Neural Network Robust Controllers Applied to Free-Floating Space Manipulators in Task Space

Tatiana de F. P. A. T. Pazelli, Marco H. Terra, Adriano A. G. Siqueira

Abstract—This paper deals with the problem of robust trajectory tracking control for free-floating manipulator systems subject to plant uncertainties and external disturbances. The system model is described through the Dynamically Equivalent Manipulator approach and the tracking problem is formulated directly in task space. Two adaptive techniques are developed considering nonlinear \( H_\infty \) controllers and artificial neural networks. Experimental results are obtained from an under-actuated three-link fixed-base planar manipulator dynamically equivalent to a two-link free-floating planar space manipulator.

I. INTRODUCTION

Space robots are featured by a dynamic coupling which causes the rotation of the main body with the coordinated motions of the arm. A number of dynamic and control problems are unique to this area due to the distinctive and complex dynamics found in many of their applications.

One of the representative types of space robotic systems, free-floating space manipulators are systems that allow the spacecraft to move freely in response to the manipulator motions in order to conserve fuel and electrical power, [6]. Several of the references related with this kind of system develop the controllers in joint space, which presents an important inconvenient for a space robot with a free-floating base. In this case, the kinematic mapping from task space to joint space, where the control is executed, becomes non-unique because of non-integrable angular momentum conservation. This may cause non-existence of the reference trajectory in joint space. In order to avoid the complications of kinematic mapping, in this work, the trajectory tracking problem is formulated directly in task space. Hence, end-effector positions can be directly controlled.

The hostile environment where a space robot operates can deteriorate its structure and physical characteristics. In view of the difficulty of taking the system back to reformulate its dynamic model, this paper aims to present a robust solution for the problem of trajectory tracking control for free-floating space manipulators subject to plant uncertainties and external disturbances. Two adaptive techniques are applied considering nonlinear \( H_\infty \) controllers and artificial neural networks. The first one applies the intelligent system to learn the dynamic behavior of the robotic system, which is considered totally unknown. The second intelligent strategy considers a well defined nominal model structure and the neural networks are applied to estimate only the behavior of parametric uncertainties and the spacecraft dynamics. Indeed, the main contribution of this paper lays on the application of these proposed methods to the task space formulation of the problem.

To deal with parametric uncertainties in controlling free-floating manipulators at task space, [1], [3], [7] and [10] applied different adaptive techniques, resorting to modelling the system as an extended arm or using inverted chain approach. In this paper, the space manipulator model is described through the Dynamically Equivalent Manipulator (DEM) approach, [5]. The authors in [5] mapped a space manipulator to a conventional fixed-base manipulator and showed that both kinematical and dynamical properties of the space manipulator system are preserved in this mapping. Since the DEM is a conventional manipulator, it can be physically built and experimentally used to evaluate control algorithms for space manipulators. Different from [1], [3], [7] and [10], in this work an experimental investigation is performed to validate the proposed control methods and a comparative study is presented.

The paper is organized as follows: Section II presents the model description through the DEM approach; the neural network nonlinear \( H_\infty \) control designs are presented in Section III; and, finally, experimental results for a two-link free-floating space manipulator are presented in Section IV.

II. MODEL DESCRIPTION AND PROBLEM FORMULATION

A. Free-Floating Space Manipulator Mapped by a Dynamically Equivalent Fixed-Base Manipulator

Consider an \( n \)-link serial-chain rigid manipulator mounted on a free-floating base and that no external forces and torques are applied on this system. Consider also the Dynamically Equivalent Manipulator (DEM) approach, [5]. The DEM is an \( n+1 \)-link fixed-base manipulator with its first joint being a passive spherical one and whose model is both kinematically and dynamically equivalent to the SM dynamics. Figure 1 shows the representation and the parameter notation for both SM and DEM manipulators. Let the SM parameters be identified by apostrophe \( (\phi', \theta', \rho', J') \), the links of the manipulators are numbered from 2 to \( n+1 \); the \( Z\)-\( Y\)-\( Z \) euler angles \( (\phi, \theta, \rho) \) represent the SM base attitude and the DEM first passive joint orientation; \( J_i \) is the joint connecting the \( (i-1) \)-th link and \( i \)-th link; \( \theta_i \) is the rotation of the \( i \)-th link around joint \( J_i \); \( C_i \) is the center of mass of the \( i \)-th link; \( L_i \) is the vector connecting \( J'_i \) and \( C_i \); \( R_i \) is the

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vector connecting \( C_i' \) and \( J_{i+1}' \); \( l_c \) is the vector connecting \( C_i \) and \( C_i' \); and \( W_i \) is the vector connecting \( J_i \) and \( J_{i+1} \).

