Decentralized Fault Detection for a Class of Large-Scale Nonlinear Uncertain Systems

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Abstract—In this paper, a decentralized fault detection scheme is developed for a class of distributed large-scale nonlinear uncertain systems. For each subsystem in the large-scale system, a fault detection estimator is designed by utilizing local measurements and certain communicated information from interconnected subsystems. Under certain assumptions, adaptive threshold for fault detection in each local subsystem is derived, and the robustness property of the decentralized fault detection scheme is investigated. A simulation example of automated highway systems is used to illustrate the effectiveness of the proposed method.

I. INTRODUCTION

A consequence of modern technological advances is the creation of distributed large-scale critical infrastructure systems, which need to operate reliably at all times, despite the possible occurrence of faulty behavior in some subsystems. Examples of such distributed large-scale infrastructure systems include power generation and distribution systems, telecommunication networks, water distribution networks, etc. The design of fault diagnosis and accommodation schemes is a crucial step in achieving reliable and safe operations of such large-scale systems.

In the last two decades, there has been a lot of research activity in the design and analysis of fault diagnosis and accommodation schemes (see, for example, [1], [4]). Most of these results are based on a centralized fault diagnosis architecture. In practice, it is very difficult to address the problem of diagnosing faults in a large-scale system with a centralized architecture because of the constraints on computational power and communication bandwidth. As a result, in recent years, there has been a significantly increasing research interest in decentralized fault diagnosis schemes (see, e.g., [2], [5], [8], [12] and the references cited therein).

In previous papers, a centralized fault detection and isolation (FDI) methodology for nonlinear uncertain systems has been developed [13], [14], [15]. This paper significantly extends the previous results by presenting a decentralized fault detection scheme for a class of large-scale nonlinear systems. In the presented decentralized fault detection architecture, a fault detection estimator is designed for each local subsystem in the large-scale system by utilizing local measurements and certain communicated information from directly interconnected subsystems. The decentralized fault detection method is presented with a rigorous analytical framework aiming at characterizing the behaviors of the diagnostic scheme. Specifically, the analysis focuses on the derivation of adaptive threshold for fault detection in each subsystem and investigation of the robustness property of the decentralized fault detection method.

The paper is organized as follows. In Section II, the problem of decentralized fault detection is formulated. Section III describes the decentralized fault detection scheme and investigates the fault detection threshold design and the robustness property with respect to modeling uncertainty for each local detection estimator. In Section IV, an application example of decentralized fault detection in automated highway systems is presented. Finally, Section V gives some concluding remarks.

II. PROBLEM FORMULATION

Consider a large-scale nonlinear dynamic system composed of M subsystems with each subsystem described by the following differential equation

\[ \dot{x}_i = A_i x_i + \zeta_i(y_i, u_i) + \phi_i(x_i, u_i, t) + \sum_{j=1, j \neq i}^{M} h_{ij}(x_j) + \beta_i(t - T_0) E_i \phi_i(y_i, u_i) \] (1)
\[ y_i = C_i x_i \] (2)

where \( x_i \in \mathbb{R}^{n_i}, \ u_i \in \mathbb{R}^{m_i} \), and \( y_i \in \mathbb{R}^{l_i} \) are the state vector, input vector, and output vector of the ith subsystem (\( n_i \geq l_i \)), respectively, for \( i = 1, \ldots, M \). Furthermore, \( \zeta_i : \mathbb{R}^{n_i} \times \mathbb{R}^{m_i} \rightarrow \mathbb{R}^{n_i}, \ \phi_i : \mathbb{R}^{n_i} \times \mathbb{R}^{m_i} \times \mathbb{R}^{l_i} \rightarrow \mathbb{R}^{n_i}, \ h_{i} : \mathbb{R}^{n_i} \times \mathbb{R}^{n_j} \rightarrow \mathbb{R}^{l_i}, \ h_{ij} : \mathbb{R}^{n_i} \rightarrow \mathbb{R}^{n_j} \) are smooth vector fields. The constant matrices \( E_i \in \mathbb{R}^{n_i \times n_i} \) and \( C_i \in \mathbb{R}^{l_i \times n_i} \), with \( q_i \leq l_i \) are of full rank, and \( (A_i, C_i) \) is an observable pair. The vector field \( \zeta_i \) appearing in (1) is the known nonlinearity of the nominal dynamics of the ith subsystem, \( \phi_i \) represents the modeling uncertainty, and \( \beta_i(t - T_0) E_i \phi_i(y, u) \) denotes the changes in the local system dynamics due to the occurrence of a fault. Specifically, \( \beta_i(t - T_0) \) is the time profile of a fault which occurs at some unknown time \( T_0 \), \( \phi_i(y_i, u_i) \) represents the nonlinear fault function, and \( E_i \) is a fault distribution matrix. Additionally, the vector field \( h_{ij} \) represents the direct interconnection between the ith subsystem and the jth subsystem, \( j \in \{1, \ldots, M\} \setminus \{i\} \). Note that likely many functions \( h_{ij} \) are identically zero in a large-scale system (i.e., many subsystems do not directly influence subsystem \( i \)).

