Decentralized Modular Control of Concurrent Fuzzy Discrete Event Systems

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Abstract—In order to analyze the event driven complex systems with uncertainties in their events and state transitions, Fuzzy Discrete Event Systems (FDES) has been proposed as an extension to the formal Discrete Event Systems theory (DES). In this paper we investigate the decentralized modular supervisory control problem of FDES with partial observation for systems which are composed of concurrently operating, multiple interacting modules with uncertainties in their events and states. The modular decentralized fuzzy supervisor consists of set of local fuzzy supervisors, one for each module and each with its own sensing and acting capabilities. Moreover, the communication is not allowed between the local fuzzy supervisors. The notion of separability for fuzzy languages is introduced and the property of a fuzzy language specification which is called separably – controllable – observable that is required for verifying the existence of modular fuzzy supervisors is defined. Some examples are presented to realize the theoretical developments.

I. INTRODUCTION

A complex event driven system which consists of multiple concurrently operating modules can be modeled by well-established Ramadge-Wonham framework of Discrete Event Systems (DES) [1]–[3]. The design of a monolithic, centralized supervisor for such a system suffers from state-explosion as mentioned in [4] when number of modules of the system increase. To reduce the space and time complexities of computation of control of complex large scale systems, decentralized modular supervisory control have been employed and several designs can be found in the literature [5]–[11]. In this approach one supervisor is assigned for each plant module and this supervisor depends only on the local module and the global specification which in turn avoids the state-explosion. When communication is allowed between local supervisors it is referred to as distributed modular control [10], [11]. Whereas in Decentralized modular control such communication is not required [5]–[9].

Most of the complex asynchronous systems suffer from uncertainty and vagueness when defining events and state transitions due to imprecision of sensors. As a result the crisp state specifications may not accurately represent the exact condition of the system at a given time. Hence the representation of events and states using possibility distributions is more appropriate for such scenarios. Extension of crisp DES theory to fuzzy DES (FDES) provides the flexibility to integrate associated uncertainties into events and state transitions [12], [13]. Supervisory control of FDES has been studied in [14] and [15] individually by extending the traditional supervisory control theory of DES. Both of these developments represent centralized versions of supervisory control under complete observation of events. Development of state feedback control for FDES is presented in [16] where authors discuss the stabilization of FDES. An extension of FDES theory involving partial observation of events is discussed separately in [17] and [18]. Note that in [15] controllability of events is considered as fuzzy where each event has a degree of being controllable, but the observability is crisp. In [14], [16], [18] although the controllability is not completely crisp, the observability is crisp. Also in [17] and [19] both observability and controllability are considered as fuzzy. The applications of FDES theory such as AIDS treatments [20], fault diagnosis in complex systems [21], behavior-based robot control [22] have proved its validity.

In this paper we discuss the decentralized modular control of FDES as an extension to its crisp version of DES, for concurrently operating interacting modules with associated uncertainties of their events and state transitions. Moreover, each fuzzy event is associated with a degree of controllability and a degree of observability as in [19] which represents more general FDES framework. We extend the notion of separability of crisp languages in DES to fuzzy languages in FDES. Then the property which is called separably – controllable – observable is defined in order to verify the existence of modular fuzzy supervisors for the system. Some examples are presented to clarify the theoretical developments.

The rest of the paper is organized as follows. In section II, we discuss some preliminaries of FDES with fuzzy observability and fuzzy controllability conditions and present fuzzy partially observation supervisory control of FDES. In section III, the decentralized modular control of concurrent FDES is established with some associated definitions. The conclusion and future research directions are presented in section IV and the proofs of the theorems are shown in Appendix.

II. PRELIMINARIES

Here we present some of the the basics of FDES theory. This includes the definitions and theories which is used to establish our decentralized modular supervisory control theory of FDES. Also some examples are presented for clarification purpose.

The fuzzy finite automaton is defined by the quadruple as \( \tilde{G} = (\tilde{Q}, \tilde{\Sigma}, \tilde{\delta}, \tilde{q}_0) \) where \( \tilde{Q} \) is set of fuzzy states, \( \tilde{\Sigma} \) represents the set of fuzzy events, \( \tilde{\delta} \) represents the transition mapping.
The transition mapping \( \delta \) is defined as: \( \delta(q, \sigma) = q \circ \sigma \).
Here "\( \circ \)" represents either Max-Min or Max-Product operation, which describes that \( G \) is modeled by either Max-Min automata or Max-Product automata respectively.

The physical possibility of a string of fuzzy events \( s \), which is made by continuation of \( n \) number of fuzzy events \( \sigma_1, \ldots, \sigma_n \), is given by (i.e. \( s = \sigma_1 \ldots \sigma_n \)):
\[
L_G(s) = L_G(\sigma_1) \ldots L_G(\sigma_n) \cap \ldots \cap L_G(\sigma_n)
\]
Here \( \cap \) represents the fuzzy-AND operation (taking minimum or product).