Considering that the DEM operates in the absence of gravity and that its base is located at the center of mass of the SM, the kinematic and dynamical parameters of the DEM can be found from the SM parameters as

\[
m_i = \frac{M_i m_i'}{\sum_{k=1}^{i-1} m_k}, \quad i = 2, \ldots, n + 1,
\]

\[
l_i = l_i', \quad i = 1, \ldots, n + 1,
\]

\[
W_i = R_i^{-1} \frac{\sum_{k=1}^{i-1} m_k'}{M_i}, \quad i = 2, \ldots, n + 1,
\]

\[
l_{c1} = \bar{l}_c \frac{\sum_{k=1}^{i-1} m_k'}{M_i}, \quad i = 2, \ldots, n + 1,
\]

where \( M_i \) is the total mass of the SM. Observe that the mass of the passive joint, \( m_1 \), is not defined by the equivalence properties.

Let the generalized coordinates \( q = [\phi \ \theta \ \rho \ \theta_2 \ \ldots \ \theta_{n+1}]^T \) be partitioned as \( q = [q_{b}' \ q_{m}']^T \), where the indexes \( b \) and \( m \) represent the passive spherical joint (base) and the active joints (manipulator), respectively. From Lagrange theory, dynamic equations of the DEM are given by

\[
\dot{M}(q_m)\ddot{q} + C(q_m, \dot{q})\dot{q} = \tau,
\]

where \( M(q_m) \in \mathbb{R}^{(n+3 \times n+3)} \) is the symmetric positive definite inertia matrix, \( C(q_m, \dot{q}) \in \mathbb{R}^{(n+3 \times n+3)} \) is the matrix of Coriolis and centrifugal forces, and \( \tau = [0 \ 0 \ 0 \ \tau_2 \ \ldots \ \tau_{n+1}]^T \) is the torque vector acting upon the joints of the DEM. Parametric uncertainties can be introduced dividing the parameter matrices \( M(q_m) \) and \( C(q_m, \dot{q}) \) into a nominal and a perturbed part

\[
\dot{M}(q_m) = \dot{M}_0(q_m) + \Delta \dot{M}(q_m)
\]

\[
C(q_m, \dot{q}) = C_0(q_m, \dot{q}) + \Delta C(q_m, \dot{q})
\]

where \( M_0(q_m) \) and \( C_0(q_m, \dot{q}) \) are nominal matrices and \( \Delta \dot{M}(q_m) \) and \( \Delta C(q_m, \dot{q}) \) are the parametric uncertainties.

### B. Problem Formulation

As we are dealing with a free-floating space manipulator, it is considered that only the active joints of the DEM are controlled, with the passive spherical joint not locked. In this case, the passive joint dynamics intervenes with the control of the manipulator active joints.

Once the DEM modelling technique locates the inertial frame origin at the center of mass of the SM, the vector of inertial position and orientation of the end-effector,

\[
p = [\theta_{ef} \ \psi_{ef} \ x_{ef} \ y_{ef} \ z_{ef}]^T,
\]

is function of free-floating base attitude and of generalized coordinates of manipulator joints, \( q_m \).

Let \( J(q) \) be the Jacobian that relates the velocities of joints coordinates, \( \dot{q} \), and the velocities of the end-effector, \( \dot{p} \):

\[
\dot{p} = J(q)\dot{q}.
\]

Considering that \( \det(J(q)) \neq 0 \), applying (3) and its derivative, \( \dot{p} = J(q)\dot{q} + J(q)\dot{q} \), to (2), dynamic equations of the DEM in task space are given by

\[
\tau = M_{ef}(q)\ddot{q} + C_{ef}(q, \dot{q})\dot{q},
\]

where

\[
M_{ef}(q) = M(q_m)J^{-1}(q),
\]

\[
C_{ef}(q, \dot{q}) = (C(q_m, \dot{q}) - M(q_m)J^{-1}(q)J(q, \dot{q}))J^{-1}(q).
\]