In this paper, we only consider the case of abrupt (sudden) faults; therefore, \( \beta_i(\cdot) \) takes on the form of a step function given by \( \beta_i(t - T_0) = 0 \) if \( t < T_0 \) and \( \beta_i(t - T_0) = 1 \) if \( t \geq T_0 \).
Assumption 1. The matrix $E_i$ in (1) satisfies the condition of $\text{rank}(C_iE_i) = q_i$, for $i = 1, \cdots, M$.

As described in [12], under Assumption 1, there exists a linear transformation of coordinates $z_i = T_i x_i = [z_{i1}^T z_{i2}^T]^T$ with $z_{i1} \in \mathbb{R}^{(n_i-t_i)}$ and $z_{i2} \in \mathbb{R}^{l_i}$, such that

- $T_i A_i^{-1} = \begin{bmatrix} A_{i1} & A_{i2} \\ A_{i3} & A_{i4} \end{bmatrix}$,
- $T_i E_i = \begin{bmatrix} 0 \\ E_{i2} \end{bmatrix}$, where $E_{i2} \in \mathbb{R}^{l_i \times q_i}$, and
- $C_i T_i^{-1} = [0 \ C_i]$, where $C_i \in \mathbb{R}^{q_i \times l_i}$ is orthogonal.

Therefore, in the new coordinate system, the model of the $i$th subsystem given by (1) is described by

$$
\dot{z}_{i1} = A_{i1} z_{i1} + A_{i2} z_{i2} + \rho_{i1}(y_i, u_i) + \eta_{i1}(z_i, u_i, t) + \sum_{j=1 \atop j \neq i}^M H_{ij}^1(z_j)
$$

$$
\dot{z}_{i2} = A_{i3} z_{i1} + A_{i4} z_{i2} + \rho_{i2}(y_i, u_i) + \eta_{i2}(z_i, u_i, t) + \beta_i(t - T_0) E_{i2} \phi_i(y_i, u_i) + \sum_{j=1 \atop j \neq i}^M H_{ij}^2(z_j)
$$

$$
y_i = C_i z_{i2},
$$

where

$$
\begin{bmatrix}
\rho_{i1}(y_i, u_i) \\
\rho_{i2}(y_i, u_i)
\end{bmatrix} = T_i G_i(y_i, u_i),
\begin{bmatrix}
\eta_{i1}(z_i, u_i, t) \\
\eta_{i2}(z_i, u_i, t)
\end{bmatrix} = T_i h_{ij}(T_i^{-1} z_j).
$$

With a more general structure of the nonlinear fault model, system (3) can be extended to

$$
\dot{z}_{i1} = A_{i1} z_{i1} + A_{i2} z_{i2} + \rho_{i1}(y_i, u_i) + \eta_{i1}(z_i, u_i, t) + \sum_{j=1 \atop j \neq i}^M H_{ij}^1(z_j)
$$

$$
\dot{z}_{i2} = A_{i3} z_{i1} + A_{i4} z_{i2} + \rho_{i2}(z_i, u_i) + \eta_{i2}(z_i, u_i, t) + \beta_i(t - T_0) f_i(y_i, u_i) + \sum_{j=1 \atop j \neq i}^M H_{ij}^2(z_j)
$$

$$
y_i = C_i z_{i2},
$$

where $f_i : \mathbb{R}^{l_i} \times \mathbb{R}^{m_i} \rightarrow \mathbb{R}^{l_i}$ is a smooth vector field representing the nonlinear fault function in each subsystem under consideration. Clearly, (3) is a special case of (4) with $f_i(y_i, u_i) = E_{i2} \phi_i(y_i, u_i)$ and with $\rho_{i2}$ being a nonlinear function of measurable variables $y_i$ and $u_i$ instead of the state vector $z_i$ and input vector $u_i$.

