Constrained Optimal Attitude Control of a Quadrotor Helicopter subject to Wind-Gusts: Experimental Studies

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Abstract—A Constrained Finite Time Optimal Controller (CFTOC) for attitude set-point maneuvers of an Unmanned Quadrotor Helicopter (UqH) operating under severe wind conditions is the subject of this article. The UqH’s nonlinear dynamics is linearized in various operating points resulting in a set of piecewise models. The CFTOC is designed for set-point maneuvers taking into account the switching between the linear models and the constraints of the actuators. The control scheme is applied in experimental studies in a prototype UqH operating in harsh weather conditions resulting from wind gusts. The UqH rejects the induced wind-disturbances while performing attitude set-point maneuvers.

Index Terms—Unmanned Aerial Vehicles, Constrained Optimal Control.

I. INTRODUCTION

Recent technological advances both in energy [1–5] and information processing have boosted the development of Unmanned Aerial Vehicles (UAV) platforms with significant mission capabilities. The rotorcraft UAVs pose a set of advantages compared to the fixed wing UAVs such as hovering, vertical takeoff and landing and aggressive maneuvering. Within the family of the rotocrafts, Unmanned quadrotor Helicopters (UqH) [6], have gained increasing attention among scientists and engineers, since quadrors outperform most of other types of helicopters on the issues of maneuverability, survivability, simplicity of mechanics and increased payload.

These comparative advantages make rotorcraft UAVs a viable technological standard for a large set of real-life applications including forest fire surveillance [7], wildlife monitoring, urban and sub-urban rescue missions [8, 9], and special military applications. In such missions, it is rather often for the UqHs to operate in harsh environments encountering powerful atmospheric disturbances. The most prominent disturbances are due to the forcible wind–gusts that lead to helicopter instabilities, performance degradation and possibly crash.

Until now in the relevant literature of UqHs, the problem of control design has been addressed using several methods such as the development of PID and LQR controllers [10, 11], Sliding Mode control [12, 13], and Nonlinear Dynamic Inversion and Backstepping [14].

This article aims to extend the previous list of control approaches for UqHs and focuses on the design and the experimental verification of a Constrained Finite Time Optimal (CFTO)–Controller to the problem of attitude control for a UqH. Apart from the experimental novelty of the presented application itself, the proposed control scheme contains additional novelties in the control design approach for a UqH. These novelties stem from the adopted switching piecewise affine (PWA) models of the UqH, the considered actuator constraints (i.e. motor saturation) and the indirect induced wind-gust disturbance rejection while satisfying certain optimality criteria.

This article is structured as follows. In Section II, the modeling approach for the attitude problem of a UqH is presented while in Section III the design and the development of the CFTO–control scheme is analyzed for the servo control problem. In Section IV, extended experimental results that prove the efficacy of the proposed scheme are presented followed by the conclusions in the last section V.

II. QUADROTOR HELICOPTER MODELING

The model of the UqH utilized in this work, assumes that the structure is rigid and symmetrical, the center of gravity and the body fixed frame origin coincide, the propellers are rigid and the thrust and drag forces are proportional to the square of propeller’s speed. The electro–mechanical structure of the UqH under study, and the relative coordinate systems are presented in Figure 1.

![Quadrotor helicopter configuration frame system](image)

Fig. 1. Quadrotor helicopter configuration frame system

The UqH’s nonlinear dynamics [14] is characterized by a set of twelve high non–linear state equations in the form:

\[
X = f(X, U) + W
\]

with \( f \) a non–linear function, \( W \) corresponds to the additive effects of the environmental (wind) disturbances, \( X \) the state vector, and \( U \) the input vector, where:

\[
X = [\phi \dot{\phi} \theta \dot{\theta} \psi \dot{\psi} z \dot{z} x \dot{x} y \dot{y}]
\]

\[
U = [U_1 \ U_2 \ U_3 \ U_4 \ \Omega_r]
\]
The control inputs in (1) are produced by the following combinations of the angular speeds of the four UqH's rotors as:

\[ U_1 = b(\Omega_1^2 + \Omega_2^2 + \Omega_3^2 + \Omega_4^2) \]
\[ U_2 = b(-\Omega_1^2 + \Omega_2^2 + \Omega_3^2 + \Omega_4^2) \]
\[ U_3 = b(-\Omega_1^2 - \Omega_2^2 + \Omega_3^2 + \Omega_4^2) \]
\[ U_4 = d(-\Omega_1^2 + \Omega_2^2 - \Omega_3^2 + \Omega_4^2) \]
\[ \Omega_r = -\Omega_1 + \Omega_2 - \Omega_3 + \Omega_4 \]

