta} v + \left[ f - \frac{g}{\delta} f - g \sigma \right]$$  \hspace{1cm} (31)

Representing $g/\delta$ as a nominal value $\beta_0$ plus maximum deviation $\Delta \beta$, and assigning the rest of Eq.(31) as disturbances $d$, we have

$$V_{uc} = (\beta_0 + \Delta \beta)v + d$$  \hspace{1cm} (32)

where, the modified control input $v$ is computed using the dynamic feedback law

$$v(s) = K_\infty(s)e(s), \quad e(t) = V_{uc,t} - V_{uc}$$  \hspace{1cm} (33)

where $K_\infty$ is a time-varying gain obtained using standard $H_\infty$ synthesis and $V_{uc,t}$ is the target ultra-capacitor voltage that results in the target SOC, $S_t$. The $H_\infty$ synthesis is carried out using MATLAB’s robust control toolbox. As mentioned at the beginning of this section, referring to Fig.7, the $H_\infty$ approach only introduces a modification to the calculation of $i_{fc,d}$, given by Eq.(30).

VII. EXPERIMENTAL TEST-STAND

We test the control strategy design developed in sections VI-A and VI-B on an experimental test-stand, shown in Fig. 8. The test-stand consists of

Fuel Cell Emulator: An SOFC system is emulated by executing a detailed mathematical model on a dSPACE® DS1103 real time processor in conjunction with a 100V/50A programmable power supply. A schematic diagram of the emulator is shown in Fig.9.

Electronic Load: A DC electronic load, shown in Fig.8, is used for power consumption. The load can draw a maximum power of 1.8kW with a maximum voltage of 60V and a maximum current of 120A.

Bidirectional DC/DC Converter: A bidirectional DC/DC converter, denoted by $C_2$ in Fig 9, is used to command the ultra-capacitor current $i_{uc}$. The converter has two control mode capabilities, current control mode or voltage control mode. We use the current control mode in our application.

Current Clampers: Measurement for $i_L$ and $i_{fc}$ are obtained using Fluke 80i-110s AC/DC current probes.

VIII. EXPERIMENTAL RESULTS

We present results of experiments carried out on the test stand described in section VII. Application of nonlinear and $H_\infty$ control is shown. In all tests, we emulate an SOFC system with $N_{cell} = 50$ connected in series. The fuel supply is assumed to follow a first order dynamics with $\dot{N}_f(s) = 1/(2s+1)\dot{N}_f(s)$. Neither control designs assume knowledge of this dynamics. The bus voltage is $V_L = 24V$ and the ultra-capacitor specifications are $C = 250F$ and $V_{max} = 16.2V$. Other salient aspects of the hybrid system, mentioned in section V, are also applicable for all experiments.

A. Nonlinear Control

In this section, we present results of the nonlinear control strategy developed in section 7. The results are presented in Fig.10. The following control parameters were chosen: $U_{ss} = 0.8$, $S_t = 0.8$, $\bar{\eta}_1 = 1$, $\bar{\eta}_2 = 0.98$, $k_s = 0.1$, $k_p = 0.2$, and $k_d = 0.004$. The terms $\delta_1$ and $\delta_2$ were chosen based on Eqs.(22) and (23), as follows:

$$\delta_1 = \begin{cases} 0 & \text{for } E_f > 0 \\ 2 & \text{for } E_f \leq 0 \end{cases} \quad \delta_2 = \begin{cases} 2 & \text{for } E_{fc} > 0 \\ -2 & \text{for } E_{fc} \leq 0 \end{cases}$$

In Fig.10, $i_L$ is subject to step changes, as shown in Fig.10(a). The corresponding variations in $i_{fc}$, $V_f$, $U$ and $\dot{N}_f$ are plotted in Figs.10(b), (c), (d) and (e) respectively. In Fig.10(b), both $i_{fc}$ and $i_{fc,d}$ are plotted together but are indistinguishable as they coincide at most instants. The ultra-capacitor SOC and current $i_{uc}$ are plotted in Figs.10(f) and (g) respectively, and the fuel cell and ultra-capacitor powers are plotted together in Fig.10(h). The results show close control of $U$ in spite of drastic transients in $i_L$. Fig.10(f) shows tight control of $S$ around the target value of 0.8.
ultra-capacitor and power-electronics components, forming a laboratory scale hybrid power grid. Both control strategies show comparable performance on this platform. In our future research, conditions which are more likely to occur in higher power systems such as saturated fuel flow, limiting fuel cell current density, limiting ultra-capacitor current, will be considered. In such cases, addition of a battery is foreseen.

REFERENCES