A fuzzy language which is generated by the corresponding fuzzy automaton is characterized by continuous occurrence of fuzzy events in the event set. For example:
\[
L = \frac{1}{\epsilon} + \frac{a_1}{\alpha} + \frac{a_2}{\beta} + \frac{a_3}{\gamma} + \frac{a_4}{\delta}
\]
Its Prefix-closure \( \tilde{L} \) is another fuzzy language which shows how the fuzzy events are continued. For example:
\[
\tilde{L} = \frac{\epsilon}{\epsilon} + \frac{a_8}{\alpha} + \frac{\beta}{\beta} + \frac{\gamma}{\gamma} + \frac{\delta}{\delta} + \frac{\alpha}{\alpha}
\]
Here \( \epsilon, \alpha, \beta, \gamma, \delta, \zeta \in \Sigma \) and \( \frac{\alpha}{\alpha} \) means the physical possibility of occurrence of \( \alpha \) is 0.8. Note that \( \tilde{L} \subseteq \tilde{L} \).
By definition \( \tilde{L} \) is a prefix-closed fuzzy language (i.e. \( \tilde{L} = \tilde{L} \)).

Assume a fuzzy sub language \( \tilde{L}_{\text{sub}} = \frac{a_2}{\beta} + \frac{a_3}{\Gamma} \) and \( \tilde{L}_{\text{sub}} \subseteq \tilde{L} \). Its Prefix-closure can be shown as follows:
\[
\tilde{L}_{\text{sub}} = \frac{\epsilon}{\epsilon} + \frac{a_8}{\alpha} + \frac{\beta}{\beta} + \frac{\gamma}{\gamma} + \frac{\delta}{\delta} + \frac{\alpha}{\alpha}
\]
Note that \( \tilde{L}_{\text{sub}} \subseteq \tilde{L} \Rightarrow \tilde{L}_{\text{sub}} \subseteq \tilde{L} \) and \( \tilde{L}_{\text{sub}}(s) \leq \tilde{L}(s) \).
It is obvious that these fuzzy languages obey the FDES properties mentioned above.

Let \( G_1 = (Q_1, \Sigma_1, \delta_1, \text{q}_0) \) and \( G_2 = (Q_2, \Sigma_2, \delta_2, \text{q}_0) \) be two fuzzy automaton. Their parallel composition, \( G_1 \parallel G_2 \) makes a new fuzzy automaton, which is given below [15].
\[
G_1 \parallel G_2 = (Q_1 \times Q_2, \Sigma_1 \cup \Sigma_2, \delta_1 \parallel \delta_2, \text{q}_0 \times \text{q}_0)
\]
Here "\( \parallel \)" operation denotes the tensor product of the two matrices. Using the events \( \sigma_1 \) and \( \sigma_2 \), an event in the combined system \( \sigma \) can be defined as follows:

\[
\sigma = \left\{ \begin{array}{ll}
\sigma_1 \otimes \sigma_2 & \text{if } \sigma \in \Sigma_1 \cap \Sigma_2 \\
\sigma_1 \otimes I_2 & \text{if } \sigma \in \Sigma_1 \setminus \Sigma_2 \\
I_1 \otimes \sigma_2 & \text{if } \sigma \in \Sigma_2 \setminus \Sigma_1
\end{array} \right.
\]
where \( I_1 \) and \( I_2 \) are unit matrices having the order of \( |Q_1| \) and \( |Q_2| \) respectively. For \( q_1 \otimes q_2 \in Q_1 \times Q_2 \) and \( \sigma \in \Sigma_1 \cup \Sigma_2 \), the transition mapping of the composition is given as:
\[
\delta_1 \parallel \delta_2(q_1 \otimes q_2, \sigma) = (q_1 \otimes q_2) \circ \sigma.
\]
Assume \( \Sigma_c \) and \( \Sigma_{uc} \) as fuzzy controllable event set and fuzzy uncontrollable event set respectively. Also \( \Sigma_o \) and \( \Sigma_{uo} \) as fuzzy observable event set and fuzzy unobservable event set respectively. All four of these are fuzzy subsets of \( \Sigma \).

We also assume that any fuzzy event \( \sigma \), \( (\bar{\sigma} \in \Sigma) \) is associated with a degree of being controllable \( (\Sigma_c(\sigma)) \) and a degree of being observable \( (\Sigma_o(\sigma)) \) as in [17], [19].
\[
\Sigma_c(\sigma) + \Sigma_{uc}(\sigma) = 1
\]
\[
\Sigma_o(\sigma) + \Sigma_{uo}(\sigma) = 1
\]

Hereafter, we do not distinguish fuzzy controllable events and fuzzy uncontrollable events separately. Each fuzzy event is associated with a degree of controllability and a degree of uncontrollability. Also each fuzzy event is associated with a degree of observability and a degree of unobservability. Note that hereafter, \( \bar{\sigma} \in \Sigma_c \) implies \( \Sigma_o(\bar{\sigma}) > 0 \).