where \( b \) is the thrust coefficient, and \( d \) is the drag coefficient, while the input \( U_1 \) is related with the total thrust and the inputs \( U_2, U_3, U_4 \) are related with the rotations of the quadrotor and \( \Omega_r \) is the overall residual angular velocity of the motors. Under a small angle approximation and other assumptions and further simplifications that can be found in [14, 15] the system can be decoupled in two independent but connected subsystems. The first one is related to the linear translations and the second one deals with the angular rotations. The attitude equations correspond to the first six equations of the nonlinear ODEs in (1); small perturbations around the operating points \( x^0_j = [0, \phi^j, 0, \theta^j, 0, \psi^j]^T, j = 1, \ldots, M \) results in the following state space form for the \( j \)th operating point:

\[ x = A_j x + B_j u + w \]

\[ x = [\delta \phi, \delta \dot{\phi}, \delta \theta, \delta \dot{\theta}, \delta \psi, \delta \dot{\psi}]^T \]

\[ u = [\delta U_1, \delta U_2, \delta U_3, \delta U_4, \delta \Omega_r]^T \]

where,

\[ A_j = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{l_{r_{\phi}} - l_{r_{\theta}}} {l_{r_{\phi}}} \psi^j & \frac{l_{r_{\phi}} - l_{r_{\theta}}} {l_{r_{\phi}}} \theta^j \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & \frac{l_{r_{\phi}} - l_{r_{\theta}}} {l_{r_{\phi}}} \psi^j & 0 & 0 & 0 & \frac{l_{r_{\phi}} - l_{r_{\theta}}} {l_{r_{\phi}}} \theta^j \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & \frac{l_{r_{\phi}} - l_{r_{\theta}}} {l_{r_{\phi}}} \psi^j & 0 & 0 & 0 & 0 \end{bmatrix} \]

\[ B_j = \begin{bmatrix} 0 & 0 & 0 & 0 & \frac{l_{r_{\phi}} - l_{r_{\theta}}} {l_{r_{\phi}}} \theta^j & \frac{l_{r_{\phi}} - l_{r_{\theta}}} {l_{r_{\phi}}} \psi^j \\ 0 & 0 & 0 & 0 & \frac{l_{r_{\phi}} - l_{r_{\theta}}} {l_{r_{\phi}}} \theta^j & \frac{l_{r_{\phi}} - l_{r_{\theta}}} {l_{r_{\phi}}} \psi^j \\ 0 & 0 & 0 & 0 & \frac{l_{r_{\phi}} - l_{r_{\theta}}} {l_{r_{\phi}}} \theta^j & \frac{l_{r_{\phi}} - l_{r_{\theta}}} {l_{r_{\phi}}} \psi^j \\ 0 & 0 & 0 & 0 & \frac{l_{r_{\phi}} - l_{r_{\theta}}} {l_{r_{\phi}}} \theta^j & \frac{l_{r_{\phi}} - l_{r_{\theta}}} {l_{r_{\phi}}} \psi^j \end{bmatrix} \]

and \( w \) is the additive external disturbance vector that affects the flight of the UqH.

The utilized parameters for the UqH attitude modeling in equations (5) and (6) are presented in Table I.

### TABLE I

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( I_{xx} )</td>
<td>Moment of Inertia of the UqH about the X axis</td>
</tr>
<tr>
<td>( I_{yy} )</td>
<td>Moment of Inertia of the UqH about the Y axis</td>
</tr>
<tr>
<td>( I_{zz} )</td>
<td>Moment of Inertia of the UqH about the Z axis</td>
</tr>
<tr>
<td>( l_a )</td>
<td>Arm length</td>
</tr>
<tr>
<td>( I_r )</td>
<td>Propeller inertia</td>
</tr>
<tr>
<td>( I_p )</td>
<td>Moment of inertia of the rotor about its axis of rotation</td>
</tr>
</tbody>
</table>

III. CONSTRAINED OPTIMAL CONTROL SCHEME

The constrained finite time optimal control scheme [16] is designed and experimentally applied for performing stabilization of the quadrotor around deviations from the nominal angles of operation (attitude control). The proposed control action affects only the angular rotations of the quadrotor \( \phi, \theta, \psi \) and their derivatives, while a feedforward controller is utilized in order to apply the necessary elevation thrust to the UqH. The overall control scheme is presented in Figure 2.

