Robust characteristic-based MPC of a fixed-bed reactor

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Abstract—In this work a model predictive control methodology is applied to a set of hyperbolic partial differential equations (PDEs) which models a chemical fixed-bed reactor. Initially, the model of the fixed-bed reactor is linearized and by the method of characteristics is transformed into the set of ODEs which is explored within the model predictive controller synthesis. We consider uncertainties present in the reactor model which are taken into account by the construction of the polytopic family of plants and subsequent robust model predictive controller synthesis which ensures input and state constraints satisfaction. The proposed robust control problem formulation and the performance of the controller have been evaluated by simulations.

Index Terms—Robust MPC, Distributed systems, Method of characteristics, Input/State Constraints

I. Introduction

The objective of this work is to study the control of a fixed bed reactor with catalyst deactivation. In order to capture all of the main “macroscopic” phenomena (i.e., reactions, diffusion, convection, and so forth), the model of a fixed bed reactor takes the form of a mixed set of partial differential, ordinary differential, and algebraic equations. In this system variables are functions of time and space. Such systems and many others (e.g., systems modeled by partial difference equations, integral equations and delay differential equations) are called distributed parameter systems (DPS) and their main characteristic is the infinite dimensional nature of the systems. The prime example of transport-reaction type of DPS in chemical engineering is a model of a fixed bed reactor with catalyst deactivation in which convective transport mechanism, for the set of operating conditions, can dominate the diffusive type of transport phenomena. In other words, a material and energy balances can be reduced to the set of hyperbolic PDEs [1]. Another important aspect of fixed-bed reactor models is the uncertainty in the physical parameters which characterize convection-reaction process system.

Over the recent years, research on control of DPS was oriented towards methodologies that deal with infinite dimensional nature of these systems. Early works in this area were on necessary conditions on stabilizability of DPS by either finite dimensional controllers [2], [3] or infinite-dimensional controller synthesis [4], [5], and on optimal control synthesis mainly concentrated on parabolic PDEs. The main theme within a functional analysis setting in the aforementioned works was that systems modeled by PDEs can be formulated in a state space form similar to that for lumped parameter systems by introducing a suitable infinite dimensional space representations and operators, and then undergo subsequent finite and/or infinite dimensional controller realizations [4], [6], [7]. For parabolic PDEs Christofides and co-workers studied nonlinear order reduction and control of nonlinear parabolic systems. For diffusion-reaction systems, which are described by parabolic PDEs, Dubljevic et. al. [8] used modal decomposition to derive finite-dimensional systems that capture the dominant dynamics of the original PDE and are subsequently used for the low dimensional controller design. Along the line of controller synthesis for parabolic PDEs a much less research has been developed for the hyperbolic PDEs within the context of optimal control strategies. Christofides et. al [9] explored distributed output feedback control of hyperbolic PDE systems. Aksikas et. al. [10] studied the solution of LQ control problem for hyperbolic system by solving an operator Riccati equation. On the other hand, for hyperbolic systems, characteristic of the hyperbolic PDEs is that the eigenvalues of the spatial differential operator cluster along vertical or nearly vertical asymptotes in the complex plane[11], which implies that the modal decomposition techniques suitable for parabolic PDEs cannot be used. In [8], Dubljevic et. al. explored the finite difference representation of model dynamics to convert the hyperbolic equations to a set of ODEs and the subsequent MPC is designed for the resulting model. For hyperbolic PDEs, the finite difference method may be numerically unstable and also results in high order systems which leads to a computationally demanding MPC.

Characteristics-based MPC is an approach for model predictive control of DPS proposed by Shang et. al. in [12] and [13]. The method of characteristic allows controller design for linear, quasilinear, nonlinear low dimensional PDEs. In this method, partial differential equations are transformed to a set of ordinary differential equations along the characteristic curves, which exactly describe the original DPS. Then the controller design can be performed on ODEs instead of PDEs without approximation. Unfortunately, the constrained model predictive control for hyperbolic systems has not been done. In a typical fixed-bed reactor shown in Fig. 1, input and output constraints should be considered. The input constraints are result of actuator limitation and the output constraints can be required by quality or safety limitations.

