Robust and LPV control of an AMB system

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Abstract— Active magnetic bearing spindles are predominantly subject to periodic disturbances due to an unbalanced mass. The frequency of these disturbances is known to be the same as the operational (rotational) speed. LTI controllers provide for good disturbance attenuation only in very limited circumstances, i.e., when the operational speed is fixed or is very low. In the event of continuously changing operating speeds, LPV controllers can be very effective for disturbance rejection. In this research we study two robust control designs and an LPV control design method to demonstrate the far reaching performance benefits of the LPV controller.

I. INTRODUCTION

The current research is aimed at supporting the development of a micro-factory, which is a desktop sized milling machine capable of 3D micro-machining, [1] and [2]. This high-end application needs very-accurate positioning of the machining-tool while spinning at high rotational speeds. It is well-known that machining spindles are subject to various disturbances that lead to vibrations. Two main sources of disturbances are (i) tool-work piece interaction leading to so-called chatter instability and (ii) presence of unbalanced mass on the spindle which leads to synchronous vibrations. Active magnetic bearings are a promising technology to enable vibration suppression in such applications. Towards this end, we design and implement robust and LPV control techniques on the MBC500 experimental setup. We use an LTI model of this system that has been obtained from input-output measurements using a predictor-based subspace identification method (PBSID$_{opt}$), [3]. Here, we elaborate on the design of two robust controllers and an LPV controller and discuss the experimental results for the MBC500 setup.

AMB systems are nonlinear, multi-variable and inherently unstable systems. A nonlinearity arises in the relationship between the electro-magnetic force, current and length of the air gap. The system (vibrational) dynamics depend on the rotational speed due to gyroscopic and electro-magnetic coupling. In general, these effects can be made negligible by using slender spindles and careful alignment of the electromagnets. Hence the essential system dynamics are captured by an LTI model for very small deviations in the current and air-gap length. It is well known that an unbalanced mass acts as a sinusoidal disturbance with a frequency equal to the rotational speed, [4] and [5].

In the literature, controllers are designed for disturbance rejection up to a certain bandwidth by using an LTI model of the AMB system (without explicitly considering a model for the disturbance). For machining applications, controllers designed with $\mu$ synthesis techniques are shown to provide better performance than decentralized PD/PID techniques in [6] and [7]. The loop shaping design procedure (LSDP) has been used to shape the open-loop singular values based on requirements for the closed-loop sensitivity in [8]. These designs exhibit limited performance when the spindle operates at changing rotational speeds (due to disturbances from mass-imbalance). When the operating speed of the spindle is known, LTI controllers can be designed to reject disturbances of that particular frequency. This is done by incorporating notch filters in the loop in [9]. Perfect disturbance rejection of a particular frequency can be achieved by incorporating a transmission-zero into the closed-loop system. The $\mathcal{Q}$-parameterization of controllers in the $H_{\infty}$ framework is exploited to precisely achieve this in [10] and [11].

Sinusoidal disturbance rejection at continuously changing operating points can be achieved by scheduling with the $\mathcal{Q}$ parameter in the $H_{\infty}$ framework. Such a gain scheduling technique is applied to a horizontally suspended AMB spindle in [12]. However, the design does not account for the rate of variation of the rotational speed and does not guarantee stability during acceleration/deceleration phase. A parameter dependent disturbance model is augmented to the AMB system dynamics (including the gyroscopic effects) to obtain an LPV model in [13]. The rotational speed is regarded as a changing parameter on which the system dynamics depend. A self-scheduled $H_{\infty}$ controller is synthesized and simulation results are presented for the LPV model. The synthesis procedure assumed a parameter independent Lyapunov function (PILF) and the resulting design is conservative. Parameter dependent Lyapunov functions (PDLF) can be used to reduce this conservatism. In [14] the entire parameter range is subdivided into a number of smaller regions and PDLFs for the smaller sets are used to synthesize LPV controllers. Additional LMI constraints are imposed to ensure a smooth switching between the parameter sets. The simulation results presented in [13] and [14] illustrate the promising disturbance rejection features of LPV control techniques.

LPV controllers have been applied to practical AMB systems only recently in [15] and [16]. Herein, the unmodeled dynamics are lumped together as an uncertainty. The size of this uncertainty is captured by means of a parameter-dependent weighting function. Design of a robust controller with such parameter dependent weighting function is transformed into an LPV synthesis problem. In our research, we focus on rejecting disturbances due to mass-imbalance, and towards this end we make use of a parameter...
dependent performance-weight for the design of an LPV controller. A fixed additive uncertainty weight is considered to account for neglected system dynamics. We begin with the design of a robust controller for high-bandwidth disturbance rejection and subsequently modify the performance weight to have a notch-filter type of behavior in the closed-loop sensitivity.