It must be noted that the Jacobian, \( J(q) \), introduces the values of spacecraft’s attitude, \( q_m \), in the dynamic equation matrices, \( M_{ef} \) and \( C_{ef} \). This does not happen when the problem is formulated in joint space, [8]. Another remark is that, in this formulation, \( M_{ef}(q) \) is not a symmetric positive definite matrix, neither \( N_{ef}(q, \dot{q}) = M_{ef}(q, \dot{q}) - 2C_{ef}(q, \dot{q}) \) is skew-symmetric. In order to preserve the characteristics of dynamics formulated in joint space, a force transformation is applied to (4), [4]:

\[
\tau = J^T(q)F,
\]

where \( F \) is a vector of generalized forces of the end-effector in inertial space. Therefore,

\[
F = M_{ef}(q)\ddot{q} + C_{ef}(q, \dot{q})\dot{q},
\]

with

\[
\dot{M}_{ef}(q) = J^{-T}(q)M_{ef}(q)J^{-T}(q) - M(q_m)J^{-1}(q)\dot{J}(q, \dot{q})J^{-1}(q),
\]

\[
\dot{C}_{ef}(q, \dot{q}) = J^{-T}(q)\dot{C}(q_m, \dot{q}) - M(q_m)J^{-1}(q)\dot{J}(q, \dot{q})J^{-1}(q).
\]

In this format the dynamic equation formulated in inertial space maintains the structure and properties found in joint space. So, \( \dot{M}_{ef}(q) \) is symmetric positive definite and \( \dot{N}_{ef}(q, \dot{q}) = \dot{M}_{ef}(q, \dot{q}) - 2\dot{C}_{ef}(q, \dot{q}) \) is skew-symmetric.

A characteristic inherited from underactuated manipulators, dealing with a system with \( n_a \) actuators leads to controlling only \( n_a \) degrees of freedom at a time. The DEM presents \( n \) active joints and, then, \( n_a = n \). So, let’s define \( \rho = [p_a^T \ p_a^T]^T \) the vector of generalized coordinates of the system, with \( p_u \in \mathbb{R}^{(6-n) \times 1} \) and \( p_a \in \mathbb{R}^{n \times 1} \), where the indexes \( u \) and \( a \) represent the passive variables (which are let free during the control procedure) and the controlled variables.
respectively. Define \( \delta = \begin{bmatrix} \delta^T_a & \delta^T_u \end{bmatrix}^T \) as a vector representing the sum of parametric uncertainties of the system and \( F_d \) as a finite energy external disturbance also introduced. Equation (6) can be partitioned as:

\[
\begin{aligned}
[F_a & \quad F_a] + \begin{bmatrix} \delta_a & \delta_d \end{bmatrix}
+ F_d =
\begin{bmatrix} M_{c,\text{sum}}(q) & M_{c,\text{sum}}(q) \\ M_{c,\text{sum}}(q) & M_{c,\text{sum}}(q) \end{bmatrix}
\begin{bmatrix} \dot{p}_u \\ \ddot{p}_u \end{bmatrix}
\end{aligned}
\]

where

\[
\begin{bmatrix}
\delta_a(q, \dot{q}, \ddot{q}, \tau_d) \\
\delta_d(q, \dot{q}, \ddot{q}, \tau_d)
\end{bmatrix}
= \begin{bmatrix}
\Delta M_{c,\text{sum}}(q) \dot{p}_u + \Delta M_{c,\text{sum}}(q) \ddot{p}_u \\
\Delta C_{c,\text{sum}}(q, \dot{q}) \dot{p}_u + \Delta C_{c,\text{sum}}(q, \dot{q}) \ddot{p}_u \\
\Delta M_{c,\text{sum}}(q) \dot{p}_u + \Delta M_{c,\text{sum}}(q) \ddot{p}_u \\
\Delta C_{c,\text{sum}}(q, \dot{q}) \dot{p}_u + \Delta C_{c,\text{sum}}(q, \dot{q}) \ddot{p}_u
\end{bmatrix},
\]