The main objective of this paper is to develop a robust decentralized fault detection scheme for the class of large-scale nonlinear dynamic systems that can be transformed into (4). Throughout the paper, the following assumptions are introduced:

Assumption 2. The functions $\eta_{i1}$ and $\eta_{i2}$ in (4), representing the unstructured modeling uncertainty, are possibly unknown nonlinear functions of $z_i$, $u_i$, and $t$, but bounded, i.e., $\forall (z_i, y_i, u_i) \in Z_i \times U_i \times U_i$, $\forall t \geq 0$,

$$
|\eta_{i1}(z_i, u_i, t)| \leq \bar{\eta}_{i1}, \quad |\eta_{i2}(z_i, u_i, t)| \leq \bar{\eta}_{i2}(y_i, u_i, t),
$$

where the constant bound $\bar{\eta}_{i1}$ and the bounding function $\bar{\eta}_{i2}(y_i, u_i, t)$ are known, and $\bar{\eta}_{i2}$ is uniformly bounded in $Y_i \times U_i \times \mathbb{R}^r$. Additionally, $Z_i \subset \mathbb{R}^{n_i}$, $U_i \subset \mathbb{R}^{m_i}$, and $Y_i \subset \mathbb{R}^{l_i}$ are compact sets of admissible state variables, inputs, and outputs, respectively.

Assumption 3. The system state vector $z_i$ of each subsystem remains bounded before and after the occurrence of a fault, i.e., $z_i(t) \in L_\infty$, $\forall t \geq 0$.

Assumption 4. The known nonlinear term $\rho_{i2}(z_i, u_i)$ in (4) satisfies the following inequality: $\forall u_i \in U_i$ and $\forall z_i, \dot{z}_i \in Z_i$,

$$
|\rho_{i2}(z_i, u_i) - \rho_{i2}(\dot{z}_i, u_i)| \leq \sigma(y_i, u_i)|z_i - \dot{z}_i|,
$$

where $\sigma(y_i, u_i)$ is a known function.

Assumption 5. The interconnection term $H_{ij}^1(z_j)$ and $H_{ij}^2(z_j)$ in (4) are uniformly Lipschitz, i.e., $\forall z_j, \dot{z}_j \in Z_j$,

$$
|H_{ij}^1(z_j) - H_{ij}^1(\dot{z}_j)| \leq \gamma_{ij}^1|z_j - \dot{z}_j|,
$$

$$
|H_{ij}^2(z_j) - H_{ij}^2(\dot{z}_j)| \leq \gamma_{ij}^2|z_j - \dot{z}_j|,
$$

where $\gamma_{ij}^1$ and $\gamma_{ij}^2$ are the known Lipschitz constants for $H_{ij}^1(z_j)$ and $H_{ij}^2(z_j)$, respectively.

Assumption 2 characterizes the class of modeling uncertainty under consideration. The bounds on the unstructured modeling uncertainty are needed in order to be able to distinguish between the effects of faults and modeling uncertainty (see [13], [14]). It is worth noting that the unstructured modeling uncertainty considered in this paper is more general than the types of uncertainty considered for decentralized fault diagnosis in the literature, which usually assume the absence of modeling uncertainty (e.g., [8]) or structured modeling uncertainty (e.g., [12]). In the case of structured models of the modeling uncertainty, to achieve robustness, it is often assumed that certain rank conditions are satisfied by the uncertainty distribution matrix. On the other hand, the utilization of structured uncertainty with additional assumptions on the distribution matrix may allow the design of FDI schemes that completely decouple the fault from the modeling uncertainty.

Assumption 3 requires the boundedness of the state variables before and after the occurrence of a fault in each subsystem. This is a technical assumption required for well-posedness since the decentralized FDI design under consideration does not influence the closed-loop dynamics and stability. It is important to note that the proposed decentralized FDI design does not depend on the structure of the decentralized controller.

Assumption 5 requires the interconnection terms between subsystems to be Lipschitz. Several examples of large-scale nonlinear systems with Lipschitz interconnection terms have been considered in literature (see, for instance, the automated highway system [8], [10] and inverted pendulums [11]).

Remark 1. The nominal system dynamics in (4) is similar to the model considered in [12]. The objective of this paper is to develop a robust fault detection scheme, while [12] presented a fault estimation method using sliding mode.
observer techniques, which requires additional assumptions on the distribution matrices of the modeling uncertainty terms \( \eta_1 \) and \( \eta_2 \) as well as the fault function \( f_i \) in (4). Moreover, in the previous papers [13], [15], centralized fault diagnosis schemes for nonlinear systems were developed. In this research work, the problem of decentralized fault diagnosis for large-scale nonlinear systems is investigated. With the interconnection terms \( H_{ij}^1(z_j) \) and \( H_{ij}^2(z_j) \) among subsystems (see (4)) and limited information being communicated, the design and analysis of a decentralized fault diagnosis method become more challenging.