II. System Description

A schematic view of the MBC500 magnetic bearing-spindle-sensor assembly is shown in Figure 1. A pair of opposing electro-magnets, and position sensors are located in the horizontal and vertical planes on either ends of the spindle. The system dynamics in the horizontal and vertical planes are assumed to be decoupled (neglecting gyroscopic/electro-magnetic coupling). Hence, the entire AMB setup is modeled by two $2 \times 2$ MIMO LTI subsystems. The identified models (via PBSID, [3]) contain the flexible modes of the spindle which are neglected to obtain reduced-order models. Robust and LPV controllers are designed for the reduced-order model in each plane. For reasons of symmetry and ease of presentation, we explain the design procedure for the horizontal plane alone. The time-domain results presented in a later section are, however, obtained with controllers implemented on both planes. The frequency response for the dynamics including flexible modes in the horizontal plane is shown by the dashed line in Figure 2. The eigenfrequencies of the flexible modes are located around 1090 rad/s and 2385 rad/s. This corresponds to rotational speeds of 10400 and 22800 rpm respectively. The reduced-order model representing only the rigid-body dynamics is shown by a solid line in the same figure.

III. Robust Control Design

For controller synthesis, the reduced-order dynamics is used as a nominal model. The neglected flexible-dynamics are accounted for by means of an additive uncertainty. The weight on this uncertainty is shown by the dash-dotted line in Figure 2. It is desired to design a robustly stabilizing (RS) controller for the set of plants given by $G_{unc} = G + W \Delta$, $\|\Delta\|_{\infty} < 1$, with a specified nominal performance (NP) determined by weight $W_y$. An $H_{\infty}$ controller $K$ is robustly stabilizing and achieves the required nominal performance for a given plant $G$, when,

$$\text{RS: } \|W \Delta K(I + GK)^{-1}\|_{\infty} \leq 1, \quad (1)$$

$$\text{NP: } \|W_y(I + GK)^{-1}\|_{\infty} \leq 1. \quad (2)$$

The interconnection shown in Figure 3 is used for two designs leading to controllers $K_1$ and $K_2$. The plant model $G$ represents the $2 \times 2$ MIMO LTI dynamics corresponding to the rigid body motion for horizontal and vertical planes. The nominal performance weight, $W_y$, is different for each of the designs and explained in the following subsections.

A. Design of $K_1$

In order to achieve disturbance rejection, the weight $W_y$ is used to obtain a desired loop-shape for the sensitivity function. We desire very good attenuation of low-frequency disturbances and allow the sensitivity to peak to 1 at a frequency of 300 rad/s. A standard first order weight $W_y = w_y(s) = \frac{s/M + \omega_B}{s + \omega_B A}$ with the overshoot $M = 1.5$, low-frequency attenuation of $A = 0.005$ and $\omega_B = 300$ is the desired bandwidth around which the sensitivity peaks up to 1. The inverse of this weight puts a frequency-wise bound on the closed-loop sensitivity and is shown by the solid line in Figure 4.
B. Design of $K_2$

When the operating speed of the spindle is fixed, a sinusoidal disturbance with a frequency equal to the rotational speed acts on the spindle. In order to suppress disturbances corresponding to a particularly frequency of $\theta$, we desire a notch in the closed-loop sensitivity. To achieve this, we use a weight on the sensitivity with an additional filter (containing an inverted notch) as $W_y = w_n w_y I$, where $w_y$ is as defined above and $w_n(s) = \frac{s^n + 2\omega n w_0}{s + \omega n w_0}$ with $\zeta$ as a damping constant.

For $\omega = 300$ rad/s the corresponding rotational speed is 2865 rpm and the disturbance falls in the high frequency range (where the sensitivity peaked to 1 in the previous design). We desire to emphasize the attenuation of disturbances of frequency $\omega$ and hence the overall bandwidth is reduced by choosing $\omega_B = 10$ and $\zeta = 0.015$. The inverse of the new weighting function $W_y$ is shown by the dashed-line in Figure 4.

![Bode Diagram](image)

**Fig. 4.** Inverse of weighting functions for controller $K_1$ and $K_2$.

C. Implementation

For the first design, when a controller $K_1$ that is obtained with a higher value of the crossover frequency on the sensitivity weight, is implemented (i.e. with $\omega_B > 300$), the system could not be stabilized. Hence we present a design with $\omega_B = 300$. For the design of $K_2$, we had to reduce this crossover frequency further to $\omega_B = 10$ for the same reason. This can be regarded as the price paid for increased attenuation at a higher frequency, in accordance with Bode’s sensitivity integral. The designs $K_1$ and $K_2$ are discretized using the standard Tustin-transformation with a sampling frequency of 10KHz.