and \( M_{c,\text{sum}} \in \mathbb{R}^{(6-n)\times(6-n)} \), \( M_{c,\text{sum}} \in \mathbb{R}^{(6-n)\times n} \), \( M_{c,\text{sum}} \in \mathbb{R}^{n\times n} \), \( C_{c,\text{sum}} \in \mathbb{R}^{(6-n)\times(6-n)} \), \( C_{c,\text{sum}} \in \mathbb{R}^{(6-n)\times n} \), \( C_{c,\text{sum}} \in \mathbb{R}^{n\times n} \), \( F_d(e) \in \mathbb{R}^{(6-n)\times 1} \) and \( F_a \in \mathbb{R}^{n\times 1} \). For simplicity of notation, the index 0 referring to the nominal system was suppressed. This composition should also preserve the properties of dynamic equation for the matrices \( M_{c,\text{sum}}(q) \) and \( C_{c,\text{sum}}(q, \dot{q}) \), namely, \( M_{c,\text{sum}}(q) = M_{c,\text{sum}}(q, \dot{q}) > 0 \) and \( N_{c,\text{sum}}(q, \dot{q}) = M_{c,\text{sum}}(q, \dot{q}) - 2C_{c,\text{sum}}(q, \dot{q}) \) is skew-symmetric.

Let \( p^d_a \in \mathbb{R}^n \) and \( p^d_d \in \mathbb{R}^n \) be the desired reference trajectory and the corresponding velocity for the end-effector controlled variables, respectively. The state tracking error is defined as

\[
\dot{x}_{ef} = \begin{bmatrix} \dot{p}_a - \dot{p}^d_a \\ \ddot{p}_a - \ddot{p}^d_a \end{bmatrix} = \begin{bmatrix} \dot{p}_u \\ \ddot{p}_u \end{bmatrix}.
\]

Also, assume that \( p^d_a \), \( \dot{p}^d_a \), and \( \ddot{p}^d_a \), belong entirely to the path independent workspace (PIW), [9], and therefore, they will not conduct to any dynamic singularity, i.e., \( \text{det}(J) \neq 0 \) throughout the path.

Consider the following state transformation, [2],

\[
\tilde{x} = T_0 x_{ef} = \begin{bmatrix} T_{11} & T_{12} \\ 0 & I \end{bmatrix} \begin{bmatrix} \dot{p}_u \\ \ddot{p}_u \end{bmatrix},
\]

where \( T_{11}, T_{12} \in \mathbb{R}^{n\times n} \) are constant matrices to be determined. From (7), (8) and (9), state space representation of the DEM is given by

\[
\begin{aligned}
\dot{x}_{ef} &= \tilde{A}_{T_{11}}(q, \dot{q}) \tilde{x}_{ef} + \tilde{B}_{T_{12}}(q) T_{11} (-\hat{F}(x_{ef}) - \tilde{E}(x_{eb}) + F_d + \delta_d + F_a) \\
&= \tilde{A}_{T_{11}}(q, \dot{q}) \tilde{x}_{ef} + \tilde{B}_{T_{12}}(q) u + \tilde{B}_{T_{12}}(q) \omega,
\end{aligned}
\]

where

\[
\tilde{A}_{T_{11}}(q, \dot{q}) = T_{10}^{-1} \begin{bmatrix} -M_{c,\text{sum}}^{-1}(q) C_{c,\text{sum}}(q, \dot{q}) \end{bmatrix},
\]

\[
\tilde{B}_{T_{12}}(q) = \begin{bmatrix} \tilde{M}_{c,\text{sum}}^{-1}(q) \end{bmatrix},
\]

\[
\hat{F}(x_{ef}) = \bar{M}_{c,\text{sum}}(q) \dot{p}_d + \tilde{M}_{c,\text{sum}}(q) \ddot{p}_d \]

\[
+ \tilde{C}_{c,\text{sum}}(q, \dot{q}) \dot{p}_d + \tilde{C}_{c,\text{sum}}(q, \dot{q}) \ddot{p}_d\]

\[
\tilde{E}(x_{eb}) = \bar{M}_{c,\text{sum}}(q) \dot{p}_u + \tilde{C}_{c,\text{sum}}(q, \dot{q}) \dot{p}_u,
\]

\[
\omega = T_{11}(\delta_a + F_d),
\]

with \( x_{ef} = [ q^T \; \dot{q}^T \; p^T_a \; \dot{p}^T_a \; \ddot{p}^T_a \; (p^d_a)^T \; (\dot{p}^d_a)^T \; (\ddot{p}^d_a)^T ]^T \).

\[
\tilde{M}_{c,\text{sum}}(q) = M_{c,\text{sum}}(q) - F_d.
\]

