Optimal Target Tracking Strategy with Controlled Node Mobility in Mobile Sensor Networks

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Abstract—This paper is concerned with target tracking using a network of collaborative sensors. The objective is to compute (online) the desired sensing and communication radii of sensors as well as their location at each time instant, such that a set of prescribed specifications are achieved. These specifications include end-to-end connectivity preservation from the target to a fixed destination, while durability of sensors is maximized and the overall energy consumption is minimized. The problem is formulated as a constrained optimization, and a procedure is presented to solve it. Simulation results demonstrate the effectiveness of the proposed techniques.

I. INTRODUCTION

Sensor networks have been envisioned as a means for gathering, processing and delivering information about the physical environment to the intended recipient(s). This type of systems have attracted much attention in both control and communication research communities in recent years [1], [2], [3]. A Mobile Sensor Network (MSN) is typically comprised of wireless mobile nodes equipped with battery-powered sensors. Such networks are known to be very effective in detecting and tracking dynamic targets, and have important civilian and military applications [4], [5].

In a MSN, each sensor communicates with a subset of sensors and uses a proper movement strategy in order to achieve certain objectives such as tracking a moving target with a trajectory which is not know a priori. The information exchange between the sensors and a proper algorithm to use the collected information in order to effectively relocate the mobile sensors are the two important components of any MSN control scheme. These components along with the capabilities of the individual sensors (in terms of power, communication range and displacement flexibility) determine the efficacy of the MSN in achieving any desired objective [6].

Recent developments in MEMs technology have provided a wealth of cheap, customizable, and embedded sensor systems capable of performing wireless communication among each other. There has been a burst of research activities in cross-layer network optimization in recent years, involving routing, flow and power control, and packet scheduling [7], [8]. The mathematical framework for the underlying optimization is based on the concept of elastic users and corresponding aggregate utility maximization; for instance, see the framework given in [9] in the context of network management. Price-based distributed algorithms concerning utility maximization for a wire-line network were developed in [10]. These algorithms assume that elastic users respond to congestion pricing signals by modifying their requirements for the bandwidth. More recent papers such as [11], [12], extended the price-based algorithms to a wireless environment. Note that wireless networks have numerous advantages in sensor applications, due mainly to the distributed nature of this type of systems.

Despite recent successes in developing efficient utility maximization algorithms, numerous issues remain open. For instance, providing diverse quality of service for different users, effective decentralization of optimal power control and packet scheduling, and most important of all, developing simple distributed algorithms to achieve robust performance in partially known environments are some of the areas that require further research. Different objective functions are introduced in the literature to evaluate the performance of the network.

In this paper, a routing strategy is presented for the relocation of mobile sensors in a network and the adjustment of their communication and sensing distance, such that a certain cost function is minimized, while the end-to-end connectivity from the target to the fixed access point (or destination point) is maintained. Various cost functions concerning individual sensors and the entire network will be considered to evaluate the performance of the network in terms of power consumption. A technique is also provided to maximize the durability of the whole network by monitoring the residual energy of individual sensors and adjusting their parameters accordingly. Simulation results elucidate the desirable characteristics of the proposed methods.

The plan of the rest of the paper is as follows. The problem is formulated in Section II, and some important assumptions and definitions are also provided which will be used later to develop the main results. An algorithm and some important theorems and lemmas are presented in Section III, as the main contributions of the paper for solving the underlying constrained and unconstrained optimization problems. Simulations are given in Section IV, and finally the concluding remarks are summarized in Section V.

II. PROBLEM FORMULATION

Consider a group of $n$ mobile sensors, each one representing a node in the sensor network, and let the coordinates
of sensor $i, i \in n := \{1, \ldots, n\}$, be denoted by $x_{si}$. Consider also a moving target and a fixed access point (also referred to as destination point), where for convenience of notation they are labeled as node 0 and node $n+1$, accordingly. In order to ensure target tracking at all times, it is required to maintain connectivity from the target to the access point continuously (in terms of sensing and communication). Furthermore, in order to accomplish the mission in the most efficient manner, it is desired that the routing cost defined as the sum of the costs (associated with the sensing and transmission power needed to communicate over the link) of each sensor involved in establishing a connected link from the target to the destination point is minimized.

One of the most desirable control strategies in MSNs is the energy-efficient strategy [5], [6]. In general, energy consumption of mobile sensors is due to communication, sensing, and movement. The optimal energy-efficient control action depends on which one of the above-mentioned energy-consuming factors is dominant [5], [13], [14].

Assumption 1. In this paper, it is assumed that the power consumption of the sensors due to movement is negligible compared to that due to communication and sensing. This is a realistic assumption for the case where the sensors are mounted on mini-wheel robots which move in an environment with approximately pure rolling [15], or the case where the target remains still for a relatively long time (and hence the sensors do not require to follow the target continuously).