To achieve desired behavior from an event driven asynchronous system which has uncertainties in its events and state transitions, the supervisory control of FDES has been discussed separately in [14] and [15]. Let \( S/\tilde{G} \) represents the fuzzy supervisor \( S \) controlling the FDES \( G \) and \( S/\tilde{G} \) is its corresponding fuzzy language. By extending the language generated by supervisor controlling the system for crisp DES presented in [3], we can define \( S/\tilde{G} \) recursively as follows:

1. \( \tilde{L}_{S/\tilde{G}}(\epsilon) = 1 \)
2. \( \tilde{L}_{S/\tilde{G}}(\tilde{s}) = \tilde{L}_{S/\tilde{G}}(\tilde{s}) \tilde{S}_{\tilde{G}}(\tilde{s}) \)

Where \( \tilde{S}_{\tilde{G}}(\tilde{s}) \) is the possibility of fuzzy event \( \tilde{s} \) being enabled by the fuzzy supervisor \( S \) after observing the fuzzy string \( \tilde{s} \). It is obvious that \( \tilde{L}_{S/\tilde{G}} \subseteq \tilde{L}_{\tilde{G}} \) and it is prefixed closed.

Note that contrast to the crisp languages in DES, hereafter we specify the fuzzy languages in FDES in their prefix-closed forms which show the way that how they are evolved over the time.

Let \( \tilde{G}_{m} \) be the prefix-closure of the fuzzy language of marked fuzzy strings of \( \tilde{G} \), which is used to represent the successfully completed operations of the system. A new fuzzy language \( \tilde{S}_{\tilde{G}} \), which is a sub language of \( \tilde{G}_{m} \) and contains marked fuzzy strings which survives under \( S/\tilde{G} \), can be achieved as follows:
\[
\tilde{L}_{S/\tilde{G}}(\tilde{s}) \subseteq \tilde{L}_{\tilde{G}} \cap \tilde{L}_{\tilde{G}_{m}}
\]

The possibility of fuzzy string \( \tilde{s} (\tilde{s} \in \tilde{G}_{m}) \) can be as follows:
\[
\tilde{L}_{S/\tilde{G}}(\tilde{s}) = \tilde{L}_{S/\tilde{G}}(\tilde{s}) \tilde{L}_{\tilde{G}_{m}}(\tilde{s}) \Rightarrow \tilde{L}_{S/\tilde{G}}(\tilde{s}) \leq \tilde{L}_{\tilde{G}_{m}}(\tilde{s})
\]

The fuzzy language \( \tilde{S}_{\tilde{G}} \) called "non-blocking" if it is exactly same to the prefix-closure of \( \tilde{S}_{\tilde{G}}(\tilde{s}) \) (i.e. \( \tilde{L}_{S/\tilde{G}} = \tilde{L}_{\tilde{S}_{\tilde{G}}(\tilde{s})} \)). This is can be further expressed as:

For any \( \tilde{s} \in \tilde{G}_{m} \): \( \tilde{L}_{S/\tilde{G}}(\tilde{s}) = \tilde{L}_{\tilde{S}_{\tilde{G}}(\tilde{s})} \)

Assume a fuzzy language specification \( \tilde{k} \) is given. Then \( \tilde{k} \) is said to be \( \tilde{L}_{\tilde{G}_{m}} \)-closed, if following inequality holds:
\[
\tilde{k}(\tilde{s}) \leq \tilde{L}_{\tilde{G}}(\tilde{s})
\]

Neglecting the observability issues of fuzzy events, the fuzzy controllability condition is given below [17].

Let \( \tilde{k} \) be the prefix closure of the fuzzy language \( \tilde{k} \subseteq \tilde{L}_{\tilde{G}} \). The physical possibility of fuzzy string \( \tilde{s} \tilde{\sigma} \) is given by \( \tilde{L}_{\tilde{G}}(\tilde{s} \tilde{\sigma}) \). Then \( \tilde{k} \) is said to be satisfying fuzzy controllability
condition with respect to $\bar{L}_G$ and $\bar{\Sigma}_{uc}$, if following inequality holds for any $\bar{s} \in \Sigma^*$ and $\bar{\sigma} \in \Sigma$,

$$k(\bar{s}) \cap \Sigma_{uc}(\bar{\sigma}) \cap \bar{L}_G(\bar{s}\bar{\sigma}) \leq k(\bar{s}\bar{\sigma})$$

**Definition 1:** The natural projection of $\bar{\sigma}$ is defined as:

$$\hat{P}(\bar{\sigma}) = \bar{\Sigma}_{uc}(\bar{\sigma}) \cdot \epsilon + \Sigma_{uc}(\bar{\sigma}) \cdot \bar{\sigma}$$

Intuitively, this means that the matrix representing the natural projection of $\bar{\sigma}$ can be achieved by multiplying each element of $"e"$ (an identity matrix) by $\Sigma_{uc}(\bar{\sigma})$ and add them together with the corresponding elements of the matrix which is made by multiplying each element of "$\bar{\sigma}$" (the event matrix) by $\Sigma_{uc}(\bar{\sigma})$.

It can be easily seen that when the unobservability of a fuzzy event $\bar{s}$ increases $\hat{P}(\bar{s})$ reaches to $\epsilon$. Then the supervisor of the closed loop system be likely unobserve $\bar{\sigma}$.

Also when the observability of the fuzzy event $\bar{s}$ increases the supervisor tends to observe $\bar{\sigma}$.