### III. ADAPTIVE NEURAL NETWORK NONLINEAR CONTROL

Define a set of \( n \) neural networks \( E_k(x, \Theta_k) \), \( k = 1, \cdots, n \), where \( x \) is the input vector and \( \Theta_k \) are the adjustable weights in the output layers. The single-output neural networks are of the form

\[
E_k(x, \Theta_k) = \sum_{i=1}^{p_k} \theta_{ki} G \left( \sum_{j=1}^{q_k} w_{kj} x_{ef} + b_{ki} \right) = \xi^T \Theta_k, \quad (11)
\]

where \( q_k \) is the size of vector \( x \), and \( p_k \) is the number of neurons in the hidden layer. The weights \( w_{kj} \) and the bias \( b_{ki} \) for \( 1 \leq i \leq p_k \), \( 1 \leq j \leq q_k \) and \( 1 \leq k \leq n \) are assumed to be constant and specified by the designer. Thus, the adjustment of neural networks is performed only by updating the vectors \( \Theta_k \). The activation function for the neurons in the hidden layer is chosen to be \( G(.) = \tanh(.) \). The complete neural network is denoted by

\[
E(x, \Theta) = \begin{bmatrix} E_1(x, \Theta_1) \\ \vdots \\ E_n(x, \Theta_n) \end{bmatrix} = \begin{bmatrix} \xi_1^T \\ \xi_2^T \\ \vdots \\ \xi_n^T \end{bmatrix} \Theta = \Xi \Theta. \quad (12)
\]

Consider a first approach where the term

\[
E^1(x_{ef}, x_{eb}) = \hat{F}(x_{ef}) + \tilde{E}(x_{eb}) - \delta_a
\]

in (10) is completely unknown regarding its structure and parameter values. The neural network defined in (12) is applied to learn the dynamic behavior of the robotic system:

\[
E^1(x_{ef}, x_{eb}) \approx \hat{E}(x, \Theta) = \Xi \Theta, \quad (13)
\]

where the input vector \( x \) should be defined as

\[
x = [ q^T \; \dot{q}^T \; p^T_a \; \dot{p}^T_a \; \ddot{p}^T_a \; (p^d_a)^T \; (\dot{p}^d_a)^T \; (\ddot{p}^d_a)^T ]^T.
\]
However, the values of $q_b$, $\dot{q}_b$, $\dot{\theta}_a$ and $\ddot{\theta}_a$ would be necessary, but they are not easy to obtain in practice. Considering that a neural network based approach is usually used when it is not possible to supply all the variables values to the system model, we have defined the vector $\mathbf{x}_e$ as
\[
\mathbf{x}_e = \left[ q_m^T \ \dot{q}_m^T \ \left( \dot{p}_a^T \right)^T \ \left( \dot{p}_a^T \right)^T \ \left( \ddot{p}_a^T \right)^T \right]^T,
\]
(14)
avoiding the necessity of any data from the free-floating base or related to passive variables. Experimental results will show the feasibility of this assumption. Defining the following optimization problem
\[
\Theta^* = \arg \min_{\Theta \in \Theta_0} \max_{\mathbf{x}_e \in \Omega_{\mathbf{x}_e}} \left\| \bar{E}(x_e, \Theta^*) - E^1(x_{e,f}, x_0) \right\|_2^2,
\]
the modified error equation (10) may be rewritten as
\[
\dot{x}_{e,f} = \tilde{A}_{T_{11}}(q, q) \dot{x}_{e,f} + \tilde{B}_{T_{11}}(q) T_{11}(F_a - E^1(x_{e,f}, x_0)) + F_d + \bar{E}(x_e, \Theta^*) - \bar{E}(x_e, \Theta^*)
\]
(15)
with
\[
u = T_{11}(F_a - \bar{E}(x_e, \Theta^*)),
\]
(16)
\[
\omega = T_{11}(\bar{E}(x_e, \Theta^*) - E^1(x_{e,f}, x_0) + F_d),
\]
(17)
where $\omega$ refers to the estimation error from the neural network system and external disturbances. Considering $u$ the control law provided by the nonlinear $\mathcal{H}_\infty$ controller, $F_a$ can be computed by
\[
F_a = \bar{E}(x_e, \Theta^*) + T_{11}^{-1}\tilde{u}.
\]
(18)
Now, consider that model structure and nominal values for the term $\bar{F}(x_{e,f})$ are well defined and available for the controller. Then, a second approach may be proposed. In this case, the neural network is applied to estimate only the behavior of parametric uncertainties and spacecraft dynamics (considered as a non-modeled dynamic):
\[
E^2(x_{e,f}, x_0) \approx \bar{E}(x_e, \Theta) = \Xi \Theta,
\]
where $E^2(x_{e,f}, x_0) = \bar{E}(x_e) - \delta_2$.