III. DECENTRALIZED FAULT DETECTION METHOD

A. Decentralized Fault Detection Estimators

Based on the subsystem model given by (4), the decentralized fault detection estimator (FDE) for each local subsystem is chosen as:

\[
\dot{\hat{z}}_{i1} = A_{i1} \hat{z}_{i1} + A_{i2} \tilde{C}_i^{-1} y_i + \rho_{i1}(y_i, u_i) + \sum_{j \neq i}^{M} H_{ij}^1(\hat{z}_j) \\
\dot{\hat{z}}_{i2} = A_{i3} \hat{z}_{i1} + A_{i4} \hat{z}_{i2} + \rho_{i2}(\hat{z}_i, u_i) + L_i(y_i - \hat{y}_i) \\
\dot{\hat{y}}_i = \tilde{C}_i \hat{z}_{i2},
\]

where \( \hat{z}_{i1}, \hat{z}_{i2}, \) and \( \hat{y}_i \) denote the estimated local state and output variables of the \( i \)th subsystem, \( i = 1, \cdots, M \), respectively. The matrix \( L_i \in \mathbb{R}^{d_i \times t} \) is a design gain matrix, and \( \hat{z}_j \triangleq [(\hat{z}_{j1})^T (\tilde{C}_j^{-1} y_j)^T]^T \) (here \( \hat{z}_{j1} \) is the estimated state vector for \( z_{j1} \) of the \( j \)th interconnected subsystem). The initial conditions are \( \hat{z}_{i1}(0) = 0 \) and \( \hat{z}_{i2}(0) = \tilde{C}_i^{-1} y_i(0) \). It is worth noting that the decentralized FDE (9) for each local subsystem is constructed based on local measurements (i.e., \( u_i \) and \( y_i \)) and the communicated information \( \hat{z}_j \) from the \( j \)th interconnected subsystem. Many decentralized estimation and fault diagnosis methods in literature allow limited communication among interconnected subsystems (see, for instance, [8], [9], [12], [2], [5]).

For each local FDE, let \( \hat{z}_{i1} \triangleq z_{i1} - \hat{z}_{i1} \) and \( \hat{z}_{i2} \triangleq z_{i2} - \hat{z}_{i2} \) denote the state estimation errors, and \( \hat{y}_i \triangleq y_i - \hat{y}_i \) denote the output estimation error. Then, before fault occurrence (i.e., for \( t < T_0 \)), we have

\[
\dot{\hat{z}}_{i1} = A_{i4} \hat{z}_{i1} + \eta_i + \sum_{j \neq i}^{M} [H_{ij}^1(z_j) - H_{ij}^1(\hat{z}_j)] \\
\dot{\hat{z}}_{i2} = A_{i4} \hat{z}_{i2} + A_{i3} \hat{z}_{i1} + \rho_{i2}(z_i, u_i) - \rho_{i2}(\hat{z}_i, u_i) \\
+ \eta_{i2}(z_i, u_i, t) + \sum_{j \neq i}^{M} [H_{ij}^2(z_j) - H_{ij}^2(\hat{z}_j)] \\
\dot{\hat{y}}_i = \tilde{C}_i (z_{i2} - \hat{z}_{i2}) = \tilde{C}_i \hat{z}_{i2},
\]

where \( A_{i4} \triangleq A_{i4} - L_i \tilde{C}_i \). Note that, since \( \tilde{C}_i \) is nonsingular, we can always choose \( L_i \) to make \( A_{i4} \) stable.

B. Adaptive Thresholds for Decentralized Fault Detection

The following lemma provides a bounding function on the state estimation error vector

\[
\hat{z}_1(t) \triangleq [(\hat{z}_{11})^T, \cdots, (\hat{z}_{M1})^T]^T
\]

for the time period before the occurrence of any faults (i.e., \( 0 \leq t < T_0 \)).

Lemma 1. Consider the system described by (4) and the fault detection estimator described by (9). Assume that there exist a pair of symmetric positive definite matrices \( P_i \) and \( Q_i \), for \( i = 1, \cdots, M \), such that the following matrix inequality

\[
A_{i1}^T P_i + P_i A_{i1} + Q_i P_i + (M - 1)I + \sum_{j \neq i}^{M} (\gamma_{ij})^2 P_j P_i < -Q_i
\]

is solvable. Then, for \( 0 \leq t < T_0 \), the state estimation error vector \( \hat{z}_1(t) \) satisfies:

\[
|\hat{z}_1|^2 \leq \frac{\sum_{i=1}^{M} (\eta_i)^2}{c \lambda_{\min}(P)} + \frac{1}{\lambda_{\min}(P)} \left( V_0 - \frac{1}{c} \sum_{i=1}^{N} (\eta_i)^2 \right) e^{-ct},
\]

where the constant \( c \triangleq \lambda_{\min}(Q)/\lambda_{\max}(P), \quad P \triangleq \text{diag}\{P_1, \cdots, P_M\}, \quad Q \triangleq \text{diag}\{Q_1, \cdots, Q_M\} \).