Assume $\hat{s} = \bar{\sigma}_1 \bar{\sigma}_2 \ldots \bar{\sigma}_n$. Let $\hat{P}(\hat{s}) = \bar{l}$ be the natural projection of $\hat{s}$. The following is obtained by considering the natural projection of each fuzzy event individually.

$$\hat{P}(\hat{s}) = \bar{l} = \hat{P}(\bar{\sigma}_1 \bar{\sigma}_2 \ldots \bar{\sigma}_n) = \hat{P}(\bar{\sigma}_1) \hat{P}(\bar{\sigma}_2) \ldots \hat{P}(\bar{\sigma}_n)$$

The fuzzy admissibility condition in [15], is extended by introducing partially observation supervisory control:

$$\Sigma_{uc}(\hat{\bar{\sigma}}) \cap \bar{L}_G(\bar{s}\bar{\sigma}) \leq \bar{S}^P(\hat{\bar{\sigma}})$$

Here $\bar{S}^P$ is the fuzzy partially observation supervisor, $\hat{P}(\hat{s}) = \bar{l}$ and $\bar{S}^P(\hat{\bar{\sigma}})$ is the possibility of $\hat{\bar{\sigma}}$ being enabled by $\bar{S}^P$ after observing fuzzy string $\bar{l}$.

The following can be derived from above:

$$\bar{L}_{\bar{S}^P/\bar{G}}(\bar{s}\bar{\sigma}) = \bar{L}_{\bar{S}^P/\bar{G}}(\bar{s}) \cap \bar{S}^P(\hat{\bar{\sigma}}) \cap \bar{L}_G(\bar{s}\bar{\sigma})$$

Where $\bar{L}_{\bar{S}^P/\bar{G}}$ is the fuzzy language generated by the fuzzy partially observation supervisor $\bar{S}^P$, controlling the system $\bar{G}$.

Assume $\hat{P}^{-1}(\hat{P}(\hat{s}))$ as a subset which gives the possibility of the natural projection of a fuzzy string, to be same as the natural projection of $\hat{s}$, (e.g. $\hat{P}^{-1}(\hat{P}(\hat{s})) = 1$).

Extending the fuzzy observability defined in [17] we can derive the following definition for fuzzy observability.

**Definition 2:** Let $k \subseteq L_G$, $\bar{s}^k \bar{\sigma} \in k$ and $\hat{s} \in \hat{P}^{-1}(\hat{P}(\hat{s}))$.

For any $\hat{s} \in \Sigma^*$ and $\bar{\sigma} \in \Sigma$, $\bar{\sigma}$ is said to be satisfying fuzzy observability condition with respect to $\bar{L}_G$, $\hat{P}$ and $\hat{\Sigma}_{uc}$, if following inequality holds:

$$k(\hat{s}) \cap \bar{L}_G(\bar{s}\bar{\sigma}) \cap \bar{k}(\hat{s}\bar{\sigma}) \cap \hat{P}^{-1}(\hat{P}(\hat{s})) \cap \hat{\Sigma}_{uc}(\hat{\bar{\sigma}}) \leq k(\hat{s}\bar{\sigma})$$

Intuitively, the possibility of fuzzy string $\bar{s}\bar{\sigma}$ belongs to $k$ is greater than or equal to the minimum (or product) of followings:

1. Possibility of $\bar{s}$ belongs to $k$
2. Physical possibility of $\bar{s}\bar{\sigma}$
3. Possibility of $\bar{s}\bar{\sigma}$ belongs to $k$
4. Possibility of $\hat{P}(\hat{s})$ to be same as $\hat{P}(\hat{s}')$
5. The degree of $\bar{\sigma}$ being controllable.

Similarly, assuming $\hat{P}(\hat{s}) = \bar{l}$ and $\bar{s}\bar{\sigma} \in \bar{k}$, an observable fuzzy supervisor can be defined as follow below for all $\bar{\sigma} \in \Sigma$:

$$\bar{S}^P(\hat{\bar{\sigma}}) \geq \bar{L}_G(\bar{s}\bar{\sigma}) \cap \bar{k}(\bar{s}\bar{\sigma}) \cap \hat{P}^{-1}(\hat{P}(\hat{s})) \cap \hat{\Sigma}_{uc}(\hat{\bar{\sigma}})$$

The terms have been described earlier.

**Definition 3:** Combining fuzzy admissibility condition and the above inequality for partial observation supervisory control, we can define the possibility of $\bar{\sigma}$ ($\bar{\sigma} \in \Sigma$) being enabled by $\bar{S}^P$ after observing $\bar{l}$, as follows (Assume $\hat{P}(\hat{s}) = \bar{l}$ and $\bar{s}\bar{\sigma} \in \bar{k}$).

Let $\mu_1 = \Sigma_{uc}(\bar{\sigma}) \cap \bar{L}_G(\bar{s}\bar{\sigma})$ and $\mu_2 = \bar{L}_G(\bar{s}\bar{\sigma}) \cap k(\bar{s}\bar{\sigma}) \cap \hat{P}^{-1}(\hat{P}(\hat{s})) \cap \hat{\Sigma}_{uc}(\hat{\bar{\sigma}})$.