In order to formulate the optimization problem, it is assumed that a link $l = (i, j)$ from node $i \in n \cup \{0\}$ to node $j \in n \cup \{n+1\} \setminus \{i\}$ exists if and only if the corresponding signal-to-interference ratio SIR exceeds certain (strictly positive) threshold $\chi$. This can be mathematically expressed as:

$$SIR_{ij} = \frac{P_{ij} \xi_{ij}}{\eta_j + \sum_{(n,k) \neq (i,j), n \neq i,j} P_{nk} \xi_{nj}} > \chi \quad (1)$$

where $P_{ij}$ is the power required to transmit information from node $i$ to node $j$, $\xi_{ij}$ is the path loss (which is defined as the reduction in power density of an electromagnetic wave as it propagates through space), from node $i$ to node $j$, $\sum_{(n,k) \neq (i,j), n \neq i,j} P_{nk} \xi_{nj}$ is the overall interference power, and $\eta_j$ is the noise power at node $j$. For simplicity, assume that the interference power is negligible, and that the noise power $\eta_j$ is equal to 1, for all $j \in n$. Then, using (1) one can find the following minimum power consumption by node $i$ for direct communication with node $j$:

$$P_{ij} = \frac{\chi}{\xi_{ij}}$$

The path loss is inversely proportional to some power of the distance $d_{ij}$ between nodes $i$ and $j$, i.e. $d_{ij}^\lambda$, for all $i, j \in n$, $i \neq j$. The potent $\lambda$ is between 2 and 4, and is closer to 4 for low-lying antennae and near-ground channels, as in typical sensor network communication [4], [16]. The communication radius of sensor $i$ at the instant $t$, denoted by $R_{ci}(t)$, is equal to the radius of the largest circle around $x_t$, such that the corresponding SIR from $x_t$ to any point inside the circle is greater than the threshold $\chi$.

On the other hand, the power required for sensing from a given distance is typically greater than the power required for communicating from the same distance. The power required for sensing from the distance $d$ is typically proportional to $d^\gamma$, where $\gamma \geq 2$, and in particular in a radar system $\gamma \geq 4$.

Assumption 2. In this paper, it is assumed that one sensor is assigned to sense the target at any given time, which will be referred to as the tracking sensor. The information is transmitted from the tracking sensor to the fixed destination point through a subset of the remaining sensors in a collaborative fashion. The tracking sensor is not fixed in general, and can be changed from time to time depending on the position of the sensors and their residual energies.

Let the tracking sensor be labeled as sensor 1 throughout the paper. The sensing radius of sensor 1 at the instant $t$, denoted by $R_{s1}(t)$, is defined as the radius of the largest circle around $x_1$, such that this sensor can sense the target anywhere inside the circle. Note that sensor $i$ can transmit the information to sensor $j$ at the instant $t$ if and only if $R_{ci}(t) \geq d_{ij}(t)$, for all $i, j \in n$. Note also that sensor 1 can sense the target at the instant $t$ if and only if $d_{10}(t) \leq R_{s1}(t)$, where $d_{10}$ denotes the distance between sensor 1 and the target.

Assumption 3. It is assumed that the target is in a reachable distance from the destination point through other sensors at all times, i.e. $x(t) \leq nR_{c,max} + R_{s,max}$, where $R_{c,max}$ is the maximum communication radius that can be covered by each sensor, $R_{s,max}$ is the maximum sensing radius that can be detected by sensor 1, and $x(t)$ is the distance between the target and destination.

Recall that the required powers for sensor $i$ ($i \in n$) to communicate information and for sensor 1 to detect information are proportional to $R_{ci}^\lambda$ and $R_{s1}^\lambda$, respectively. On the other hand, by assumption the movement power is negligible compared to the above-mentioned communication and sensing powers. Thus, for any $i, j \in \{2, \ldots, n\}:

$$\frac{\text{power consumption of sensor } i}{\text{power consumption of sensor } j} = \frac{R_{ci}^\lambda}{R_{cj}^\lambda} \quad (2)$$

and the following cost function (which reflects the overall instantaneous power consumed by all sensors) is to be minimized at any time $t > 0$:

$$J_P(t) = \alpha R_{s1}(t) + \sum_{i \in k(t)} R_{ci}^\lambda(t) \quad (3)$$

subject to the condition $R_{s1}(t) + \sum_{i \in k(t)} R_{ci}(t) \geq x(t)$ for some $k(t) \subset n$. Furthermore, the constraints given below need to be satisfied for all $t > 0$:

1. $0 \leq R_{ci}(t) \leq R_{c,max}$, $\forall i \in k(t)$
2. $0 \leq R_{s1}(t) \leq R_{s,max}$

where $\alpha$ is a constant coefficient used to normalize the sensing power with respect to the communication power.
While minimizing power consumption is of great importance in MSNs, in many applications it is more desirable that the sensor with the smallest residual energy consume the smallest amount of power at each instant, in order to maximize the lifespan of the sensors, and hence the durability of the overall network. In this type of applications, the following performance index is used instead of (3):

$$J_D(t) = \alpha \theta_1(t) R_{s1}^\lambda(t) + \sum_{i \in k(t)} \theta_i(t) R_{ci}^\lambda(t)$$  \hspace{1cm} (4)

subject to the condition $R_{s1}(t) + \sum_{i \in k(t)} R_{ci}(t) \geq x(t)$, for any time $t > 0$, where $\theta_i$’s are strictly positive weighting functions chosen appropriately to make the power consumption of each sensor consistent with the corresponding residual energy, and $\theta_1$ is the weighting function for the sensing power of sensor 1 (which is, by assumption, assigned to sense the target).

**Definition 1.** Given a MSN satisfying Assumptions 1 and 3:

- The minimization problem with the performance index $J_P$ and the constraints (i) and (ii) will hereafter be referred to as constrained power optimization problem (CPOP), and the corresponding minimum cost will be denoted by $J_P^*$. 
- The minimization problem with the performance index $J_P$ and relaxed constraints (i), (ii) will hereafter be referred to as unconstrained power optimization problem (UPOP), and the corresponding minimum cost will be denoted by $J_P^\prime$. 
- The minimization problem with the performance index $J_D$ and the constraints (i) and (ii) will hereafter be referred to as constrained durability optimization problem (CDOP), and the corresponding minimum cost will be denoted by $J_D^*$. 
- The minimization problem with the performance index $J_D$ and relaxed constraints (i), (ii) will hereafter be referred to as unconstrained durability optimization problem (UDOP), and the corresponding minimum cost will be denoted by $J_D^\prime$. 

**Definition 2.** Consider the weighting functions $\theta_i(t)$ in (4), and let $m \in \mathbb{n}$ be a given integer. The following definition will prove convenient in the development of the main results:

$$\sigma_m(t) = \sum_{i=1}^{m} \left( \theta_i(t) \right)^{\frac{1}{\lambda}}$$  \hspace{1cm} (5)

**Remark 1.** Note that since $\theta_i$ is strictly positive for all $i \in \mathbb{n}$, it is straightforward to conclude that $\sigma_{j+1}(t) > \sigma_j(t)$, for all $j \in \{1, \ldots, n-1\}$.

Before proceeding to the next section, let *Holder’s inequality* [17] (which will be used later to prove some of the results) be introduced here.

*Holder’s Inequality:* Given the strictly positive parameters $p$ and $q$ with the property $\frac{1}{p} + \frac{1}{q} = 1$, then the following inequality:

$$\sum_{i=1}^{n} |x_i y_i| \leq \left( \sum_{i=1}^{n} |x_i|^p \right)^{\frac{1}{p}} \left( \sum_{i=1}^{n} |y_i|^q \right)^{\frac{1}{q}}$$  \hspace{1cm} (6)

holds for all $(x_1, x_2, \ldots, x_n), (y_1, y_2, \ldots, y_n) \in \mathbb{C}^n$.

In the next section, effective techniques are proposed to solve the optimization problems introduced earlier at any given time $t$. For convenience of notation, the time argument from the variables $x, R_{s1}, R_{ci}, \sigma_i, \theta_i$ and the set $k$ will be omitted in the development of the main results.

**III. MAIN RESULTS**

Consider a MSN satisfying Assumptions 1, 2 and 3. The following lemma is used to solve UDOP.

**Lemma 1.** Let $J_D$ be the minimum cost in UDOP after setting the sensing radius of sensor 1 to $R_{s1}$. Then:

$$J_D = \frac{(x - R_{s1})^\lambda}{\sigma_n^{\lambda-1}} + \alpha \theta_{s1} R_{s1}^\lambda$$  \hspace{1cm} (7)

**Proof:** Choose $x_1 = \theta_1^\frac{1}{\lambda} R_{s1}, y_1 = \theta_1^{-\frac{1}{\lambda}}, p = \lambda$ and $q = \frac{\lambda}{\lambda - 1}$. Then, using Holder’s inequality along with Remark 1 (and on noting that $\exists k \subset \mathbb{n}$, such that $R_{s1} + \sum_{i \in k} R_{ci} \geq x$), one can write:

$$\sum_{i \in k} \theta_i R_{ci}^\lambda \geq \frac{\left( \sum_{i \in k} R_{ci} \right)^\lambda}{\sigma_k^{\lambda-1}} \geq \frac{\left( \sum_{i \in k} R_{ci} \right)^\lambda}{\sigma_n^{\lambda-1}} \geq \frac{(x - R_{s1})^\lambda}{\sigma_n^{\lambda-1}}$$

for some $k \subset \mathbb{n}$. It follows immediately from (6) that $J_D \geq \frac{(x - R_{s1})^\lambda}{\sigma_n^{\lambda-1}} + \alpha \theta_{s1} R_{s1}^\lambda$, and that the inequality turns to an equality by choosing $R_{ci} = \frac{(x - R_{s1})}{\theta_i^{\frac{1}{\lambda}} \sigma_n}$. This completes the proof.  \hspace{1cm} •