Similarly, $x_e$ is defined by (14), the optimal approximation parameters vector is given by
\[
\Theta^* = \arg \min_{\Theta \in \Theta_0} \max_{x_e \in \Omega_{x_e}} \left\| \bar{E}(x_e, \Theta^*) - E^2(x_{e,f}, x_0) \right\|_2^2,
\]
and the modified error equation (10) may be rewritten as
\[
\dot{x}_{e,f} = \tilde{A}_{T_{12}}(q, q) \dot{x}_{e,f} + \tilde{B}_{T_{12}}(q) T_{12}(F_a - \bar{F}(x_{e,f})) + \omega + \dot{\Theta} \bar{E}(x_e, \Theta^*) - \bar{E}(x_e, \Theta^*)
\]
(19)
\[
= \tilde{A}_{T_{12}}(q, q) \dot{x}_{e,f} + \tilde{B}_{T_{12}}(q) T_{12}(F_a - \bar{F}(x_{e,f})) + \tilde{B}_{T_{12}}(q) T_{12}(\bar{E}(x_e, \Theta^*) - E^2(x_{e,f}, x_0) + F_d)
\]
(20)
\[
= \tilde{A}_{T_{12}}(q, q) u + \tilde{B}_{T_{12}}(q) \omega,
\]
(21)
where $\omega$ refers to the estimation error from the neural network system and external disturbances. Considering $u$ the control law provided by the nonlinear $\mathcal{H}_\infty$ controller, $F_a$ can be computed by
\[
F_a = \bar{F}(x_{e,f}) + \tilde{E}(x_e, \Theta^*) + T_{11}^{-1}\tilde{u}.
\]
(22)
Regarding the nonlinear $\mathcal{H}_\infty$ control solution based on the game theory, let
\[
\tilde{u} = -R^{-1}B^T T_0 \tilde{x}_{e,f}
\]
(23)
be the optimal control input, with $B = [I \ 0]^T$, and $T_0$ being the solution of the following algebraic equation
\[
\begin{bmatrix}
0 & K \\
K & 0
\end{bmatrix} - T_0^2 B \left( R^{-1} - \frac{1}{\gamma^2} I \right)^{-1} B^T T_0 + Q = 0,
\]
(24)
such that $K > 0$, $R < \gamma^2 I$ and
\[
T_0 = \begin{bmatrix}
T_{11} & T_{12} \\
0 & I
\end{bmatrix} = \begin{bmatrix}
R_1^T Q_1 & R_1^T Q_2 \\
Q_1^T & Q_2^T
\end{bmatrix},
\]
(25)
considering $R_1$ the result of the Cholesky factorization
\[
R_1^T R_1 = \left( R^{-1} - \frac{1}{\gamma^2} I \right)^{-1}
\]
and the positive definite symmetric matrix $Q$ factorized as
\[
Q = \begin{bmatrix}
Q_1^T & Q_{12} \\
Q_{12}^T & Q_2^T
\end{bmatrix}.
\]
(26)
The matrices $Q$ and $R$ are defined by the designer preserved the restrictions.

Thus, considering the stability analysis developed by [2], the adaptive neural network nonlinear $\mathcal{H}_\infty$ control solution for the two different approaches is stated as follows.

Let $E(x_e, \Theta)$ be a set of $n$ neural networks defined by (12) with $x_e$ being a vector of available data defined by (14) and $\Theta$ being a vector of adjustable parameters. Given a desired disturbance attenuation level $\gamma > 0$ and matrices $Z = Z^T > 0$, $Q = Q^T > 0$, $P_0 = P_0^T > 0$, $Z_0 = Z_0^T > 0$, and $R = R^T < \gamma^2 I$, the following performance criterion
\[
\int_0^T \left( \dot{x}_{e,f}^T Q \dot{x}_{e,f} + \tilde{u}^T R \tilde{u} \right) dt \leq \dot{x}_{e,f}(0) R_0 \dot{x}_{e,f}(0) + \Theta^T(0) Z_0 \Theta(0)
\]
(27)
\[
+ \gamma^2 \int_0^T (\omega^T \omega) dt,
\]
(28)
where $\tilde{\Theta} = \Theta - \Theta^*$ denotes the neural parameter estimation error, is satisfied, for any initial condition, if there exists a dynamic state feedback controller
\[
\Theta = \beta(t, \dot{x}_{e,f}) = -Z^{-T} \Xi^T T_{11} B^T T_0 \dot{x}_{e,f},
\]
(29)
\[
F_a = F_a(t, \dot{\Theta}, \dot{x}_{e,f}) = \Xi \Theta - T_{11}^{-1} R^{-1} B^T T_0 \dot{x}_{e,f},
\]
(30)
solution of the adaptive neural network nonlinear $H_{\infty}$ control problem subject to (15); or, for the second approach, if there exists a dynamic state feedback controller

