Reactive Target-tracking Control with Obstacle Avoidance of Unicycle-type Mobile Robots in a Dynamic Environment

Jiangmin Chunyu, Zhihua Qu, Eytan Pollak and Mark Falash

Abstract—This paper proposes a new reactive control design for unicycle-type mobile robots with limited sensor range to track targets while avoiding static and moving obstacles in a dynamically evolving environment. With the relative motion among the mobile robot, targets, and obstacles formulated in polar coordinates, kinematic control laws achieving target-tracking and obstacle avoidance are synthesized using Lyapunov based technique, and more importantly, the proposed control laws also account for possible kinematic control saturation constraints. Simulation examples are presented to illustrate the effectiveness of the proposed control.

I. INTRODUCTION

It is a basic requirement that mobile robots can safely explore and move within dynamic environments for most real-world applications. Considering mobile robots are often subject to nonholonomic constraints, to achieve this goal, the central and difficult studies can be classified into the following two regimes.

A. Target Tracking Control of Nonholonomic Systems

The target tracking control objective is to make the system asymptotically follow a desired trajectory. The desired trajectory must be feasible, that is, it has been planned to satisfy nonholonomic constraints. Especially when a desired trajectory stops at some point in the configuration space, the tracking control problem reduces to the stabilization problem, also called the regulation problem.

Control of nonholonomic systems has drawn great attention due to its practical importance and theoretical challenges. A comprehensive introduction to nonholonomic systems modeling, analysis, and control can be found in [1]. By Brockett theorem [2], nonholonomic systems can not be asymptotically stabilized around a fixed point under any smooth (or even continuous) time-independent state feedback control law in cartesian coordinates. Therefore, tracking control problem and regulation problem are usually treated separately using different approaches.

Most existing methods dealing with regulation problem use one of these two strategies: time-implicit but discontinuous feedback control laws [3] and time-varying continuous controls [4]. Tracking control designed using the backstepping method is shown to ensure global asymptotic stability [5]. In [6], a unifying design framework is proposed by investigating uniform complete controllability of time-varying systems. The proposed controls are globally asymptotically stabilizing, in simple closed forms, time-varying and smooth, and near-optimal.

B. Obstacle Avoidance of Nonholonomic Systems

Obstacle avoidance has been extensively studied at the navigation system level (path planning/trajectory planning). In comparison with standard motion planning approaches such as graph methods and potential field methods, which are proposed to deal with geometrical constraints, more specifically, holonomic systems in the presence of static obstacles, motion planning of nonholonomic wheeled mobile robots in dynamic environments is more challenging and important in the area of mobile robotics. Nonholonomic path planners [7] are proposed based on Reeds and Shepp’s results on shortest paths of bounded curvature in the absence of obstacles. With obstacles modeled as polygons, obstacle avoidance and curvature constraint are taken into account by offsetting each polygon. A feasible path is then obtained by using a sequence of such optimal path segments as those in [8]. In [9], using ideas from fluid mechanics, a collision free path is calculated satisfying minimum curvature constraint. This method assumes the environment is static and known a priori. An analytical nonholonomic trajectory generation algorithm is proposed in [10]. A family of parameterized polynomial trajectories are derived to make all the resulting trajectory candidates feasible. Then, the free parameter(s) representing the family are confined into appropriate intervals so that collision avoidance criteria are met.

Obstacle avoidance can also be solved directly in the kinematics/dynamics controller, which is normally called avoidance control. The dynamic window approach is used [11], [12]. A search space is defined, consisting only of the admissible velocities and accelerations of the robot within a short time interval. Then the commands controlling the velocities and accelerations of the robot are chosen by maximizing an objective function associated with target tracking and obstacle avoidance. The concepts of collision cones and velocity obstacles are introduced in [13], [14] respectively, which are widely used to design avoidance control [15], [16]. The underlying idea is that obstacle avoidance is achieved if the robot velocity is selected such that its velocity relative to the obstacles’ motion does not enter the corresponding collision cones/velocity obstacles. Avoidance control is also
proposed combining potential field methods and sliding mode control [17], [18]. A gradient-tracking based sliding mode controller for the mobile robot is proposed to achieve target tracking and obstacle avoidance. Moreover, potential field based formation control of multiple mobile robots are studied in [19], [20].