**Lemma 2.** The function

$$f(R_{s1}) = \sigma_n^{\lambda-1} \alpha \gamma \theta_{s1} R_{s1}^\gamma - \lambda (x - R_{s1})^{\lambda-1}$$  \hspace{1cm} (8)

has exactly one real-positive root over $[0, x]$.

**Proof:** The proof is omitted due to space restrictions and may be found in [18].  \hspace{1cm} •

**Definition 3.** Let the sensing radius and communication radii obtained by solving the UDOP be denoted by $R_{s1}^*$ and $R_{ci}^*$, $i \in \mathbb{n}$, respectively. Furthermore, denote the sensing radius and communication radii obtained by solving the CDOP with $R_{s1}^\prime$ and $R_{ci}^\prime$, $i \in \mathbb{n}$, respectively.

**Theorem 1.** The solution of the UDOP is unique, and is characterized by:

$$f(R_{s1}^*) = 0, \quad R_{ci}^* = \frac{(x - R_{s1}^*)}{\theta_i^{\frac{1}{\lambda}} \sigma_n}, \quad i \in \mathbb{n}$$
Proof: The proof follows by taking derivative of $J_D$ and using the results of Lemmas 1 and 2.

Remark 2. It is implied from Theorem 1 and the proof of Lemma 1 that $k = n$, in the solution of UDOP. Furthermore, the sum of the sensing radius of sensor 1 and the communication radii of all sensors at any instant time is equal to the distance between the target and destination point at that time. This means that in the optimal strategy all sensors will be functional, and will be located on distinct points on the straight line connecting the target to the destination. It is also implied that $\theta_j R_j^{\lambda-1} = \theta_i R_i^{\lambda-1}, \forall i, j \in n$. In other words, the solution of UDOP has the following property:

$$\frac{\theta_j}{\theta_i} = \frac{R_j^{\lambda-1}}{R_i^{\lambda-1}}, \forall i, j \in n \quad (8)$$

From Lemma 2, Theorem 1, and the uniqueness of the solution of UDOP, one can conclude that

$$\sigma_{n-1}^{\lambda-1} \alpha \gamma_1 (R^*_n)^{\gamma-1} = \lambda (x - R^*_n)^{\lambda-1} \quad (9)$$

$$(x - R^*_n)^{\lambda-1} = (x - R^*_n - R^*_s)^{\lambda-1} \quad (10)$$

$$\frac{\theta_i^{\lambda-1} \sigma_n}{\theta_1^{\lambda-1} \sigma_1} = \frac{\theta_i^{\lambda-1} \sigma_n}{\theta_i^{\lambda-1} \sigma_{n-1}} \quad (11)$$

where $R^*_n$ in (10) and (11) is the optimal value for the communication radius of sensor $n$ in the UDOP, and $R^*_s$ is the optimal sensing radius for sensor 1, as introduced in Theorem 1.

Lemma 3. Consider a MSN with the optimal parameters $R^*_i$, $i \in n$, introduced in Definition 3. These optimal parameters have the following properties at all times:

i) if $\theta_i = \theta_j$, then $R^*_i = R^*_j$;

ii) if $\theta_i < \theta_j$, then $R^*_i \geq R^*_j$;

iii) if $\theta_i > \theta_j$, then $R^*_i \geq \theta_i R^*_j$;

iv) if $\theta_i < \theta_j$ and $\theta_i R^*_j < \theta_i R^*_i$, then $R^*_i = R^*_i$.

Proof: The proof is omitted due to space restrictions and may be found in [18].

Definition 4. Define $R^*_i$, $i \in n$, as the new communication radii obtained by solving the UDOP after setting the sensing radius of sensor 1 to $R^*_1$. Let $q$ and $l$ be the indices of the smallest and largest $\theta_i$, $i \in n$, i.e. $\theta_q = \min_{i \in n} \theta_i$ and $\theta_l = \max_{i \in n} \theta_i$.

Remark 3. It can be concluded from Lemma 3 that $R^*_q \geq R^*_l$ for all $i \in n$. Also, from Remark 2, $R^*_i \geq R^*_s$ for all $i \in n$.

The following two lemmas will be used in the proof of Theorem 2.