IV. LPV DESIGN

The LTI designs are limited in their performance to very specific frequency range, low frequency for $K_1$, and centered around $\theta$ for $K_2$. In order to reject disturbances due to mass-imbalance over a wide frequency range, we advocate the use of LPV controllers that are dependent on the rotational speed. This can be imposed by considering a parameter dependent state-space realization of $W_y(\theta)$ is obtained from series-interconnection of $w_y$ and $w_n(\theta)$ where $w_y$ is the same as used in the design of $K_2$ and we use

$$w_n(\theta) = \begin{bmatrix} -2\theta \zeta & -\theta & 2\theta (1-\zeta) \\ \theta & 0 & 0 \\ 1 & 0 & 1 \end{bmatrix}.$$  \hspace{1cm} (3)

For frozen values of the parameter, i.e., $\theta(t) = \omega$, $w_n$ represents the same LTI transfer function that is used in the design for $K_2$. The interconnection of Figure 5 containing an LTI plant $G$, a weight $W_\Delta$ and an LPV weight $W_y(\theta)$ can be re-written as the following open-loop LPV system.

$$\begin{pmatrix} \dot{x} \\ z \end{pmatrix} = \begin{bmatrix} A(\theta(t)) & B_1(\theta(t)) & B_2 \\ C_1(\theta(t)) & D_{11}(\theta(t)) & D_{12} \\ C_2 & D_{21} & 0 \end{bmatrix} \begin{bmatrix} x \\ w \\ u \end{bmatrix}. \hspace{1cm} (4)$$

Here $x \in \mathbb{R}^n$ is the state, $w \in \mathbb{R}^{n_w}$ is the uncontrolled disturbance input, $u \in \mathbb{R}^{n_u}$ is the controlled input, $y \in \mathbb{R}^{n_y}$ is the measured output and $z \in \mathbb{R}^{n_z}$ is the controlled output.

We desire an LPV output feedback controller for internal stabilization of the closed-loop and a guaranteed $L_2$ gain not larger than $\gamma$, i.e., $\|z\|_2 \leq \gamma \|w\|_2$. The structure of such a controller depending on $\theta$, $\dot{\theta}$ is taken to be

$$\begin{pmatrix} \dot{x}_K \\ u \end{pmatrix} = \begin{bmatrix} A_K(\theta(t),\dot{\theta}(t)) & B_K(\theta(t)) \\ C_K(\theta(t)) & D_K(\theta(t)) \end{bmatrix} \begin{bmatrix} x_K \\ y \end{bmatrix}. \hspace{1cm} (5)$$

A. LPV Controller Synthesis

The synthesis technique used in this research is based on the algorithm presented in [17]. We use the matrix inequalities resulting from the projection lemma (Theorem 1). These are relaxed to obtain synthesis LMI conditions. A solution to the LMIs enables scheduling of the controller based on real-time measurement of the parameter $\theta$. For the subsequent discussion, we assume $D_{12}$ and $D_{21}$ to be of full column and row rank respectively. Let $N_R$ and $N_S$ be the bases of the null spaces of $[B_2^2 D_{12}^T 0]$ and $[C_2 D_{21} 0]$ respectively.
Theorem 1: For the open-loop LPV system (4), there exists an LPV controller of the form given by (5) that enforces global asymptotic stability and an $L_2$ gain smaller than $\gamma$ for the closed-loop system, whenever there exists parameter dependent matrices $R(\theta)$ and $S(\theta)$, satisfying the semi-infinite dimensional matrix inequalities in (6) and (7) (due to $\theta$, $\dot{\theta}$), for all admissible pairs $(\theta, \dot{\theta}) \in \mathcal{P} \times \mathcal{P}$. Hence, the authors in [17] recommend to choose $R$ and $S$, such that the

$$\dot{N}_R^T F_1(\theta) N_R < 0, \quad N_S^T F_2(\theta) N_S < 0$$

(6)

$$\begin{pmatrix} R(\theta) & I \\ I & S(\theta) \end{pmatrix} \geq 0,$$

(7)

$$F_1 =\begin{pmatrix} A(\theta) R(\theta) + R(\theta) A^T(\theta) & -\dot{\theta} \frac{\partial R}{\partial \theta}(\theta) \\ -\dot{\theta} \frac{\partial S}{\partial \theta}(\theta) & R(\theta) C_1^T(\theta) & B_1(\theta) \\ C_1(\theta) R(\theta) & -\gamma I & D_{11}(\theta) \\ B_1^T(\theta) & D_{11}^T(\theta) & -\gamma I \end{pmatrix},$$

$$F_2 =\begin{pmatrix} A^T(\theta) S(\theta) + S(\theta) A(\theta) & S(\theta) B_1(\theta) & C_1^T(\theta) \\ B_1^T(\theta) S(\theta) & -\gamma I & D_{11}^T(\theta) \\ C_1(\theta) & D_{11}(\theta) & -\gamma I \end{pmatrix},$$

Proof: A detailed proof is given in [17].