Proof: For the \( i \)th subsystem, let us consider a Lyapunov function candidate \( V_i = \hat{z}_{i1}^T P_i \hat{z}_{i1} \), where \( P_i \) is a symmetric positive definite matrix. The time derivative of \( V_i \) along the solution of (10) is given by

\[
\dot{V}_i = \hat{z}_{i1}^T (A_{i1}^T P_i + P_i A_{i1} + 2 \hat{z}_{i1} P_i \eta_i(z_i, u_i, t) + 2 \hat{z}_{i1} P_i \sum_{j \neq i}^{M} [H_{ij}^1(z_j) - H_{ij}^1(\hat{z}_j)].
\]

Note that

\[
z_{i2} - \hat{z}_j = \left[ \begin{array}{c} z_{i2} - z_{i1} \\ z_{i2} - \hat{z}_{i2} - C_{j1}^{-1} y_j \end{array} \right] = \left[ \begin{array}{c} \hat{z}_{i2} \\ 0 \end{array} \right].
\]

Therefore, based on (7) and (17), we have

\[
2 \hat{z}_{i1}^T P_i \sum_{j \neq i}^{M} [H_{ij}^1(z_j) - H_{ij}^1(\hat{z}_j)] \\
\leq 2 |P_i \hat{z}_{i1}| \sum_{j \neq i}^{M} (\gamma_{ij})^2 |z_{j1} - \hat{z}_{j1}| = \sum_{j \neq i}^{M} 2 (\gamma_{ij})^2 |P_i \hat{z}_{i1}| |\tilde{z}_{j1}| \\
\leq \sum_{j \neq i}^{M} (\gamma_{ij})^2 |z_{i1}^T P_i \hat{z}_{i1} + \hat{z}_{i1}^T P_i \hat{z}_{i1} + |\eta_i|^2.
\]

Additionally, we have

\[
2 \hat{z}_{i1}^T P_i \eta_i \leq 2 |P_i \eta_i| |\hat{z}_{i1}^T P_i \hat{z}_{i1} + |\eta_i|^2.
\]
By using (16), (18), and (19), we obtain
\[
\dot{V}_i \leq \tilde{z}_i^T \left[ A_{i1}^T P_i + P_i A_{i1} + P_i P_i + \sum_{j=1 \atop j \neq i}^{M} (\gamma_{ij})^2 P_i P_i \right] \tilde{z}_i + |\eta_{i1}|^2 + \sum_{j=1 \atop j \neq i}^{M} \tilde{z}_i^T \tilde{z}_{j1}.
\] (20)

Now, let us consider the following overall Lyapunov function candidate for the large-scale system: \( V = \sum_{i=1}^{M} V_i = \sum_{i=1}^{M} \tilde{z}_i^T P_i \tilde{z}_i \). Note that
\[
\sum_{i=1}^{M} \sum_{j=1 \atop j \neq i}^{M} \tilde{z}_j^T \tilde{z}_{j1} = (M-1) \sum_{i=1}^{M} \tilde{z}_i^T \tilde{z}_{i1}.
\] (21)

Therefore, from (20), (21), and (14), we have
\[
\dot{V} \leq \sum_{i=1}^{M} \tilde{z}_i^T \left[ A_{i1}^T P_i + P_i A_{i1} + P_i P_i + (M-1)I \right] \tilde{z}_i + \sum_{i=1}^{M} |\eta_{i1}|^2
\leq -\sum_{i=1}^{M} \tilde{z}_i^T Q_i \tilde{z}_i + \sum_{i=1}^{M} |\eta_{i1}|^2
= -\tilde{z}_1^T Q \tilde{z}_1 + \sum_{i=1}^{M} |\eta_{i1}|^2.
\]
where \( \tilde{z}_1 \) and the matrix \( Q \) are defined in (13) and Lemma 1, respectively. By using the Rayleigh principle and the definition of \( V(t) \), we have
\[
\dot{V} \leq -\lambda_{\text{min}}(Q)|\tilde{z}_1|^2 + \sum_{i=1}^{M} |\eta_{i1}|^2 \leq -cV + \sum_{i=1}^{M} |\eta_{i1}|^2.
\]

Based on the previous inequality, it can be easily shown that
\[
V(t) \leq \left( \frac{1}{c} \sum_{i=1}^{M} |\eta_{i1}|^2 \right) + \left( V(0) - \frac{1}{c} \sum_{i=1}^{M} |\eta_{i1}|^2 \right) e^{-ct}.
\]

Note that we can always choose a constant \( V_0 \) such that \( V(0) < V_0 \). Thus, based on the definition of \( V(t) \), the Rayleigh principle, and Assumption 2, the proof of (15) can be immediately concluded.