Lemma 4. Consider a MSN with the optimal parameters $R^*_i$, $i \in n$, introduced in Definition 4. If $R^*_q > R^*_s$, then $R^*_q = R^*_s$.

Proof: It is known that $\sum_{i=1}^{n} R^*_i = \sum_{i=1}^{n} R^*_i = x - R^*_s$, and that $R^*_q \leq R^*_s < R^*_q$. This implies that $\exists z \in n$ such that $R^*_z > R^*_z$. Now, it follows from Remark 2 that $\theta_q R^*_q \lambda-1 < \theta_q R^*_q \lambda-1 = \theta_l R^*_l \lambda-1 < \theta_l R^*_l \lambda-1$, and hence part (iv) of Lemma 3 yields that $R^*_q = R^*_s$.

Remark 4. It is straightforward to show that if $R^*_q \leq R^*_s$, then $R^*_q = R^*_s$, $\forall i \in n$. Therefore, for the case when $R^*_q = R^*_s$, one can conclude that $R^*_q = R^*_s$.

Lemma 5. Consider a MSN with the optimal parameters $R^*_s$, $R^*_i$, $i \in n$, introduced in Definition 4. If $R^*_s > R^*_s$, then $R^*_q \geq R^*_s$.

Proof: The proof is omitted due to space restrictions and may be found in [18].

Theorem 2. Consider the UDOP and assume that $R^*_q \geq R^*_s$, then $R^*_q = R^*_s$.

Proof: If $R^*_q > R^*_s$, then according to Lemma 5 $R^*_q \geq R^*_s$. Thus, it results from Lemma 4 and Remark 4 that $R^*_q = R^*_q$. If, on the other hand, $R^*_q \leq R^*_s$, then it follows from Remark 2 that $R^*_q \geq R^*_q \geq R^*_q$. One can therefore conclude that $R^*_q = R^*_q$.

Lemma 6. Consider the UDOP and set the communication radii of all but one sensor, say sensor $n$, to $R^*_c, i \in \{1, \ldots, n-1\}$. Let the communication radius of sensor $n$ be chosen as $R^*_n < R^*_c$, and solve the new UDOP (with the above-mentioned parameter setting) for $n-1$ remaining sensors with $x_{new} = x - R^*_n$. Then the new optimal radii $R^*_s$ and $R^*_i$ have the following properties:

i) $R^*_s > R^*_1$;

ii) $R^*_i > R^*_i, i = 1, \ldots, n-1$.

Proof: The proof is omitted due to space restrictions and may be found in [18].

The following lemma is the key to prove one of the important features of the UDOP.

Lemma 7. Consider the UDOP and assume that the optimal communication radii of $k$ sensors (say, sensors $n - k + 1, \ldots, n$) are greater than or equal to $R_{n-1}$. Set $R^*_c = R^*_c$, for all $i \in \{n - k + 1, \ldots, n\}$, and solve the new unconstrained optimization problem for $n - k$ sensors and $x_{new} = x - k R^*_c$. Then $R^*_s \geq R^*_s$, where $R^*_s$ is the optimal sensing radius of sensor 1 in the new unconstrained optimization problem setting.

Proof: The proof is omitted due to space restrictions and may be found in [18].

Theorem 3. Consider the UDOP and assume that $R^*_s > R^*_s$; then $R^*_s = R^*_s$.

Proof: Assume that $R^*_s < R^*_s$. Let $R^*_c = R^*_c$. If $R^*_c > R^*_c, i = n - k + 1, \ldots, n$ and $R^*_c < R^*_c, i = 1, \ldots, n - k$. Let $g$ be the index of the smallest $\theta_i, i \in \{1, \ldots, n - k\}$. From parts (i) and (ii) of Lemma 3, it can be deduced that $R^*_q = \max_{i \in \{1, \ldots, n-k\}} R^*_c$ since $R^*_c < R^*_c, \forall i \in \{1, \ldots, n - k\}$, and according to Theorem 2 (for the new unconstrained optimization problem with $x_{new} = x - k R^*_c$ and $n - k$ sensors to be optimized) it is known
that $R_{cg}^* < R_{c,max}$. Therefore, it can be concluded from Remark 2 that $R_{ci}^* < R_{c,max}, \forall i \in \{1, \ldots, n-k\}$. Hence, in the new problem setting the constrained and unconstrained optimizations both lead to the same result for the $n-k$ sensors. Thus, from Lemma 1:

$$J^* = \frac{(x - k R_{c,max} - \tilde{R}_{ci}^*)^\lambda}{(\sigma_{n-k})^{\lambda-1}} + \alpha \theta_{s1} \tilde{R}_{s1}^* \gamma$$

$$+ R_{c,max}^\lambda \sum_{i=n-k+1}^{n} \theta_i$$

(12)