For any $\bar{\sigma} \in \Sigma$,

$$\bar{S}^P(\hat{\bar{\sigma}}) = \begin{cases} 
\mu_1, & \text{if } \mu_1 \geq \mu_2 \text{ and } \mu_1 \geq k(\bar{s}\bar{\sigma}) \\
\mu_2, & \text{if } \mu_2 > \mu_1 \text{ and } \mu_2 \geq k(\bar{s}\bar{\sigma}) \\
k(\bar{s}\bar{\sigma}), & \text{otherwise}.
\end{cases}$$

Intuitively, this explains the possibility of a fuzzy event $\bar{s}$ being enabled by the partially observation supervisor $\bar{S}^P$, according to the possibility of $\bar{\sigma}$ being uncontrollable and the possibility of $\bar{\sigma}$ being controllable.

**Theorem 1:** Fuzzy controllability and Fuzzy observability Theorem:

There exists a non-blocking fuzzy partially observation supervisor $\bar{S}^P$ for the system $G$ such that $\bar{k}(\bar{s}) = \bar{L}_{G,\bar{P}}(\bar{s},\bar{m}(\bar{s}))$ and $\bar{L}_{G,\bar{P}}(\bar{G}) = \bar{k}(\bar{s})$ if and only if following conditions are hold:

1. $\bar{k}$ is fuzzy controllable with respect to $\bar{L}_G$ and $\bar{\Sigma}_{uc}$.
2. $\bar{k}$ is fuzzy observable with respect to $\bar{L}_G$, $\hat{P}$ and $\hat{\Sigma}_{uc}$.
3. $\bar{k}$ is $\bar{L}_{G,\bar{P}}$-closed.

Proof: See Appendix.

**Example 1:** Fig. 1 represents the fuzzy automaton which is used to model a system with two fuzzy states. Let $\bar{\Sigma}_{uc}(\bar{\alpha}) = 0.2$, $\bar{\Sigma}_{uc}(\bar{\beta}) = 0.3$, $\bar{\Sigma}_{uc}(\bar{\alpha}) = 0.2$ and $\bar{\Sigma}_{uc}(\bar{\beta}) = 0.1$.

![](image)

**Fig. 1:** A fuzzy automaton modeling a system with two fuzzy states

The fuzzy languages $\bar{L}_G$ and $\bar{k}$ are given by,

$$\bar{L}_G = \frac{1}{\epsilon} + \frac{0.8}{\alpha} + \frac{0.7}{\beta} + \frac{0.8}{\alpha\beta} + \frac{0.7}{\alpha\beta} + \frac{0.7}{\alpha} + \frac{0.7}{\beta} + \ldots$$

$$\bar{k} = \frac{1}{\epsilon} + \frac{0.3}{\alpha} + \frac{0.7}{\beta} + \frac{0.2}{\alpha\beta} + \frac{0.3}{\alpha\beta} + \frac{0.7}{\beta}$$

As stated earlier a fuzzy language specification in FDES must be given in its prefix-closed form which shows the possibility distribution of fuzzy strings of which the supervised system is required to achieve.

It is easy to verify by definitions that $\bar{k}$ is fuzzy controllable and fuzzy observable. Note that $\hat{P}^{-1}(\hat{P}(\epsilon)) = \bar{\Sigma}_{uc}(\bar{\alpha})$ and $\hat{P}^{-1}(\hat{P}(\epsilon'))(\bar{\beta}) = \bar{\Sigma}_{uc}(\bar{\beta})$.

The following $\bar{S}^P$ is capable of achieving $\bar{k}$.

$$\bar{S}^P_{\alpha} = \frac{k(\bar{s})}{\alpha} + \frac{k(\bar{\sigma})}{\beta} \Rightarrow 0.3 + 0.7$$

$$\bar{S}^P_{\beta} = \frac{\mu_1}{\alpha} + \frac{\mu_2}{\beta} \Rightarrow 0.3 + 0.7$$
III. DECENTRALIZED MODULAR CONTROL OF CONCURRENT FUZZY DISCRETE EVENT SYSTEMS

A modular decentralized supervisory control architecture for FDES is discussed in this section. Assume a group of $n$ modules concurrently processing and distributed over an area where each module has different sites and no communication is allowed between modules. The local behavior of the $i^{th}$ module is modeled by fuzzy automaton $\hat{G}_i$. The composite plant which represents the global behavior of all the modules, is modeled by parallel composition of all fuzzy automaton (i.e. $\hat{G}_1||\ldots||\hat{G}_n$). Fig. 2 shows the fuzzy automaton $\hat{G}_1$ and $\hat{G}_2$ used to model the local behaviors of two such modules.

Fig. 2. The fuzzy automaton $\hat{G}_1$ and $\hat{G}_2$

Fig. 3 shows the corresponding parallel composition used to model the behavior of the composite plant where $n = 2$.

As the centralized supervisory control of the composite system increases the complexity of supervisor synthesis, decentralized modular supervisory control is preferred. Let $\Sigma_i$ be the set of fuzzy events of $\hat{G}_i$ and $\Sigma$ be the set of fuzzy events of $\hat{G}_1||\ldots||\hat{G}_n$. It is known that $\Sigma = \Sigma_1\cup\ldots\cup \Sigma_n$. Note that the natural projection is defined here as $P_i : \Sigma^* \rightarrow \Sigma_i$ (i.e. each local supervisor sees its own set of fuzzy events).