![UqH CFTO–control Structure](image)

**Fig. 2.** UqH CFTO–control Structure

For each operating point \( x^0_j \), the UqH is assumed to operate within a regime specified by certain boundaries of its angular rates, or

\[ \gamma^j - \Delta \gamma^j = \gamma^j \leq \gamma^j + \Delta \gamma^j = \gamma^j + \Delta \gamma^j, \gamma \in \{ \phi, \theta, \psi \} \] (7)

In a similar manner, there are constraints related to the control inputs and the differential angles, denoted as

\[ \Delta \gamma^j \leq \delta \gamma^j \leq \Delta \gamma^j, \gamma \in \{ \phi, \theta, \psi \} \] (10)

Let the matrix \( H_i \) be a zeroed 2 × 11 matrix except for its \( i \)-th column, which is equal to \([1, -1]^T \). Then the previous bounds can be cast in a more compact form as:

\[ [H_1] \quad [H_2] \quad \ldots \quad [H_{11}] \] (11)

Assume that the system is sampled with a sampling period of \( T_s \). Let the discrete equivalent of its PWA–description be:

\[ x_{k+1} = A_j^* x(k) + B_j^* + w^* \] (12)

where \( x_k (u_k) \) corresponds to state (control) value of the \( k \)-th sample and \( w^* \) corresponds to the discrete expression of the additive disturbance \( w \).
The CFTOC-design problem consists of computing the optimum control vector sequence \( \tilde{u} = [u_k u_{k+1} ... u_{k+N-1}]^T \), where \( N \) corresponds to the prediction horizon that minimizes the following cost function:

\[
J_N(x_k, x') = [x_{k+N} - x']^T P [x_{k+N} - x'] + \\
\sum_{m=0}^{N-1} [u_{k+m}]^T R [u_{k+m}] + [x_{k+m} - x']^T Q [x_{k+m} - x']
\]

where \( P = \tilde{P}^T \geq 0 \), \( R = R^T > 0 \) and \( Q = Q^T \geq 0 \) are the weighting matrices.

The predicted final state \( x_{k+N} \) is typically constrained to a predefined set \( x_{k+N} \in \mathcal{X} \) which is dictated by certain stability and feasibility requirements. The solution to the CFTOC problem [16–20] is a continuous control action of the form:

\[
u_k = F^T x_k + G_t \text{ if } x_k \in \tilde{R}_t
\]

where \( \tilde{R}_t, t \in \{1, ..., \text{max}\} \) corresponds to a convex polyhedron \( (\tilde{R}_t \in \mathbb{R}^n) \) calculated by the algorithm. The \( \text{max} \) number of polyhedra is similarly specified by the algorithm. In general the higher the number of the PWA–systems along with the large dimension of the state vector and the number of constraints, the more complicated the solution is.

This complexity increases significantly with the value of the prediction horizon \( N \), and the number \( \text{max} \) of the convex polyhedra (regions) grows up (usually) exponentially. For the on–line implementation of the controller the number \( \text{max} \) of the regions is not only a measure of the controller’s complexity but also affects its implementation typically precomputed in a look–up table.

IV. EXPERIMENTAL STUDIES

The proposed control scheme was applied to the experimental platform of a Dragonflyer quadrotor helicopter [21], while some modifications have been made in order to increase the UqH’s capabilities and to allow a computer–aided feedback control of the UqH’s attitude. The programming environment was NI’s LabView, while the measuring of the UqH states was provided by the utilization of an Xsens MTi-G [22] Attitude and Heading Reference System (AHRS), with a wireless (IEEE 802.15.4) data link between the AHRS and the computer unit. The radio control link between the base station, where the controller resides, and the UqH was achieved using an NI USB 6229–board, connected with a FUTABA 6EXP radio transmitter, operating in training mode, and sending the control commands computed from the control algorithm. The induced wind gust velocities were measured using a rotary vane anemometer [23]. The main components of the experimental set–up are presented in Figure 3, where the UqH had been attached to a Heli-Safe flight test stand modified in order to allow only attitude control. The wind–gust disturbances were generated using an electric fan.