Define:

$$T := \frac{1}{2} \min \{R_{s,max} - \tilde{R}_{s1}^*, \theta_i \frac{\lambda}{\lambda-1} \sigma_{n-k} \ (R_{c,max} - \tilde{R}_{cg})\}$$

Note that $T$ is a positive value. Consider now the following values for the communication and sensing radii for an arbitrary $\beta \in (0, T)$:

$$\tilde{R}_{s1}^* = \tilde{R}_{s1} + \beta$$

$$\tilde{R}_{ci} = R_{c,max}, \ i = n-k+1, \ldots, n$$

As a result:

$$j^* = \frac{(x - k R_{c,max} - \tilde{R}_{ci}^* - \beta)^\lambda}{(\sigma_{n-k})^{\lambda-1}} + \alpha \theta_{s1} (\tilde{R}_{s1}^* + \beta) \gamma$$

$$+ R_{c,max}^\lambda \sum_{i=n-k+1}^{n} \theta_i$$

(13)

Define $\Delta J(\beta) = J^* - J^*$; since $\beta \in (0, T)$, thus $\tilde{R}_{s1} + \beta < R_{s,max} \leq \tilde{R}_{s1}$, and According to Lemma 7, $R_{s1}^* \leq \tilde{R}_{s1}^*$. As a result, $\tilde{R}_{s1} + \beta < \tilde{R}_{s1}^*$. Furthermore, it can be shown (by using Lemma 2 and Theorem 1) that $\tilde{R}_{s1}^*$ is the unique positive real root of the following equation over the interval $[0, x - k R_{c,max}]$:

$$f(R) = \frac{\lambda}{\lambda-1} \alpha \gamma \theta_{s1} R^\gamma - 1 - \lambda (x - k R_{c,max} - R)^{\lambda-1}$$

(14)

where $f$ is the dual of the function $f$ (introduced in Lemma 2) for the newly defined optimization problem. On the other hand, one can show that $\tilde{f}(R)$ in (14) is strictly increasing with respect to $R$ over $[0, x_{new}]$. Therefore, $f(\tilde{R}_{s1}^* + \beta) < f(\tilde{R}_{s1}^*) = 0$. By taking the derivative of $\Delta J(\beta)$ with respect to $\beta$ and considering the inequality $f(\tilde{R}_{s1}^* + \beta) < 0$, one can verify that $\frac{d \Delta J}{d \beta} > 0$, and hence from $\Delta J(0) = 0$ it can be deduced that $J^* > J^*$ which contradicts the minimality of $J^*$ in (12). This means that

$$\tilde{R}_{s1}^* = R_{s,max}$$

The following algorithm can be used to solve the underlying constrained optimization problem systematically, in order to find the optimal communication and sensing radii at all times.

**Algorithm 1.**

1) Choose $\zeta = n$

2) Sort the $\theta_i$’s in a descending order, and let them be represented as $\theta_{i1} \geq \theta_{i2} \geq \ldots \geq \theta_{in}$

3) Find the real positive root of the following equation (with respect to $R$) over $[0, x]$, and denote it with $R_p$:

$$\left[ \sum_{i=1}^{\zeta} \left( \frac{1}{\theta_i} \right) \right]^{\lambda-1} \alpha \gamma \theta_{s1} R^\gamma - 1 - \lambda (x - R)^{\lambda-1} = 0$$

4) Set $\omega = 0$

5) Set $\tilde{R}_{s1}^* = \min \{R_p, R_{s,max}\}$

6) Let $\tilde{R}_{ci} = \tilde{R}_{s1} - \lambda \left( \sum_{i=1}^{\zeta} \frac{1}{\theta_i} \right)^{\frac{1}{\lambda}}$, $j = 1, \ldots, \zeta$

7) Set $x = x - \omega R_{s,max}$ and $\zeta = \zeta - \omega$

8) If $\zeta \neq 0$ and $\omega \neq 0$, then go to step 3

9) If $\zeta = 0$, then $\tilde{R}_{s1}^* = x - n R_{c,max}$

10) End

**Remark 5.** The real-time implementation of Algorithm 1 requires that the cooperating sensors share certain information. For instance, all sensors would require the information of the weighting functions $\theta_i(t)$ for all $i \in k(t)$ in the CDOP problem, in order to minimize the cost function (4). On the other hand, it is known that due to the connectivity preserving property of Algorithm 1 from sensor 1 to the destination point, a unidirectional multi-hop communication link is always available in the network. In addition, it is assumed that the destination point is equipped with a transmitter capable of sharing the received data with all sensors (a realistic assumption in most sensor networks). This means that all sensors will have the required information about all other sensors, throughout the process.