By extending the modular decentralized control for DES discussed in [6], [7] to FDES, we can derive the fuzzy language generated by composite plant as a representation of the fuzzy languages generated by each local plant, which is shown as follows:

$$L_{\hat{G}_1||\ldots||\hat{G}_n} = P^{-1}_1(L_{\hat{G}_1}) \cap \ldots \cap P^{-1}_n(L_{\hat{G}_n}).$$

Note that the inverse projection of a fuzzy language $L$ (i.e. $P^{-1}(L)$) is a set of fuzzy languages that each natural projection is $L$. Let $P^{-1}(L)(s)$ be the possibility of the string $s$ belongs to the inverse projection of $L$. Then the above equality implies that for any $s \in \Sigma^*$,

$$L_{\hat{G}_1||\ldots||\hat{G}_n}(s) = P^{-1}_1(L_{\hat{G}_1})(s) \cap \ldots \cap P^{-1}_n(L_{\hat{G}_n})(s).$$

Intuitively, possibility of string $s$ belongs to fuzzy language generated by the composite plant is equal to the minimum of the possibilities of $s$ belong to inverse projections of each fuzzy language.

Extending the separability of the languages in DES presented in [5], [7], we can provide following definition for the separability of fuzzy languages in FDES.

**Definition 4:** Let $\Sigma = \Sigma_1 \cup \ldots \cup \Sigma_n$ and a fuzzy language $\tilde{k}$ ($\tilde{k} \subseteq \Sigma^*$) is said to be separable with respect to $\{\Sigma_1, \ldots, \Sigma_n\}$, if there exists a set of fuzzy languages $k_i$ ($k_i \subseteq \Sigma_i$) where $i \in \{1, \ldots, n\}$ for any $s \in \Sigma^*$ such that:

$$k(s) = P^{-1}_1(k_1)(s) \cap \ldots \cap P^{-1}_n(k_n)(s).$$

Intuitively, this says that if a fuzzy language is separable then its prefix-closure can be decomposed to several prefix-closed sub languages.

Note that in DES separability of a crisp language does not necessarily imply that its prefix-closure is also decomposable to several prefix-closed sub languages (refer Lemma 7 in [7]). But in FDES as it is required to specify a fuzzy language in its prefix-closure form, the notion of separability is discribed using prefix-closed fuzzy languages and fuzzy sub languages.

For example let $n = 2$, $\Sigma_1 = \{\tilde{a}, \tilde{b}\}$, $\Sigma_2 = \{\tilde{\alpha}, \tilde{\beta}\}$.

Assume followings,

$$\tilde{k_1} = \frac{1}{5} + \frac{0.7}{\tilde{a}} + \frac{0.5}{\tilde{b}},$$

$$\tilde{k_2} = \frac{3}{7} + \frac{0.3}{\tilde{a}} + \frac{0.6}{\tilde{b}}.$$

Let $\tilde{k} = \frac{1}{5} + \frac{0.7}{\tilde{a}} + \frac{0.5}{\tilde{b}} + \frac{0.3}{\tilde{a}} + \frac{0.6}{\tilde{b}} + \frac{0.3}{\tilde{a}} + \frac{0.3}{\tilde{a}} + \frac{0.3}{\tilde{a}} + \frac{0.3}{\tilde{a}} + \frac{0.6}{\tilde{b}} + \frac{0.6}{\tilde{b}} + \frac{0.5}{\tilde{b}}.$$

It is easy to verify that the language $\tilde{k}$ is separable with respect to $\{\Sigma_1, \Sigma_2\}$.

The distributed local knowledge of $\tilde{k}$ at $i^{th}$ module is represented by $\tilde{P}_i(\tilde{k})$. Followings are straightforward.

$$\tilde{P}_1(\tilde{k}) = \frac{1}{5} + \frac{0.7}{\tilde{a}} + \frac{0.5}{\tilde{b}} + \frac{0.3}{\tilde{a}} + \frac{0.3}{\tilde{a}} + \frac{0.6}{\tilde{b}}.$$

$$\tilde{P}_2(\tilde{k}) = \frac{1}{5} + \frac{0.3}{\tilde{a}} + \frac{0.6}{\tilde{b}} + \frac{0.5}{\tilde{b}}.$$

Let $\tilde{P}_i(\tilde{k})(s)_{\text{max}}$ represents the highest degree of possibility of $s$ which is belongs to $\tilde{P}_i(\tilde{k})$. For example,

$$\tilde{P}_1(\tilde{k})(\tilde{a})_{\text{max}} = 0.7, \tilde{P}_1(\tilde{k})(\tilde{b})_{\text{max}} = 0.5$$

$$\tilde{P}_2(\tilde{k})(\tilde{a})_{\text{max}} = 0.3, \tilde{P}_2(\tilde{k})(\tilde{b})_{\text{max}} = 0.6.$$

Note that $\tilde{k}_i = \tilde{P}_i(\tilde{k})$ when the condition $\tilde{k}_i(s)_{\text{max}} = \tilde{P}_i(\tilde{k})(s)_{\text{max}}$ is hold. Then separability indicates that the global fuzzy language specification $\tilde{k}$ can be recovered by combining all local fuzzy language specifications given by $\tilde{P}_1(\tilde{k})...\tilde{P}_n(\tilde{k})$ correspond to modules $\hat{G}_1...\hat{G}_n$, when the condition $\tilde{k}_i(s)_{\text{max}} = \tilde{P}_i(\tilde{k})(s)_{\text{max}}$ holds.