C. Outline of this paper

In this paper, we restrict our attention to a reactive control solutions to position tracking and obstacle avoidance of a class of most studied nonholonomic systems: unicycle-type mobile robots. The polar representation is employed to synthesize the controls due to the following two reasons: (1) The polar representation naturally provides a better measure of progress towards a target position and the distance to the obstacle; (2) By introducing the polar coordinate transformation, it is shown that control laws can be readily developed.

The polar representation has been firstly introduced in [21], adopted for regulation problem [22] and employed to solve the tracking control problem [23]. However, to the best of our knowledge, we are the first to combine polar coordinate transformation and Lyapunov-like analysis to solve the aforementioned multi-objective control problem (position tracking and obstacle avoidance). The main contributions of this paper are: (1) An easily implementable reactive control algorithm for solving the position tracking and obstacle avoidance of unicycle-type is derived; (2) The proposed control algorithm takes into consideration the possible kinematic control saturation constraints.

The remainder of this paper is organized as follows: Section 2 formulates the problems of tracking target and avoiding the static/moving obstacles. Section 3 proposes a novel reactive control design for a single unicycle-type mobile robot to achieve target-tracking and collision avoidance based upon polar description of the relative motions. Section 4 presents examples and their simulations to demonstrate the effectiveness of proposed controls. Section 5 concludes the paper and suggests some future research directions.

II. PROBLEM FORMULATION

The kinematics of a unicycle-type mobile robot is subject to the nonholonomic no-slip constraint of form

\[ A(q)\dot{q} = 0 \quad \text{with} \quad A(q) = \begin{bmatrix} -\sin \theta & \cos \theta & 0 \end{bmatrix}, \]

where \( q = [x \ y \ \theta]^T \) are states defined in the configuration space. \((x,y)\) is the center position and \( \theta \) is the orientation.

A basis \( G(q) \) of the null space of \( A(q) \) is given by

\[ G(q) = \begin{bmatrix} \cos \theta & 0 & 0 \\ \sin \theta & 0 & 0 \\ 0 & 1 \end{bmatrix}. \]

Therefore, the kinematic model can be written as follows

\[ \dot{q} = G(q)U, \quad (1) \]

where \( U = [v \ \omega]^T \) is called kinematic control. Especially, \( v \) is the linear velocity and \( \omega \) is the angular velocity.

The sign of linear velocity \( v \) will determine forward or backward motion of the vehicle. The target and obstacles are also supposed to satisfy the kinematic model (1). Subscripts \( r, g \) and \( o \) indicate the vehicle, target and obstacle respectively.

![Fig. 1. Illustration of target-tracking with collision avoidance in a 2D dynamic environment](image)

Referring to Figure 1, \( X_GO_GY_G \) is the global inertial reference frame. The relative motion between the robot and its target in polar coordinates is represented by

\[ \rho_{rg} = \sqrt{(x_r-x_g)^2 + (y_r-y_g)^2}, \quad (2) \]

and

\[ \phi_{rg} = \arctan2(y_g-y_r, x_g-x_r). \quad (3) \]

Clearly, \( \rho_{rg} \) represents the distance between the robot and target. And \( \phi_{rg} \) is the line-of-sight angle.

To derive the tracking error kinematics in polar coordinates, differentiating (2) and (3) on both sides, we can obtain

\[ \dot{\rho}_{rg} = -\cos\phi_{rg}(\dot{x}_r-\dot{x}_g) - \sin\phi_{rg}(\dot{y}_r-\dot{y}_g), \quad (4) \]

and

\[ \dot{\phi}_{rg} = -\cos\phi_{rg}(\dot{y}_r-\dot{y}_g) + \sin\phi_{rg}(\dot{x}_r-\dot{x}_g). \quad (5) \]