Now, we analyze the state estimation error \( \tilde{z}_{i2}(t) \) of the \( i \)th subsystem. For \( t < T_0 \), the solution of (11) is given by
\[
\tilde{z}_{i2}(t) = \int_0^t e^{A_{i1}(t-\tau)} \left[ A_{i3} \tilde{z}_{i1}(\tau) + \eta_{i2}(z_i, u_i, t) \right] d\tau + \int_0^t e^{A_{i1}(t-\tau)} [\rho_{i2}(z_i, u_i) - \rho_{i2}(\tilde{z}_i, u_i)] d\tau + \int_0^t e^{A_{i1}(t-\tau)} \sum_{j=1 \atop j \neq i}^{M} [H_{ij}(z_j) - H_{ij}(\tilde{z}_j)] d\tau.
\]
Therefore, for each component of the output estimation error, i.e., \( \tilde{y}_{ip}(t) \equiv \tilde{C}_{ip} \tilde{z}_{i2}(t) \), \( p = 1, \cdots, l_i \), where \( \tilde{C}_{ip} \) is the \( p \)th row vector of matrix \( \tilde{C}_i \), by applying the triangle inequality, we have
\[
|\tilde{y}_{ip}(t)| \leq \int_0^t \tilde{C}_{ip} e^{A_{i1}(t-\tau)} \sum_{j=1 \atop j \neq i}^{M} \left[ H_{ij}^2(z_j) - H_{ij}^2(\tilde{z}_j) \right] d\tau + \int_0^t \tilde{C}_{ip} e^{A_{i1}(t-\tau)} \left[ A_{i3} \tilde{z}_{i1}(\tau) + \eta_{i2} \right] d\tau + \int_0^t \tilde{C}_{ip} e^{A_{i1}(t-\tau)} \left[ \rho_{i2}(z_i, u_i) - \rho_{i2}(\tilde{z}_i, u_i) \right] d\tau.
\]
Based on (8) and (17), we have
\[
|H_{ij}^2(z_j) - H_{ij}^2(\tilde{z}_j)| \leq \gamma_{ij}^2 |\tilde{z}_j|.
\]
Therefore, we obtain
\[
|\tilde{y}_{ip}(t)| \leq k_{ip} \int_0^t e^{-\lambda_{ip}(t-\tau)} \left( ||A_{i3}|| |\tilde{z}_{i1}(\tau)| + |\eta_{i2}| + \sum_{j=1 \atop j \neq i}^{M} \gamma_{ij}^2 |\tilde{z}_j| + \sigma(y_i, u_i) |\tilde{z}_{i1}(\tau)| \right) d\tau.
\]

where \( k_{ip} \) and \( \lambda_{ip} \) are positive constants chosen such that \( |\tilde{C}_{ip} e^{A_{i1} t}| \leq k_{ip} e^{-\lambda_{ip} t} \) (since \( A_{i4} \) is stable, constants \( k_{ip} \) and \( \lambda_{ip} \) satisfying the above inequality always exist [3]). By letting
\[
\theta_i = [\gamma_{i2}^2, \cdots, \gamma_{i(i-1)}^2, ||A_{i3}|| + \sigma(y_i, u_i)], \quad \gamma_{ij} \triangleq \gamma_{ij}^2 \quad \text{for } j \neq i,
\]
(22)

(that is, the components of \( \theta_i \) are given by \( \theta_{ii} = ||A_{i3}|| + \sigma(y_i, u_i) \), and \( \theta_{ij} \triangleq \gamma_{ij}^2 \) for \( j \neq i \)), the previous inequality can be rewritten as
\[
|\tilde{y}_{ip}(t)| \leq k_{ip} \int_0^t e^{-\lambda_{ip}(t-\tau)} \left( \sum_{j=1 \atop j \neq i}^{M} \theta_j |\tilde{z}_{j1}(\tau)| + |\eta_{i2}| \right) d\tau
\leq k_{ip} \int_0^t e^{-\lambda_{ip}(t-\tau)} \left( \theta_1 |\tilde{z}_{11}(\tau)| + |\eta_{i2}| \right) d\tau.
\]

Now, based on (23) and (15), we obtain
\[
|\tilde{y}_{ip}(t)| \leq k_{ip} \int_0^t e^{-\lambda_{ip}(t-\tau)} \left[ |\theta_1| \chi(\tau) + |\eta_{i2}| \right] d\tau.
\]

where
\[
\chi(t) \triangleq \left\{ \frac{1}{c} \left( \sum_{i=1}^{M} (\bar{\eta}_{i1})^2 + (c \tilde{V}_0 - \sum_{i=1}^{M} (\bar{\eta}_{i1})^2) e^{-ct} \right) \right\}^{1/2}
\]
with \( \tilde{c} \triangleq c \lambda_{\text{min}}(P) \).