**Definition 5:** Consider $k_i \subseteq \Sigma_i$. Then a fuzzy language $\tilde{k}$
is said to be separably – controllable – observable with respect to $\cup_{i=1}^{n} \Sigma_i$ if following three conditions are satisfied.  
1. $k$ is separable with respect to $\{\Sigma_1, \ldots, \Sigma_n\}$. 
2. $k_i$ is fuzzy controllable. 
3. $k_i$ is fuzzy observable. 

Suppose a global fuzzy language specification $\tilde{k}$ for composite plant $\{\tilde{G}_1, \ldots, \tilde{G}_n\}$ is given and we want to verify whether there exist a set of modular partially observation supervisors $\{\tilde{S}_1^P, \ldots, \tilde{S}_n^P\}$ for concurrent systems $\{\tilde{G}_1, \ldots, \tilde{G}_n\}$ such that $L_{\tilde{S}_i^P/G_i}\{s\} = \tilde{k}(s) = k_i(s)$. 

The following theorem is presented for supervisory control of decentralized modular FDES. 

**Theorem 2**: Modular partially observation Supervisory Control Theorem for FDES: 

Assume a concurrent modular system $\{\tilde{G}_1, \ldots, \tilde{G}_n\}$ with local fuzzy event sets $\{\Sigma_1, \ldots, \Sigma_n\}$, with respective local projections $\{P_1, \ldots, P_n\}$, and a set of local fuzzy language specifications $k_i$ ($k_i \subseteq \Sigma_i^*$). Let $\Sigma = \Sigma_1 \cup \ldots \cup \Sigma_n$ and the global fuzzy language specification to be $k$ ($k \subseteq \Sigma^*$, $k \neq \emptyset$). 

Also assume the fuzzy events in $\Sigma_i$ are partially observable to $\tilde{S}_i^P$. There exists a set of modular partially observation supervisors $\{\tilde{S}_1^P, \ldots, \tilde{S}_n^P\}$ such that for any $s \in \Sigma^*$: 

$\tilde{L}_{\tilde{S}_i^P/G_i}\{s\} = \tilde{k}(s)$ and $\tilde{L}_{\tilde{S}_i^P/G_i}\{s\} = k_i(s)$, if and only if, 

1. $k$ is separably – controllable – observable
2. $k_i$ is $L_{\tilde{S}_i}\{s\}$-closed.

where $i \in \{1, \ldots, n\}$. 

Proof: See appendix. 

IV. CONCLUSION 

Modular control of decentralized discrete event systems where control is distributed between the set of local modules to avoid state space-explosion problem, represents control specification of the composite plant where group of modules working concurrently. As most real world scenarios are imprecise and vague in nature, events and state transitions of such a system can be better modeled by using FDES. 

In this paper we have established the decentralized modular control of FDES with a more general setting where each event has a degree of observability and a degree of controllability. The notion of separability is introduced for fuzzy languages and the concept of separately-controllably-observable is also introduced as an existence condition for modular decentralized fuzzy supervisors. 

This theoretical development still needs to validate experimentally by testing in real time distributed and modular plants with associated uncertainties in their events and states. Investigations on communication issues of decentralized modular control of FDES will be an interesting research area.

**APPENDIX I** 

**PROOF OF THEOREMS** 

**Theorem 1** proof. 

$\Rightarrow$ Consider the followings: 

1) The base case string of length $= 0$. By definition $L_{\tilde{S}_i^P/G_i}(\epsilon) = 1$, $\tilde{k}(\epsilon) = 1$. Assume the condition is true for fuzzy string $\tilde{s}$ and $|\tilde{s}| \leq n$. Then $L_{\tilde{S}_i^P/G_i}(\tilde{s}) = \tilde{k}(\tilde{s})$. 

2) Consider $\tilde{\sigma}$ as a non-null fuzzy event so that $|\tilde{s} \cdot \tilde{\sigma}| = n + 1$. 

We know: $L_{\tilde{S}_i^P/G_i}(\tilde{s}\tilde{\sigma}) = L_{\tilde{S}_i^P/G_i}(\tilde{s}) \cap L_{\tilde{S}_i^P/G_i}(\tilde{\sigma}) = \tilde{k}(\tilde{s}) \cap \tilde{k}(\tilde{\sigma}) \geq \bar{\tilde{k}}(\tilde{s}) \cap \bar{\tilde{k}}(\tilde{\sigma})$.