The PDE models of most transport-reaction processes are uncertain. A typical source of model uncertainty is unknown or partially known time varying process parameters like reaction rate. Presence of uncertain variables and unmodeled
dynamics, if not taken into account in the controller design, may lead to poor performance of the controller or even to closed-loop instability. For designing a robust controller, plant is assumed to be lied in a set which can be characterized in some quantitative way. The objective of robust control design is to ensure that some performance specification is met by the designed controller, as long as the plant remains within the specific set. When constraints on states and controls are present, in addition to robust stability, it is necessary to ensure the constraint satisfaction under model parameter uncertainty [14], [15].

In this work the problem of controlling a fixed-bed catalytic reactor with uncertain reaction kinetics is addressed. We formulated a robust constrained characteristic-based MPC problem which is based on initial transformation of the set of hyperbolic PDEs by method of characteristics into the set of the ODEs with certain structural features which are explored within the robust model predictive control algorithm construction. We also assume that the uncertainties within a system arise from the uncertain kinetics because of catalyst deactivation and also fluctuations in the flow rate and can be described by polytopic systems. Finally, the robust model predictive controller is realized as a quadratic optimization function which is based on the nominal model and the uncertainties are captured through the evolution of constraints.

II. MODEL DESCRIPTION

The chemical transport-reaction process considered is a catalytic fixed-bed reactor. A schematic diagram of this reactor is shown in Fig.1. The dynamics of a fixed-bed reactor can be described by partial differential equations derived from mass and energy balances. To model the reactor, a plug-flow pseudo-homogeneous model is considered. Moreover, we consider an one-spatial dimension model with no gradients in temperature and concentration in the radial direction. In the simplified system considered here, a lumped reaction kinetics equation was assumed and has the following form (see [16]):

\[ r_A = k(t)e^{(-\frac{E}{RT})}C_A^{m_1}C_H^{n_2} \]  

Under the above mentioned assumptions, the dynamics of the process are described by the following energy and mass balance partial differential equations (PDE’s).

\[ \epsilon \frac{\partial C_A(z, t)}{\partial t} = -v \frac{\partial C_A(z, t)}{\partial z} - \rho_B r_A(z, t) \]  

\[ \frac{\partial T(z, t)}{\partial t} = -\frac{\partial T(z, t)}{\partial z} + \frac{\rho_B \Delta H_r}{\rho C_p} r_A(z, t) \]

Initial and boundary conditions are:

\[ C_A(0, t) = C_{A,in}, \quad C_A(z, 0) = C_{A0}(z), \]
\[ T(0, t) = T_{in}, \quad T(z, 0) = T_0(z) \]  

The control objective is to regulate the outlet temperature of the reactor, therefore output variable is defined by:

\[ y(t) = \delta_T(z)T(z, t) \]

In the equations above, \( C_A, T, \epsilon, \rho_B, \rho, C_p, E, \Delta H_r, v, l \) denote the reactant concentration, the temperature, the porosity of the reactor packing, the catalyst density, the fluid density, the heat capacity, the activation energy, the enthalpy of reaction, the superficial velocity, and the length of the reactor respectively. \( C_H \) is the hydrogen concentration. It is assumed that hydrogen is the excess reactant, therefore its concentration is constant. \( k \) is the pre-exponential factor.

Catalysts lose their activity with time. Basic causes for this loss are active site poisoning by the impurities in the feed, active site coverage by coke, and pore mouth blockage. The rate of catalyst deactivation and its ultimate life are dependent on both the feedstock characteristics and operating conditions [17]. Moreover, deactivation rate depends on structural properties of pores, for example, their diameter and also their site densities. There is not a unique model for describing the catalyst deactivation. Probabilistic approach can be used to formulate the rate of deactivation which results in uncertain parameters in the system [18]. In this work we assume that the pre-exponential factor is an uncertain variable to cover the uncertainty of the reaction rate resulting from catalyst deactivation.