Therefore, based on the above analysis, we have the following

Decentralized Fault Detection Decision Scheme: The decision on the occurrence of a fault (detection) in the \( i \)th subsystem is made when the modulus of at least one component of the output estimation error (i.e., \( \tilde{y}_{ip}(t) \)) generated by the local FDE exceeds its corresponding threshold \( \nu_{ip}(t) \).
given by

\[ \nu_{ip}(t) \triangleq k_{ip} \int_{0}^{t} e^{-\lambda_{ip}(t-\tau)} \left[ \gamma_{i}(\tau) + \eta_{i2} \right] \, d\tau. \]  

The fault detection time \( T_d \) is defined as the first time instant such that \( |\tilde{y}_{ip}(T_d)| > \nu_{ip}(T_d) \), for some \( T_d \geq T_0 \) and some \( p \in \{1, \ldots, l_i\} \), that is,

\[ T_d \triangleq \inf \left\{ t \geq 0 : |\tilde{y}_{ip}(t)| > \nu_{ip}(t) \right\}. \]

The above design and analysis is summarized by the following result:

**Theorem 1 (Robustness):** For the nonlinear system (4), the decentralized fault detection decision scheme, characterized by the fault detection estimator (9) and adaptive thresholds (26) for each subsystem, guarantees that there will be no false alarms before fault occurrence (i.e., for \( t \leq T_0 \)).

**Remark 2.** It is worth noting that \( \nu_{ip}(t) \) given by (26) is an adaptive threshold for fault detection, which have obvious advantage over a fixed one. Moreover, the threshold \( \nu_{ip}(t) \) can be easily implemented using linear filtering techniques [13], [14]. Additionally, the constants \( V_0 \) in (25) is (possibly conservative) bound for the unknown initial conditions \( V(0) \).

However, note that, since the effect of this bound decreases exponentially (i.e., it is multiplied by \( e^{-\alpha t} \)), the practical use of such a conservative bound will not affect significantly the performance of the decentralized fault detection algorithm.

**IV. APPLICATION TO AUTOMATED HIGHWAY SYSTEMS**

With increasing traffic congestion, there has been a significant interest in the development of automated highway systems (AHS). In AHS, vehicles are driven automatically with onboard controllers to avoid human error that caused many of today’s automobile accidents. Spooner and Passino [10] described the car-following problem in which only tracking information is available (as opposed to information about lead and other subsequent vehicles) to each following vehicle. To ensure the safe and reliable operations of AHS, the development of fault diagnosis technologies is particularly important.

As described in [10], the dynamics of the \( i \)-th vehicle in the car-following system may be expressed as follows:

\[
\begin{align*}
\dot{v}_i &= v_i - v_{i-1} \\
\dot{\psi}_i &= \frac{1}{m_i} (-D_i v_i^2 - d_i + \xi_i) \\
\dot{\xi}_i &= \frac{1}{\tau_i} (-\xi_i + u_i) \\
y_i &= \begin{bmatrix} \psi_i + \omega v_i \\ \xi_i \end{bmatrix},
\end{align*}
\]

where \( \psi_i \) is the distance between the \( i \)-th and the \( (i-1) \)-th vehicle, \( v_i \) is the \( i \)-th vehicle’s velocity, and \( \xi_i \) is the driving/breaking force applied to the \( i \)-th vehicle. The control input is represented by \( u_i \) (if \( u_i > 0 \), then it represents a throttle input, and if \( u_i < 0 \), then it represents a brake input). The system output \( y_i \) allows for a velocity-dependent intervehicle spacing due to the \( \varpi v_i \) term (\( \varpi \) is a positive constant). As the velocity of the \( i \)-th vehicle increases, the distance between the \( i \)-th vehicle and \( (i-1) \)-th vehicle should increase. Additionally, \( m_i \) is the vehicle mass, \( D_i \) is the aerodynamic drag, \( d_i \) and \( \tau_i \) are respectively the constant frictional force and the engine/break time constant. In this simulation example, we consider the case of 3 vehicles (i.e., \( M = 3 \)). As in [10], [12], the following simulation parameters are used: \( m_i = 1300 \text{kg} \), \( D_i = 0.3 \text{Ns}^2/\text{m}^2 \), \( d_i = 100 \text{N} \), \( \tau_i = 0.2 \text{s} \), and \( \varpi = 0.4 \).