Assume $\tilde{s}\tilde{\sigma} \in \tilde{S}_i$ and $\tilde{\sigma} \in \Sigma$ and $P(\tilde{s}) = \tilde{\sigma}$. 

From **Definition 3**: 

1. Consider the first scenario where $\tilde{S}_i^P(\tilde{s}) = \tilde{S}_i^P(\tilde{s}) \cap \tilde{L}_G(\tilde{s})$. Substituting with above expression and fuzzy controllability condition yields to, 

$\Rightarrow L_{\tilde{S}_i^P/G_i}(\tilde{s}\tilde{\sigma}) = \tilde{k}(\tilde{s}) \cap \tilde{S}_i^P(\tilde{s}) \cap \tilde{L}_G(\tilde{s}) \leq \tilde{k}(\tilde{s})$. 

Consider the second scenario where $\tilde{S}_i^P(\tilde{s}) = \tilde{L}_G(\tilde{s}) \cap \tilde{k}(\tilde{s}) \cap \tilde{P}(\tilde{s}) \cap \tilde{G}(\tilde{s})$. Substituting with above expression and fuzzy observability condition yields to, 

$\Rightarrow L_{\tilde{S}_i^P/G_i}(\tilde{s}\tilde{\sigma}) = \tilde{k}(\tilde{s}) \cap \tilde{L}_G(\tilde{s}) \cap \tilde{k}(\tilde{s}) \cap \tilde{P}(\tilde{s}) \cap \tilde{G}(\tilde{s})$. 

Substituting as above we have, 

$\tilde{k}(\tilde{s}) \leq \tilde{k}(\tilde{s}) \cap \tilde{S}_i^P(\tilde{s}) \cap \tilde{L}_G(\tilde{s}) \Rightarrow \tilde{k}(\tilde{s}) \leq \tilde{L}_{\tilde{S}_i^P/G_i}(\tilde{s})$. 

Therefore we have proved that $\tilde{k}(\tilde{s}) = \tilde{L}_{\tilde{S}_i^P/G_i}(\tilde{s})$ holds for any $\tilde{s} \in \Sigma^*$ and $\tilde{\sigma} \in \Sigma$ where $|\tilde{s} \cdot \tilde{\sigma}| = n + 1$. This completes the proof of the induction step. 

Assume $L_{\tilde{S}_i^P/G_i}(\tilde{s}\tilde{\sigma}) = \tilde{k}(\tilde{s})$. 

1) Proof of fuzzy controllability condition holds: 

We know $L_{\tilde{S}_i^P/G_i}(\tilde{s}) = L_{\tilde{S}_i^P/G_i}(\tilde{s}) \cap \tilde{S}_i^P(\tilde{s}) \cap \tilde{L}_G(\tilde{s})$. 

With fuzzy admissibility condition this yields to, 

$\Rightarrow L_{\tilde{S}_i^P/G_i}(\tilde{s}) \geq \tilde{L}_{\tilde{S}_i^P/G_i}(\tilde{s}) \cap \tilde{S}_i^P(\tilde{s}) \cap \tilde{L}_G(\tilde{s})$. Also we know that $\tilde{L}_{\tilde{S}_i^P/G_i}(\tilde{s}) = \tilde{k}(\tilde{s})$ and $L_{\tilde{S}_i^P/G_i}(\tilde{s}) = \tilde{k}(\tilde{s})$. 

Therefore we have proved that $\tilde{k}(\tilde{s}) = \tilde{L}_{\tilde{S}_i^P/G_i}(\tilde{s})$. 

The fuzzy controllability condition holds.
2) Proof of fuzzy observability condition also holds:

Let \( \tilde{L}_{S/G} (\tilde{s}) \cap \tilde{S}_P \cap \tilde{G}(\tilde{s}) \). From the definition of the observable fuzzy supervisor, \( \tilde{S}_P (\tilde{s}) \leq \tilde{L}_G (\tilde{s}) \). Consequently, fuzzy observable and \( \tilde{G} \). This completes the proof.

3) To prove \( \tilde{L}_{G,m} \) closure also holds let's extend the definition of \( \tilde{L}_{S/G} \) for partial observation scenario:

\[
\tilde{L}_{S/G} (\tilde{s}) = \tilde{L}_{S/G} (\tilde{s}) \cap \tilde{G}, \tilde{m} (\tilde{s})
\]

Then \( \tilde{k}(\tilde{s}) \) is separable.

Since \( \tilde{L}_{S/G} \) is the language generated by partially observation supervisor \( \tilde{S}_P \) for the plant \( \tilde{G} \), it is fuzzy controllable, fuzzy observable and \( \tilde{G}, \tilde{m} \)-closed. So \( \tilde{k} \) is fuzzy observable, fuzzy observable and \( \tilde{G}, \tilde{m} \)-closed according to Theorem 1, because \( \tilde{k}(\tilde{s}) = \tilde{L}_{S/G} (\tilde{s}) \) and \( \tilde{k}(\tilde{s}) = \tilde{L}_{S/G} \).

\[
\tilde{k}(\tilde{s}) = \tilde{L}_{S/G} (\tilde{s}) \cap \tilde{G}, \tilde{m} (\tilde{s})
\]

This completes the proof.

REFERENCES