$$\hat{\Theta} = \beta(t, \hat{x}_{ef}) = -Z^{-T}Z^TT_{11}B^TT_0\hat{x}_{ef},$$

(29)

$$F_a = F_a(t, \hat{\Theta}, \hat{x}_{ef}) = \hat{F}(x_{ef}) + Z\Theta - T_{11}^{-1}R^{-1}B^TT_0\hat{x}_{ef},$$

(30)
solution of the adaptive neural network nonlinear $H_{\infty}$ control problem subject to (20).

For both approaches, the torques applied upon the active joints are given by

$$\tau_a = J_a^T F_a,$$

(31)

where

$$J(q) = \begin{bmatrix} J_{uu} & J_{ud} \
J_{du} & J_{dd} \end{bmatrix}.$$

IV. RESULTS

For validation and comparison purposes, the proposed adaptive $H_{\infty}$ control solutions are applied to a free-floating, planar, two-link space manipulator system. The corresponding DEM is an experimental fixed-base, three-link, planar manipulator, UArmII (Underactuated Arm II). Its first joint is configured as passive, such that $q_n = [q_2 \ q_3]^T$ are the joints to be controlled. The experimental manipulator UArmII is composed by a DC motor in each joint, a break and an optical encoder with quadrature decoding used to measure joint positions. Joint velocities are obtained by numerical differentiation and filtering. The dynamic nominal parameters of the SM and of the DEM are given in Table I.

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<th>TABLE I DYNAMIC PARAMETERS</th>
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<td><strong>SM Parameters</strong></td>
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<td>Body</td>
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<td>Base</td>
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<td>Link 2</td>
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<td>Link 3</td>
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| **DEm Parameters** |  |
| Body | $m_1$ | $l_1$ | $W_1$ | $l_3$ |
| Body | (kg) | (kg m²) | (m) | (m) |
| Link 1 | 1.3552 | 0.008251 | 0.203 | 0 |
| Link 2 | 0.850 | 0.0075 | 0.203 | 0.096 |
| Link 3 | 0.625 | 0.006 | 0.203 | 0.077 |

A trajectory tracking task is defined for the space manipulator end-effector. The Cartesian positions $p_a = [x_{ef} \ y_{ef} \ z_{ef}]^T$ of the end-effector are chosen to be the controlled variables, while its orientation $\phi_{ef}$ is left free. The reference trajectory is defined as a semi-circle starting at the end-effector initial position (set by $q(0) = [0^o \ 20^o \ -40^o]^T$) and characterized by radius $= 5$ cm. The angles that determine the semi-circle reference trajectory follows a fifth degree polynomial with $t_f = 3s$ (time defined for the task execution). During the experiment, a limited disturbance, initializing at $t = 1s$, was introduced in the following form: $\tau_d = \begin{bmatrix} 0.12e^{-2t} \sin(2\pi t) \\ 0.08e^{-2t} \sin(2\pi t) \end{bmatrix}$.

Compared to the torque applied in case that none disturbance is inserted, the disturbance $\tau_d$ presents considerable energy. It is also considered that the DEM model is subject to parametric uncertainties, since UArmII is a real robot acting on a real environment.