Consider now the problem of minimizing the sum of the power consumed by all sensors, which is a special case of the underlying optimization problem. In this case, all of the $\theta_i$’s ($i \in n$) are equal to 1. Note that the smaller the total consumed power at each instant in a given interval, the smaller the total consumed energy in that interval.

**Theorem 4.** Consider the CPOP and denote its minimum cost with $J^*_p$. Let $R_p$ be the unique real positive root of $f(R_{s1}) = n \alpha \gamma R_{s1}^\gamma - 1 - \lambda (x - R_{s1})^{\lambda-1}$ over $[0, x]$ (see Lemma 2). Then $J^*_p$ is equal to:

i) $J^*_p = \frac{(x - R_{s1})^\lambda}{n^{\lambda-1}} + \alpha R_{s1}$

ii) $J^*_p = \frac{(x - R_{s1})^\lambda}{n^{\lambda-1}} + \alpha R_{s1}$

iii) $J^*_p = n R_{s1}^\lambda + \alpha (x - n R_{s1})^{\lambda-1}$

if $R_p \leq R_{s,max}$ and $\frac{1}{n} (x - R_p) \leq R_{c,max}$

if $R_p > R_{s,max}$ and $\frac{1}{n} (x - R_p) \leq R_{c,max}$

if $R_p \leq R_{s,max}$ and $\frac{1}{n} (x - R_p) > R_{c,max}$

**Proof:** The proof is omitted due to space restrictions and may be found in [18].

**Remark 6.** Note in Theorem 4 that the inequalities $R_p > R_{s,max}$ and $\frac{1}{n} (x - R_p) > R_{c,max}$ together do not constitute a valid case, according to Assumption 3.
Remark 7. To run Algorithm 1 in a given time interval, it is required first to specify the tracking sensor. The selection criteria can include, for example, the distance between the sensors and the target, and the residual energy of the sensors. Note that a sensor that is very close to the target and has a high level of residual energy would be more desirable, to increase the reliability and durability of target tracking by the corresponding sensor. In particular, it is straightforward to show that if a sensor with highest residual energy is selected as the tracking sensor, it would be more desirable for the CPOP problem.

IV. SIMULATION RESULTS

Example 1. Consider a sensor network consisting of 6 sensors mounted on mini-wheel robots. Assume that the sensor parameters are given by $R_{c,\text{max}} = 20$ m, $R_{s,\text{max}} = 6$ m, $\lambda = 3.2$, $\gamma = 6.5$ and $\alpha = 2$. Let the initial residual energy of the sensors be chosen as uniformly distributed random numbers between 150 KJ and 900 KJ. It is assumed here that a sensor with high initial energy is selected as the tracking sensor, and that this sensor will not change throughout the mission (the latter one is mainly for simplicity of analysis, as stated earlier). Two different scenarios are investigated here.

![Fig. 1. Residual energy of sensors in Example 1, under the life-span maximization strategy (first scenario).](image1)

Scenario 1: In the first scenario, it is desired that all sensors work cooperatively for a long period of time. For this purpose, the residual energy of all sensors needs to be monitored at all times, and the power consumption of each sensor is to be adjusted accordingly, so that the life-span of every sensor in the network becomes more or less the same as shown in Fig. 1. To this end, the sensing and communication radii are chosen in such a way that if the residual energy of one sensor, say sensor $i$, at one instant is $k$ times greater than that of another sensor, say sensor $j$, then the rate of energy consumption (power consumption) by sensor $i$ must be $k$ times greater than that by sensor $j$. It is desired now to choose the values of $\theta_i$’s at every time instant such that the above objective is achieved by minimizing $J_D^p$. As noted from (2) and (8), if at the time $t$ the residual energy of sensor $i$ is $k$ times greater than that of sensor $j$, then $\theta_i$ should be set $k^{\lambda-\alpha}$ times greater than $\theta_i$ at that time instant, for all $i, j \in n$. This condition is satisfied by choosing $\theta_i = (\text{residual energy of sensor } i)^{\frac{1}{\lambda-\alpha}}$, for any $i \in n$.

Remark 8. To implement scenario 1, $\theta_{s1}$, $x$ and $\theta_i$’s, $\forall i \in n$, are required to be shared, and then Algorithm 1 is to be applied. Two computational schemes can be envisaged here: centralized and distributed. In a centralized scheme, all shared information is transmitted to the destination point, and $R_{s1}$ and $R_{c1}$’s are derived subsequently along with the location of each sensor. In a distributed scheme, on the other hand, $\theta_2$, ..., $\theta_n$ are sent to the destination point while $\theta_{s1}$, $x$ and $\theta_i$ are shared through the unidirectional link from sensor 1 to the sensor with the smallest distance from the destination point. Any sensor in MSN uses Algorithm 1 separately to find the optimal parameters.