Substituting (1) into (4) and (5) and following from the trigonometric identities, the error system in the polar coordinates can be written as

\[ \begin{cases} \dot{\rho}_{rg} = -v_r\cos(\theta_r-\phi_{rg}) + v_g\cos(\theta_r-\phi_{rg}) \\ \dot{\phi}_{rg} = -v_r\sin(\theta_r-\phi_{rg}) + v_g\sin(\theta_r-\phi_{rg}) \end{cases}. \quad (6) \]

Similarly, the relative motion between the robot and the \( i \)th obstacle in the polar coordinates can be formulated as

\[ \begin{cases} \dot{\rho}_{oir} = -v_{oir}\cos(\theta_{oir}-\phi_{oir}) + v_r\cos(\theta_r-\phi_{oir}) \\ \dot{\phi}_{oir} = -v_{oir}\sin(\theta_{oir}-\phi_{oir}) + v_r\sin(\theta_r-\phi_{oir}) \end{cases}. \quad (7) \]

In the subsequent section, (6) and (7) will be used to design the controller for the robot. Without loss of generality, \( \theta, \phi_{oir}, \text{ and } \phi_{rg} \) fall into \([-\pi, \pi)\).

In order to have a well-defined problem, we assume the followings throughout the paper:
Assumption 1: the mobile robot under consideration is represented by a 2-D circle with the center at \((x_r(t), y_r(t))\) and of radius \(R_r\). The range of its sensors is also described by a circle centered at \((x_r(t), y_r(t))\) and of radius \(R_i\). Meanwhile, the \(i\)th obstacle is represented by a 2-D circle with the center at \((x_o(i)(t), y_o(i)(t))\) and of radius \(R_{oi}\).

Assumption 2: The mobile robot has the following velocity saturation constraints:

\[
|v_r| \leq \bar{v}_r \quad \text{and} \quad |\omega_r| \leq \bar{\omega}_r, \tag{8}
\]

where \(\bar{v}_r > 0\) is the maximum linear velocity. And \(\bar{\omega}_r\) is the maximum angular velocity. Furthermore, the robot is supposed to have a superior maneuvering capability given by

\[
\bar{v}_r \geq k_1 \bar{v}_r \quad \text{and} \quad \bar{v}_r \geq k_2 \bar{v}_ai \quad (k_1, k_2 > 2). \tag{9}
\]

Then the control objectives of this paper can be summarized as follows:

- Globally uniform bounded position tracking, i.e. provided that \(\lim_{t \to +\infty} \rho_{air}(t) > R_{oi}\), there exists \(D > 0\) such that \(\lim_{t \to +\infty} \rho_{rg}(t) \leq D, \forall (x_{r}(t_0), y_{r}(t_0)) \in \mathbb{R}^2;\)
- Obstacle avoidance, i.e. there exists \(D_o > 0\) such that \(\rho_{air}(t) > (R_{io} + R_r), \forall t \geq t_0\) provided that \(\rho_{air}(t_0) \geq D_o.\)

III. TARGET-TRACKING AND OBSTACLE AVOIDANCE CONTROL LAW SYNTHESIS

In this section, we derive a reactive switching control using Lyapunov-type analysis that guarantees collision avoidance and tracking of a target for a single robot. Specifically, the kinematic control \(v_r\) is chosen to be a constant speed \(v_r\). The reason is twofold: (1) observing form (6) and (7), kinematic control \(\omega_r\) plays a vital role as to multi-objective of target tracking and obstacle avoidance; and (2) this type of control is simple from theoretical development aspect and requires less control effort as well. The detailed design and proofs are explored as follows.

Let us first begin with some definitions:

Definition 3.1: Let \(\mathbb{R}_+^\Delta \doteq [0, +\infty)\), a \(C^1\) function \(P_a : \mathbb{R}_+ \rightarrow \mathbb{R}_+\), is called an attractive potential field function on \(\mathbb{R}_+\) if the following conditions hold:

1) \(P_a(\rho_{rg}) = 0\) and \(\nabla P_a(\rho_{rg}) = 0\) iff \(\rho_{rg} = 0;\)
2) \(P_a(\rho_{rg}) < \bar{P}_a\) and \(0 < \nabla P_a(\rho_{rg}) < \bar{F}_a\) when \(\rho_{rg} \neq 0;\)

where \(\bar{P}_a > 0\) is the upper bound of \(P_a(\rho_{rg})\) and \(\bar{F}_a > 0\) is the upper bound of \(\|\nabla P_a(\rho_{rg})\|\).