Introducing additional constraints

$$N_R^T \frac{\partial^2 F_1(\theta)}{\partial \theta^2} N_R \geq 0, \quad N_S^T \frac{\partial^2 F_2(\theta)}{\partial \theta^2} N_S \geq 0,$$

(10)

that turn out to be parameter independent, imposes convex constraints on the variables. Owing to this convexity, it is sufficient for the matrix inequalities (6), (7) and (10) to be satisfied at the generator-vertices of the convex set. This leads to a minimization problem in $\gamma$ subject to a set of LMI s in the matrix variables $R_0$, $R_1$, $S_0$, and $S_1$.

The synthesis LMIs at the generators can be solved off-line for the variables $\gamma$, $R_0$, $R_1$, $S_0$, and $S_1$ for a fixed rotational speed range. To avoid numerical trouble, we minimize the cost function: $\gamma + \epsilon \text{Tr} \left( R \left( \frac{\theta + \dot{\theta}}{2} \right) \right) + \epsilon \text{Tr} \left( S \left( \frac{\theta + \dot{\theta}}{2} \right) \right)$, where $\epsilon$ is a small positive number. When the real-time evaluation of $\dot{\theta}$ is available, $R(\theta)$ and $S(\theta)$ are constructed. Subsequently the system matrices for the LPV controller can be computed by following the steps in [17].

**Step 1**: Define $D_{cl} = D_{11} + D_{12} D_K D_{21}$ and compute $D_K$ by solving

$$\sigma_{\max}(D_{cl}) < \gamma.$$

**Step 2**: Compute $\hat{B}_K$ and $\hat{C}_K$ that satisfy

$$\begin{pmatrix} 0 & D_{21} \\ D_{21}^T & -\gamma I & D_{cl}^T \end{pmatrix} \left( \begin{array}{c} \hat{B}_K^T \\ * \end{array} \right) = \begin{pmatrix} C_2 \\ B_1^T & S \end{pmatrix},$$

(12)

$$\begin{pmatrix} 0 & D_{cl}^T \\ D_{cl} & -\gamma I \end{pmatrix} \left( \begin{array}{c} \hat{C}_K^T \\ * \end{array} \right) = \begin{pmatrix} B_2^T \\ C_1 & R + D_{12} \hat{C}_K \end{pmatrix}.$$

(13)

**Step 3**: Compute $\hat{A}_K$ as

$$\hat{A}_K = -(A + B_2 D_K C_{cl})^T + \begin{pmatrix} SB_1 + \hat{B}_K D_{21} & (C_1 + D_{12} D_{21})^T \\ -\gamma I & D_{cl}^T \end{pmatrix}^{-1} \begin{pmatrix} B_1 + B_2 D_{21} D_{21}^T \\ D_{cl} & -\gamma I \end{pmatrix} \begin{pmatrix} C_1 R + D_{12} \hat{C}_K \end{pmatrix},$$

(14)

**Step 4**: It is convenient to take $N = I - SR$ and $M = I$, such that they satisfy

$$I - SR = NM^T.$$

(15)

**Step 5**: The LPV controller is constructed from

$$A_K(\theta) = N^{-1} \Pi_A, \quad B_K(\theta) = N^{-1} \Pi_B,$$

(16)

$$C_K(\theta) = (\hat{C}_K - D_K C_2 R) M^{-T},$$

(17)

where,

$$\Pi_A = \left( S \frac{\partial k}{\partial \theta} - S(A - B_2 D_K C_2) R \right. + \hat{A}_K - \hat{B}_K C_2 R - S B_2 \hat{C}_K \left. \right) M^{-T},$$

(18)

$$\Pi_B = (\hat{B}_K + S B_2 D_K).$$

(19)

**B. Discretization of LPV controller**

For real-time implementation of the designed LPV controller, we discretize the controller based on a trapezoidal approximation given in detail in [18]. A computationally efficient implementation can be obtained by using $\Pi_A(\theta)$ and $\Pi_B(\theta)$ given above. Omitting the parameter dependence due to space restrictions, the following discretization of the LPV controller is used.