Remark 9. It is to be noted that the power consumption of sensor 1 is not due solely to communication, and part of it is resulted by sensing. Furthermore, the relation (8) is not necessarily valid in constrained optimization. However, since the coefficients $\theta_i$ ($i \in n$) are tuned online, the strategy described above is still effective in increasing the life-span of the network in both constrained and unconstrained optimization problems.

Fig. 2. (a) The location of the target, destination point, and sensors at $t = 24$ min (b) The residual energy of each sensor at $t = 24$ min.

In Fig. 1, the residual energy of sensors is plotted versus time, and it shows that all sensors run out of energy at $t =$...
102 min simultaneously. Furthermore, it is worth mentioning that in Fig. 1, there is a significant drop in the residual energy of sensor 1 from \( t = 75 \) min to \( t = 77 \) min. This is due to the fact that the distance between the target and destination point becomes close to \( nR_{c,\text{max}} + R_{s,\text{max}} = 126 \) m in this time interval (see also Fig. 3(a)), which forces every sensor to deploy its maximum allowable communication radius, and more importantly, sensor 1 to use its maximum allowable sensing radius.

Figures 2(a), 3(a) and 4(a) depict the location of target and sensors under the first scenario in the \( x - y \) plane in three different time instants \( t = 24, 76 \) and \( 90 \) min, respectively. As it can be observed from Figures 2(a), 3(a) and 4(a), the proposed strategy aligns all sensors on a straight line. Their exact location on the line as well as their communication and sensing radii are computed online, according to the residual energy of every sensor in the network. The boundary of the region where the signal transmitted by each sensor can be received is marked by a colored solid circle in these figures. Similarly, the boundary of the sensing region corresponding to sensor 1 is marked by a blue dotted circle. The residual energy of each sensor in the above time instants is plotted in Figures 2(b), 3(b) and 4(b), accordingly. It is to be noted that the communication radii of the sensors are proportional to their residual energy at \( t = 24 \) min, but they are all equal at \( t = 76 \) min (although different sensors have distinct residual energy at this time instant). This is due to the fact that the distance between the target and destination point is nearly 126 m at this instant, i.e. the farthest distance the communication and sensing capabilities of all sensors can cover. Hence, the maximum radii for communication and sensing need to be adopted by the sensors, regardless of their residual energy. Moreover, since the distance between the target and destination point is about 83 m at \( t = 90 \) min, not all sensors need to use their maximum communication and sensing radii. In this case, the proportionality between the communication radius and the residual energy holds except for sensor 6, which is the sensor with the smallest distance from the destination point. This is due to the fact that the sensors cannot exceed the maximum communication radius of \( R_{c,\text{max}} \). As a result of this constraint, in this example the communication radius of sensor 6 is equal to 20 m.

Scenario 2: It is now desired to minimize the sum of power consumption of all sensors. In this case, one can use Algorithm 1 and Theorem 5 to find the optimal solution for the underlying constrained optimization problem. Figure 5 depicts the residual energy of the sensors versus time using the optimal strategy in this case. As it can be seen from this figure, all sensors do not run out of energy at the same time.

Remark 10. To apply Algorithm 1 in this scenario, the value...
of $x$ only needs to be transmitted through the unidirectional link from sensor 1 to the sensor with the smallest distance from the destination point. The optimization procedure will then be performed by each sensor separately.

By comparing Figures 1 and 5, it can be observed that the cooperative operation of the sensors in the first scenario lasts 46% longer than that of the same sensors in the second scenario (note that by definition cooperative operation requires that the residual energy of every sensor in the network be nonzero). Another comparison of the two strategies is provided in Fig. 6. This figure shows the sum of residual energies of all sensors versus time in the first scenario (solid curves) and second scenario (dotted curves). It can be seen from this figure that after 70 minutes of the network operation, the total residual energy of the sensors in the second scenario is approximately 3.8% more than that in the first scenario. In other words, the total energy consumption of the sensors in the second scenario is less than that of the sensors in the first scenario.

Fig. 5. Residual energy of the sensors in Example 1, under the total energy consumption minimization strategy (second scenario).

Fig. 6. The total residual energy of all sensors in both scenarios.

V. CONCLUSIONS

An algorithm is proposed in this paper to solve a constrained optimization concerning a mobile sensor network. The cost function takes into account the power consumption of overall network in order to accomplish the target tracking objective in an efficient fashion. A strategy is also provided to maximize the durability of the sensors by monitoring the residual energy of the individual sensors, and then adjusting their parameters and relocating them accordingly. The proposed relocation scheme in this case ensures a uniform consumption of the remaining energy of each sensor, such that all sensors run out of energy at the same time. The algorithm guarantees end-to-end connectivity from the target to the fixed access point, which is crucial in order to achieve the tracking of a moving target. Simulation results illustrate the efficacy of the proposed techniques.

REFERENCES