Definition 3.2: A \(C^1\) function \(P_r : \mathbb{R}_+ \rightarrow \mathbb{R}_+\), is called a repulsive potential field function on \(\mathbb{R}_+\) if the following conditions hold:

1) \(P_r(\rho_{air}) \geq \bar{P}_r\) if \(\rho_{air} \leq (R_{io} + R_r);\)
2) \(P_r(\rho_{air}) \in [0, \bar{P}_r]\) and \(\nabla P_r(q) \in [-\bar{F}_r, 0]\) if \(\rho_{air} > (R_{io} + R_r);\)

where \(\bar{P}_r > 0\) is the upper bound of \(P_r\) and \(\bar{F}_r > 0\) is the upper bound of \(\|\nabla P_r(\rho_{air})\|\).

In what follows, the properties of above defined potential functions will be used for Lyapunov argument.

A. Target-tracking Control Design

As shown in Figure 2, the idea of tracking control design is to steer orientation angle \(\theta_r\) to track line-of-sight \(\phi_{rg}\) as soon as possible with consideration of the angular velocity saturation \(\bar{\omega}_r\). Therefore, the robot’s tracking control is given by

\[
v_r = \begin{cases} v_r & \text{if } \rho_{rg} \leq D, \\ 0 & \text{otherwise}. \end{cases}
\]

\[
\omega_r = \text{sat} \left( -k\Phi(\theta_r - \phi_{rg}) + \phi_{rg}, \bar{\omega}_r \right), \tag{10}
\]

where \(D\) is the position tracking bound, \(k\) is a positive constant gain. And function \(\Phi(\alpha)\) determines the direction of rotation, which is designed to be

\[
\Phi(\alpha) = \begin{cases} 2\pi + \alpha & \text{if } \alpha \leq -\pi, \\ \alpha - 2\pi & \text{if } \alpha > \pi, \\ \alpha & \text{otherwise}. \end{cases} \tag{11}
\]

And \(\text{sat}(u, \bar{u})\) is a saturation function defined as

\[
\text{sat}(u, \bar{u}) \doteq \text{sgn}(u(t)) \cdot \min(|u(t)|, \bar{u}). \tag{12}
\]

Theorem 1: Consider system (1) under control (10) and suppose assumptions 1 and 2 hold. Let \(D > \frac{(\bar{v}_r + \bar{v}_g)}{\bar{\omega}_r}\), then there exists \(k \geq 0\) such that \(\lim_{t \to +\infty} \rho_{rg}(t) \leq D, \forall (x_{r}(t_0), y_{r}(t_0)) \in \mathbb{R}^2\). Moreover, the tracking error bound \(D\) can be reduced by increasing the value of the angular velocity saturation \(\bar{\omega}_r\).

Proof: In order to prove the tracking error is uniformly bounded by \(D\), we confine our attention to the case \(\rho_{rg}(t) > D\). In this case, it follows from (6) that

\[
\phi_{rg} \leq -\frac{(v_r + v_g)}{D}. \tag{13}
\]

To avoid the saturation of control action \(\omega_r\), we can chose

\[
k \leq \frac{\bar{\omega}_r - (\bar{v}_r + \bar{v}_g)}{\pi}. \tag{14}
\]

Substituting (13) and (14) into (10), we can claim that \(\omega_r\) avoids to violate the saturation bound. Hence

\[
\omega_r = -k\Phi(\theta_r - \phi_{rg}) + \phi_{rg}. \tag{15}
\]
Consider the function \( \Theta(\alpha) \) given by
\[
\Theta(\alpha) = \begin{cases} 
2\pi + \alpha & \text{if } \alpha \leq -\pi, \\
2\pi - \alpha & \text{if } \alpha > \pi, \\
\alpha & \text{otherwise}.
\end{cases}
\] (16)