Let \( x_{i1} = \psi_i \), \( x_{i2} = v_i \), \( x_{i3} = \xi_i \), and the state vector of the \( i \)-th vehicle \( x_i = [x_{i1}, x_{i2}, x_{i3}]^T \), for \( i = 1, 2, 3 \). Then, the dynamics of the 3-vehicle following system are given by

\[
\begin{align*}
\dot{x}_i &= \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 1300 \\ 0 & 0 & -5 \end{bmatrix} x_i + \begin{bmatrix} 0 \\ -\frac{10}{13} - \frac{0.3}{1300} \end{bmatrix} \\
&\quad - \begin{bmatrix} x_{(i-1)2} \\ 0 \\ 0 \end{bmatrix} + \varphi_i(x_{i1}, u_i, t) + \beta_i \phi_i(y_i, u_i) \\
y_i &= \begin{bmatrix} 1 & 0 & 4 \\ 0 & 0 & 1 \end{bmatrix} x_i.
\end{align*}
\]

Note that the effects of modeling uncertainty \( \varphi_i \) and faults \( \phi_i \) have been included in the above model. Specifically, here the modeling uncertainty \( \varphi_i \) is assumed to include: (i) a disturbance signal in the form of \( \theta_i \sin(t) \) with \( |\theta_i| \leq 0.5 \) in the state equation of \( \varphi_2 \) and \( \varphi_3 \), respectively; (ii) up to 3% inaccuracy in the engine/break time constant \( \tau_i \) of all vehicles. In the actual simulation results, \( \theta_i = 0.4 \) and 2% inaccuracy in \( \tau_i \) are used.

By using a linear transformation of coordinates \( z_i = [z_{i1}^T, z_{i2}^T]^T = T_i x_i \) with \( T_i = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.04 & 0 \\ 0 & 0.01 \end{bmatrix} \), the state space model in the new coordinate system is

\[
\begin{align*}
\dot{z}_i &= \begin{bmatrix} -2.5 & 25 & 0 \\ -0.25 & 2.5 & 3.077 \times 10^{-5} \\ 0 & 0 & -5 \end{bmatrix} z_i + \begin{bmatrix} 0 \\ 5 \end{bmatrix} \begin{bmatrix} -0.0031 - 0.0058 (z_{i2} - 0.1 z_{i1}) \\ 25 (z_{i1} - 0.1 z_{i1}) \\ -25 (z_{i1} - 0.1 z_{i1}) \end{bmatrix} + \eta_i + \beta_i f_i \\
y_i &= \begin{bmatrix} 0 & 0 & 10 \\ 0 & 0 & 1 \end{bmatrix} z_i.
\end{align*}
\]

The above model is clearly in the form of (4). Based on the modeling uncertainty described above. We have \( \eta_{i1} = 0.5 \) and \( \eta_{i2} = 0.05 + 0.102 |u_i| \). A decentralized fault detection estimator is constructed for each vehicle by utilizing the method presented in Section III. The initial conditions of the vehicles are chosen to be \( x_i = [0, 10, 0]^T \). Decentralized stabilizing controllers for the vehicles are designed, such that, the velocity of first vehicle tracks a reference signal of \( 10 + 5 \text{sint} \text{MPH} \), and each of the two other vehicles follows the previous one with controlled distance. The gain matrix \( L_i \) of the FDE is chosen such that the poles of matrix \( A_{i4} \)
are located at -1.5 and -1.7, respectively. Consequently, the related design constants are $k_{i1} = k_{i2} = 1$, $\lambda_{i1} = -1.5$, and $\lambda_{i2} = -1.7$. Additionally, we choose the matrices $P_i = [0.5 \ 0 \ 0; 0.5 \ 0 \ 0; 0.0 \ 0.5 \ 0]$, which results in $c = 0.6752$.

Figure 1 shows the simulation results when a process fault in the form of $\phi_2(y, u) = [0, -0.3\varphi_2/m_2, 0]^T$ occurs to the second vehicle at $T_0 = 10$ second. As can be seen, the fault is successfully detected at approximately $T_d = 12.58$ second. Figure 2 shows the simulation results when an actuator fault in the form of $\phi_3(y, u) = [0, 0, -0.1u_3/\tau_3]^T$ occurs to the third vehicle at $T_0 = 10$ second. Again, the fault is timely detected.

The robustness of the detection scheme is investigated. An application example of automated highway system is used to show the effectiveness of the algorithm. The extension of the presented diagnostic method to include faults in the communication link between interconnected subsystems is one interesting topic for future research. Another direction for future research is to consider large-scale nonlinear systems with more general nonlinearities and interconnection terms.

References


V. Conclusions

In this paper, a decentralized fault detection scheme is presented for a class of large-scale nonlinear uncertain systems. Under certain assumptions, adaptive thresholds are designed for decentralized fault detection in each subsystem.