The parameters of the modified UqH considered were re–calculated using Computational Fluid Dynamics software using as a basis the technical data in [24] and are set as \( I_{xx} = I_{yy} = 5.0 \cdot 10^{-3} \text{kg} \cdot \text{m}^2, I_{zz} = 8.9 \cdot 10^{-3} \text{kg} \cdot \text{m}^2, I_a = 0.21 \text{m}, \) and \( J_r = 5.5100 \cdot 10^{-3} \text{kg} \cdot \text{m}^2 \). The tuning parameters of the CFTO–controller were \( \tilde{P} = 10^6 \cdot I_6, R = 100 \cdot I_1, \) and \( Q = 1000 \cdot I_6, \) and the prediction horizon was set to \( N = 4 \).

The constraints on the inputs have been computed based on the physical parameters of the system and specifically: a) the maximum angular velocity of the motors [25], b) the thrust factor in hovering set as \( b = 2.8 \cdot 10^{-5} \text{N} \cdot \text{sec}^{-2} \), and c) the drag factor in hovering set as \( d = 8 \cdot 10^{-7} \text{N} \cdot \text{m} \cdot \text{sec}^{-2} \).

Based on the above values the following constraints on the inputs can be set as \( 0 \leq U_1 \leq 11.23, [U_2] \leq 5.61, [U_3] \leq 5.61, [U_4] \leq 0.16 \). The state constraints were \( |x_i^{\phi}| \leq \frac{\pi}{2} \text{ rad}, i = 1, 3, |x_i^{\theta}| \leq \pi \text{ rad}, i = 5 \) and \( |x_i^{\psi}| \leq 1 \text{ rad/sec}, i = 2, 4, 6 \).

In the rest of the article, the modeling formulations and the controller design approach for the set-point control problem are presented, extended with the obtained corresponding experimental results that prove the efficiency of the proposed control scheme. The proposed controllers were tested: a) in conditions of laminar air flow, b) in conditions under the presence of severe wind–gusts, and c) with different modeling approaches using various–PWA systems (\( M = 1 \) and \( M = 3 \)) and for different set–points. In all these cases the sampling period was set to \( T_s = 0.5 \text{sec.} \) During all experiments the utilized PWA systems were calculated using the modeling parameters listed in Table II.

<table>
<thead>
<tr>
<th>PWA No.</th>
<th>Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( \phi = \theta = \psi = 0 )</td>
</tr>
<tr>
<td>2</td>
<td>( \phi = \theta = \psi = 0.1 )</td>
</tr>
<tr>
<td>3</td>
<td>( \phi = \theta = \psi = -0.1 )</td>
</tr>
</tbody>
</table>

For a set–point selected as \( x' = [-0.25 0 0 0 0 0]^T \) the response of the UqH’s Euler Angles in the absence of wind–gusts for \( i \in \{1, 3\} \) PWA systems and a prediction horizon \( N = 4 \) is shown in Figure 4. Once the UqH has reached steady–state, a forcible three directional wind–gust (with a directional magnitude of \( x = -1.31 \text{m/s}, y = 3.84 \text{m/s} \), and \( z = 1.65 \text{m/s} \)) is applied and the corresponding response is shown in Figure 5. As it can be observed, the CFTO–control effectively achieves the desired set–point while it also manages to attenuate the effects of these harsh environmental
disturbances. In addition, it must be pointed out that the utilization of more PWA systems results in better response characteristics such as higher maneuvering velocity and smaller steady-state error.

In a similar manner, if the desired set-point is set at \(x_r = [0 \ 0 \ 0 \ 0 \ 0.25]^T\) (which is a set-point at yaw) the experimental studies prove once again the efficacy of the proposed CFTO–control scheme, as shown in Figures 6, 7 in the absence and the presence of wind–gusts respectively. As it was presented in the previous experimental cases the utilization of more PWA systems improves the UqH rotation angles response.

V. CONCLUSIONS

In this paper, an innovative CFTO–control scheme for the attitude control of an UqH under the influence of wind gusts and the existence of physical constraints has been presented. The resulting controller has been validated in experimental studies with a prototype UqH modeled as a set of switching linear PWA systems. The presented experimental results indicate the efficacy of the proposed controller both in attitude set-point maneuvers and wind–gust disturbance attenuation.
Fig. 7. Comparison of Rotation Angles Responses for a set-point $x' = 0 \ 0 \ 0 \ 0 \ 0.025^T$ subject to $x(1.31m/s), y(3.84m/s)$ and $z(1.65m/s)$ directional Wind Gust.

REFERENCES