Differentiating (16) on both sides and following from (15), it is straightforward to verify that
\[
\dot{\Theta}(\theta - \phi_r) = -k\Theta(\theta - \phi_r)
\] (17)

Then let us pick the Lyapunov-like function candidate as
\[
V = P_o(\rho_r) + \frac{1}{2} F_o (\rho_r + \phi_g) \left( \frac{3}{\pi} \right)^2 \Theta^2 (\theta - \phi_r).
\] (18)

It follows from (6), (10), and (17) that
\[
\dot{V} = \frac{\partial P_o}{\partial \rho_r} (\rho_r + \phi_g) - \dot{F}_o (\rho_r + \phi_g) (\frac{3}{\pi})^2 \Theta^2 (\theta - \phi_r).
\] (19)

In the case \( \Theta(\theta - \phi_r) > \frac{\pi}{3} \), following from (19) and (16), we can obtain
\[
\dot{V} \leq \frac{\partial P_o}{\partial \rho_r} (\rho_r + \phi_g) - \dot{F}_o (\rho_r + \phi_g) (\frac{3}{\pi})^2 \Theta^2 (\theta - \phi_r) < 0.
\] (20)

On the other side, in the case \( \Theta(\theta - \phi_r) < \frac{\pi}{3} \), following from (19), under assumption 2 (\( \rho_r > 2\phi_g \)), we can obtain
\[
\dot{V} \leq \frac{\partial P_o}{\partial \rho_r} (\rho_r + \phi_g) - \dot{F}_o (\rho_r + \phi_g) (\frac{3}{\pi})^2 \Theta^2 (\theta - \phi_r) < 0.
\] (21)

Adding (20) and (21) together, we can show that \( \dot{V} < 0 \) as long as \( \rho_r(t) > D \). Noting (14), \( D \) can be chosen smaller by increasing \( \phi_g \), which implies a smaller tracking error bound \( D \) can be achieved with a larger \( \phi_g \).

**Remark 1:** Considering the regulation problem, without loss of generality, let us choose \( q_g = [0, 0, 0] \). Thus (6) reduces to
\[
\begin{align*}
\dot{\rho}_g &= -v_r \cos(\theta - \phi_g), \\
\dot{\phi}_g &= -\frac{v_r \sin(\theta - \phi_g)}{\rho_g}, \\
\dot{\theta}_r &= \omega_r.
\end{align*}
\] (22)

Compared with (1), (22) is still a nonholonomic system defined in polar coordinates, which can also be rewritten into the standard driftless nonholonomic form as
\[
\begin{bmatrix}
\dot{\rho}_g \\
\dot{\phi}_g \\
\dot{\theta}_r
\end{bmatrix} =
\begin{bmatrix}
-cos(\theta - \phi_g) & 0 & 0 \\
-sin(\theta - \phi_g) & 0 & 0 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
v_r \\
\frac{v_r}{\rho_g} \\
\omega_r
\end{bmatrix}.
\] (23)

Correspondingly, we redesign the tracking control which is given by
\[
vg = \operatorname{sat}(\min\{\rho_g, 1\} \cdot k_v, v_r), \\
\omega_r = \operatorname{sat}(-k_P (\theta - \phi_g) + \phi_g, \omega_r),
\] (24)

where \( k_v < \frac{\omega_r}{\pi} \) is a positive constant.

Considering (22) under control (24), let \( k \leq \frac{\omega_r}{\pi} \). Asymptotic position tracking can be concluded by essentially the same proof as shown above.

**B. Obstacle Avoidance Control Design**

As shown in Figure 3, the idea of avoidance control design is to steer orientation angle \( \theta \) to track line-of-sight \( \phi_{oir} \) as soon as possible with consideration of the angular velocity saturation \( \omega_r \). Therefore, the robot’s avoidance control is given by
\[
v_r = v_r, \omega_r = \operatorname{sat}(-k_o (\theta - \phi_{oir}) + \phi_{oir}, \omega_r),
\] (25)

where \( k_o \) is a positive constant.