The level of disturbance attenuation defined for the proposed nonlinear $H_{\infty}$ controllers is $\gamma = 2$. The selected weighting matrices are shown in Table II. For the nonlinear $H_{\infty}$ controllers via neural network proposed, let $n = 2$ be the size of $p_a$ determined by the number of joints of the space manipulator (active joints in DEM), which define the size of $x_e$, $q_n = 10$. Define $E(x_e, \Theta) := \begin{bmatrix} E_1(x_e, \Theta_1) \ E_2(x_e, \Theta_2) \end{bmatrix}^T$ with $p_7 = 7$ neurons in the hidden layer, the bias vector $b_k = [-3 -2 -1 0 1 2 3]$ and the weighting matrix for the first layer $\Omega_1 = [\Theta_e^T] = [1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1]$. The uncertain vector $\Theta$ is defined as $\Theta = [\Theta_1 \ \Theta_2]^T$, with $\Theta_1 = [\theta_{11} \ \cdots \ \theta_{17}]$ and $\Theta_2 = [\theta_{21} \ \cdots \ \theta_{27}]$, and the matrix $\Sigma$ can be computed with $\xi_1^T = [\xi_{11} \ \cdots \ \xi_{17}]$ and $\xi_2^T = [\xi_{21} \ \cdots \ \xi_{27}]$.

<table>
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<th>TABLE II SELECTED WEIGHTING MATRICES</th>
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<tr>
<td>$\gamma = 2$</td>
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<td>NN $H_{\infty}$ (1)</td>
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<td>NN $H_{\infty}$ (2)</td>
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In order to clearly identify the controllers actuation, Figures 2 and 3 illustrate the results obtained without adding disturbances (nominal case) and the results for the disturbed situation (disturbed case) from both the proposed controllers. A red mark identifies, in Figures 2(b) and 3(b), the instant when the disturbance begins ($t = 1s$).

![End-effector trajectory](image1)

![Joint torque](image2)

Fig. 2. Adaptive Neural Network Nonlinear $H_{\infty}$ (1) - End-effector trajectory and applied torques: (a) nominal case; (b) disturbed case.

By comparing Figures 2(a) and 3(a) to Figures 2(b) and 3(b), respectively, the robustness characteristic of the applied
Adaptive Neural Network Nonlinear $\mathcal{H}_\infty$ (2) - End-effector trajectory and applied torques: (a) nominal case; (b) disturbed case.

The $\mathcal{H}_\infty$ criterion can be verified. The graphical results illustrate that both the applied controllers reject disturbance efficiently and attenuate its effect in the trajectory tracking task.

As the same value of $\gamma$ was applied to both the proposed controllers, two performance indexes are used to numerically compare the controllers in this application: the $L_2$ norm of the state vector, $L_2^2[\mathbf{x}_e]$, and the sum of the applied torques, $E[\tau_a]$, [8]. The mean value of five experimental results obtained for each controller is shown in Table III.

<table>
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<th>TABLE III PERFORMACE INDEXES</th>
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<tr>
<td>Nominal case</td>
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<td>$L_2^2[\mathbf{x}_e]$</td>
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<tr>
<td>$\mathcal{H}_\infty$ (1)</td>
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<td>$\mathcal{H}_\infty$ (2)</td>
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The results presented by the adaptive neural networks approaches exhibit their efficiency in estimating the effect of uncertainties, and mainly, the non-modeled dynamics of the spacecraft. For the first approach, it is noted that some energy has been spent in attenuating the disturbance effects. On the other hand, in this experimental investigation, the second strategy proposed, which resorts to a known nominal model for the arm dynamics, keeps its error avoidance capacity without raising its torque effort. A comparison between the proposed strategies, by graphical and numerical results, with and without external disturbances, indicates the better tracking performance of the second strategy. However, a negotiation between error tolerance and energy consumption should be considered concerning the application requirements and the presence or not of strong disturbances.

V. CONCLUSION

This paper presented an experimental investigation on the motion control of a free-floating space manipulator subject to parametric uncertainties and external disturbances performed by two different methods of nonlinear $\mathcal{H}_\infty$ controllers. The first approach for adaptive neural network $\mathcal{H}_\infty$ controller applied the intelligent system to learn the dynamic behavior of the robotic system, which is considered totally unknown. The second approach combined the knowledge of the dynamic model with the intelligent adaptive tool, joined the best characteristics of both strategies. The $\mathcal{H}_\infty$ control law was applied to attenuate the effects of estimation errors and external disturbances. It must be emphasized that none of the proposed $\mathcal{H}_\infty$ approaches demands measured acceleration or velocity values from the free-floating base. The problem formulation developed directly in task space, using the DEM approach to model the space robot dynamics, prevented from kinematic mapping complications. Experimental results showed the effectiveness of the proposed control strategies for the considered application.

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REFERENCES