$$z_{k+1} = (I - SR - \frac{T_k}{2} \Pi_A)^{-1} (I - SR + \frac{T_k}{2} \Pi_A) z_k + \sqrt{T_k} (I - SR - \frac{T_k}{2} \Pi_A)^{-1} \Pi_B y_k,$$

(20)

$$u_k = \sqrt{\frac{T_k}{2}} C_K (I - SR - \frac{T_k}{2} \Pi_A)^{-1} (I - SR) z_k + \left( \frac{T_k}{2} C_K (I - SR - \frac{T_k}{2} \Pi_A)^{-1} \Pi_B + D_K \right) y_k.$$

(21)

**C. Implementation**

The LPV controller ($K_{LPV}$) is synthesized for $\theta \in [2, 300]$ and $\dot{\theta} \in [-50, 50]$. The synthesis problem consists of 12 LMIs in 685 variables and is solved using LMI Lab in MATLAB. In practical systems, the parameter rate of variation can seldom be measured accurately. Hence, the authors in [17] recommend to choose $R$ and $S$, such that the
controller is independent of the parameter rate of variation. We performed simulations and experiments to assess the system performance with $K_{LPV}$ that is constructed in two ways, i.e., including and excluding (i.e. $\dot{\theta} = 0$) the rate of variation. The system performance remained the same. Hence, we used a construction with $\dot{\theta} = 0$ due to computational advantages. The controller matrices are updated at a frequency of $100$ Hz whereas the controlled-input is computed at $10K$ Hz.

V. EXPERIMENTAL RESULTS

For designs in the horizontal (HOR.) and vertical (VER.) planes, the values of $\gamma$ (performance index) are given below.

<table>
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<tr>
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<th>HOR.</th>
<th>VER.</th>
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<tbody>
<tr>
<td>$K_1$</td>
<td>2.55</td>
<td>2.69</td>
</tr>
<tr>
<td>$K_2$</td>
<td>2.77</td>
<td>2.95</td>
</tr>
<tr>
<td>$K_{LPV}$</td>
<td>2.95</td>
<td>3.22</td>
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The values of $\gamma$ for $K_{LPV}$ are slightly larger than those of $K_2$ indicating there is some conservatism in the synthesis of $K_{LPV}$, hence the LPV controller is suboptimal for a fixed rotational speed. The controllers are implemented on the dSPACE DS1103 board. In order to evaluate the performance of the controllers, we rotate the AMB spindle from standstill to 2800 rpm in 28 seconds, with each controller in three separate experiments. The displacements on either ends of the spindle are measured from startup to 30 seconds. The sensitivities of the flexible model in closed-loop with $K_1$ and $K_2$ are shown in Figure 6. Figure 7 shows the spindle displacements with each of the controllers $K_1$, $K_2$ and $K_{LPV}$, at the left end in the horizontal plane. The displacements in both horizontal and vertical planes are shown as a function of time in Figures 9, 10 and 11 for each of the controllers.

The controller $K_1$ achieves very good suppression for low-frequency disturbances (< 1000 rpm) and a deteriorated performance for high speeds, which is expected from the high gain values in the sensitivity plot. With controller $K_2$, the closed loop sensitivity has a notch centered at a frequency of 300 rad/s (or 2864 rpm) and this is clearly reflected in the increased attenuation around 30 seconds (when the disturbance frequency is around 300 rad/s). The disturbance attenuation is worse for $K_2$ in comparison to $K_1$ at low frequencies, due to higher gain in the sensitivity. The LPV design ($K_{LPV}$) clearly out-performs both the robust designs by suppressing disturbances in the entire frequency range of interest. For frozen values of the parameter i.e., $\dot{\theta}(t) = \theta$ and $\theta \in [2, 300]$, frequency response of $K_{LPV}(\theta)$ is shown in Figure 8. The high-controller gains (inverted notch) correspond to the disturbance frequencies, as desired in the LPV synthesis.

VI. RECOMMENDATIONS

The dynamics of the MBC 500 system contains lightly damped modes. We observe that $H_\infty$ controllers designed with a model containing these lightly damped modes are unstable. The unstable $H_\infty$ controller offers better performance but is considerably challenging to start up. Further we observe that an LPV controller designed on the same lines as discussed in this paper for such a model is also unstable. The unstable controllers promise performance benefits that aid in
operating beyond the first critical speed. Our current research efforts are focused on implementation of these unstable LTI and LPV controllers and experimental evaluation of their performance.

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REFERENCES