**Theorem 2:** Consider system (1) under control (25) and suppose assumptions 1 and 2 hold. There exists a pair \( (D_o, k_o) \) such that \( \rho_{oir}(t) > (R_{oir} + R_t), \forall t \geq t_0 \) provided that \( \rho_{oir}(t_0) > D_o + (R_{oir} + R_t) \).

**Proof:** Following from (17), we have
\[
\Theta(\theta(t) - \phi_{oir}(t)) = \Theta(\theta(t_0) - \phi_{oir}(t_0)) e^{-k_o(t - t_0)}.
\] (26)

To prove the obstacle avoidance, let us consider the worst case that \( \Theta(\theta(t_0) - \phi_{oir}(t_0)) = \pi \), \( v_r = v_r \), and \( v_{oir} = v_{oir} \). Let \( \tau = \frac{\ln 3}{k_o} \), it follows from (26) that
\[
\Theta(\theta(t) - \phi_{oir}(t)) \leq \frac{\pi}{3} \quad (t \geq t_0 + \tau).
\] (27)

Under the assumption 2 (\( \rho_r > 2\phi_g \)), substituting (27) into (7), we can obtain
\[
\rho_{oir}(t) < 0 \quad (t \geq t_0 + \tau).
\] (28)

Therefore, following from (25) and (28), a conservative design to achieve obstacle avoidance boils down to solve a pair \( (D_o, k_o) \) satisfying the following inequality
\[
D_o - \frac{\ln 3}{k_o} (\rho_r + \phi_g) > 0.
\] (29)

To avoid the saturation of control action \( \omega_r \), following from (7) and (25), \( (D_o, k_o) \) should also satisfy the inequality
\[
k_o \pi + \frac{v_r + v_{oir}}{D_o - \frac{\ln 3}{k_o} (\rho_r + \phi_g)} < \omega_r.
\] (30)

For an instance, \( (D_o, k_o) \) can be given by
\[
k_o = \frac{\omega_r}{2\pi} \quad \text{and} \quad D_o = (2 + 2\pi \ln 3) \frac{(\rho_r + \phi_g)}{\omega_r}.
\] (31)
Let us pick the Lyapunov-like function candidate as
\[ E = P_r (\rho_{oir}) + \frac{1}{2} \Theta^2 (\theta_r - \phi_{oir}). \]  
(32)

It follows from (7), (17), and (25) that
\[ E = \frac{\partial P_r}{\partial \rho_{oir}} (-v_{t,0} \cos (\theta_{oir} - \phi_{oir}) + v_r \cos (\theta_r - \phi_{oir})) \]
\[ -k_o \Theta^2 (\theta_r - \phi_{oir}). \]  
(33)

Note that the set \( \{ (\rho_{oir}, \phi_{oir}, \theta_r) | \Theta (\theta_r - \phi_{oir}) \leq \frac{\pi}{2} \} \) is an invariant set (from (27)). Following from (33), \( E \) is non-increasing. Thereby we can come to the conclusion \( \rho_{oir}(t) > (R_{io} + R_f), \forall t \geq t_0 \) provided that \( \rho_{oir}(t_0) \geq D_0 + (R_{io} + R_f) \).

C. Switching Strategy

In the above two subsections, tracking control and avoidance control are suggested respectively. The switching strategy is that the avoidance control (25) is activated when \( \rho_{oir} \leq D_0 + (R_{io} + R_f) \); otherwise, the tracking control (10) is activated. It is obvious that the transient process will occur when \( \Theta (\phi_{oir} - \phi_{oir}) > \frac{\pi}{2} \) and \( \rho_{oir} \leq D_0 + (R_{io} + R_f) \). Similar to the local minima problem inherent in potential field methods, a natural problem of the proposed switching strategy is to identify whether the transient process will last forever. A graphical exploration is given below to address this issue.

As shown in Figure 4, to avoid the obstacle, the bold arrows represent the desired orientation angles when \( \rho_{oir} \leq D_0 + (R_{io} + R_f) \). Otherwise, the mobile robot tries to align its heading with the thin arrows which point to the target. Therefore, the worst case is \( \Theta (\phi_{oir} - \phi_{oir}) = \pi \) and \( \rho_{oir} \leq D_0 + (R_{io} + R_f) \). Considering the following simulation scenario in which \((x_r(t), y_r(t)) = (0, 0), (x_o(t), y_o(t)) = (15, 15), \) and \((x_g(t), y_g(t)) = (25, 25) \). The obstacle and target are static. The simulation parameters are the same as discussed in Section 5. As shown in Figure 5. The robot can make a detour and converge to the goal successfully.

Remark 2: We discuss the applicability of the proposed algorithms to the differential drive vehicle, whose control inputs are the angular velocities \( \omega_{R} \) and \( \omega_{L} \) of the right and left wheel respectively. Let \( r \) be the wheel radius and \( d \) be the axis length. Then a one-to-one mapping into the driving and steering velocities \( v_r \) and \( \omega_{t} \) is given by
\[ v_r = r (\omega_{R} + \omega_{L}) / 2, \quad \omega_{t} = r (\omega_{R} - \omega_{L}) / d. \]  
(34)

In view of the bounded velocity of the velocity of the motors, each wheel can achieve a maximum angular velocity \( \Omega \). Therefore, following from (34), to extend the controls (10) and (25) to differential drive vehicles boils down to the following condition:
\[ r (\omega_{R} + \omega_{L}) = 2 \cdot v_r; \quad r (\omega_{R} - \omega_{L}) \in [0, d \cdot \omega_{t}]. \]
which can be further simplified into
\[ v_r \geq \frac{d \cdot \omega_{t}}{2}; \quad \Omega \geq \frac{2v_r + d \cdot \omega_{t}}{2r}. \]

IV. SIMULATIONS

This section describes the simulation results of a unicycle robot. The parameters used for these simulations are: \( R_r = 1 \) m, \( R_s = 2 \) m, \( R_{io} = 1 \) m, \( k = k_o = \frac{3}{\pi} \), \( D = 0.1 \) m, and \( D_s = 1 \) m. In addition, the initial location of the vehicle is \((0, 0), \theta_r(0) = 0 \) rad, \( v_r(0) = 0.2 \) m/s, and \( \omega_t(0) = 0 \) rad/s. And the bounds on the linear velocity and angular velocity are \( v_r = 0.2 \) m/s and \( \omega_{t} = 0.6 \) rad/s.

The desired trajectory \((x_g(t), y_g(t))\) is given by
\[
\begin{align*}
x_g(t) &= \frac{\sqrt{2}}{2} x'(t) + \frac{\sqrt{2}}{2} \sin (0.15x'(t)) \\
y_g(t) &= \frac{\sqrt{2}}{2} x'(t) - \frac{\sqrt{2}}{2} \sin (0.15x'(t))
\end{align*}
\]  
(35)
where \( x'(t) = 0.09t \). It can be easily verified that \( v_r < 0.1 \) m/s.

1) Target Tracking and Collision Avoidance with Static Obstacles: There are three static obstacles \((4, 4), (4, 1), (15, 15), \) and \((25, 20, 1) \). The simulation result is shown in Figure 6.

\[1\text{Data format:(center position, radius). For example, (4, 4) denotes the center position. The radius is 1 m.}\]
When $T = 200$ s, the target falls behind the robot. Therefore, the conflict between target tracking and obstacle avoidance results in the circular-like movement of the unicycle robot.

2) Target Tracking and Collision Avoidance with Moving Obstacles: Compared with example 1, three moving obstacles of radius being 1 m are also considered. The simulation result is shown in Figure 7. In this simulation, $v_{ol}(t) = 0.1$ m/s.

V. CONCLUSIONS

In this paper, we proposed a systematic approach for unicycle-type robot to achieve multiple objectives for tracking virtual command vehicle and collision avoidance. Examples through simulation confirm the effectiveness of Lyapunov design of multi-objective control for the unicycle-type robot proposed in Section 3. Future research will consider more complex nonholonomic vehicle models to accommodate a larger class of mobile robots.