In practice there are always limitations on the values of input and output variables. Violation of these constraints may cause safety and economical problems. In this work, it is assumed that the following input and output constraints should be satisfied:

\[ v_{min} \leq v(t) \leq v_{max} \]
\[ y_{min} \leq y(t) \leq y_{max} \]

III. CHARACTERISTICS-BASED MPC

The method of characteristics is a technique for solving hyperbolic partial differential equations [12]. The idea behind the method of characteristics is that every hyperbolic PDE has a characteristic curve associated with it, along which dynamics evolve and as a result, the hyperbolic PDE can be represented as an equivalent system of ODEs.

Consider a quasilinear system of first-order equations with two dependent variables \( \nu_1, \nu_2 \) and two independent variables \( t \) and \( z \).

\[ \frac{\partial \nu_1}{\partial t} + a_1 \frac{\partial \nu_1}{\partial z} = f_1(\nu_1, \nu_2, u) \]
\[ \frac{\partial \nu_2}{\partial t} + a_2 \frac{\partial \nu_2}{\partial z} = f_2(\nu_1, \nu_2, u) \]  

\[ \text{Fig. 1. Schematic diagram of Fixed-Bed reactor} \]
if coefficients $a_1 \neq a_2$, the system has two different characteristics determined by:

Characteristic $C_1$ : \[
\frac{dz}{dt} = a_1
\]

Characteristic $C_2$ : \[
\frac{dz}{dt} = a_2
\]

along these characteristics dynamic of the system is described by:

\[
\begin{align*}
\frac{d\nu_1}{dt} &= f_1(\nu_1, \nu_2, u) \quad \text{along characteristic } C_1 \\
\frac{d\nu_2}{dt} &= f_2(\nu_1, \nu_2, u) \quad \text{along characteristic } C_2
\end{align*}
\]

Then, by using the method of characteristics, the set of partial differential equations (5) is transformed to a set of ODEs along the characteristic curves. This set of ODEs can be used to predict the system’s evolution. Let us assume that the model equations are linearized about the steady state profile of the reactor. Steady state profile of temperature and concentration are shown in Fig. 2.

It should be noted that for hyperbolic systems with constant space-derivative coefficient, the linearization can be done before and after applying the method of characteristics. Since in this case study $v$ is the input variable, both space derivative and reaction terms are nonlinear. Thus the linearization should be performed before applying the method of characteristic.

Then the resulting linearized system is:

\[
\begin{bmatrix}
\frac{\partial C_A}{\partial t} \\
\frac{\partial T}{\partial t}
\end{bmatrix} =
\begin{bmatrix}
\frac{\nu_{ss}}{\epsilon} & 0 \\
0 & \frac{\partial C_A}{\partial z}
\end{bmatrix}
\begin{bmatrix}
\frac{\partial C_A}{\partial t} \\
\frac{\partial T}{\partial z}
\end{bmatrix}
+ \begin{bmatrix}
M_{11}(t, z) & M_{12}(t, z) \\
M_{21}(t, z) & M_{22}(t, z)
\end{bmatrix}
\begin{bmatrix}
C_A \\
T
\end{bmatrix}
+ \begin{bmatrix}
B_1 \\
B_2
\end{bmatrix} u
\]

where the functions $M_{ij}$ are given by:

\[
\begin{align*}
M_{11} &= -n_1\rho Bk(t)e^{-\frac{E}{RT}}C_{A_{ss}}^{n_1-1}C_{H}^{n_2}, \\
M_{12} &= \frac{E}{RT}\rho Bk(t)e^{-\frac{E}{RT}}C_{A_{ss}}^{n_1}C_{H}^{n_2}, \\
M_{21} &= n_1\rho B\Delta H_{r}k(t)e^{-\frac{E}{RT}}C_{A_{ss}}^{n_1-1}C_{H}^{n_2}, \\
M_{22} &= -\frac{E}{RT}\rho B\Delta H_{r}k(t)e^{-\frac{E}{RT}}C_{A_{ss}}^{n_1}C_{H}^{n_2}
\end{align*}
\]

and

\[
B_1 = \frac{1}{\epsilon} \frac{\partial C_{A_{ss}}}{\partial z}, \quad B_2 = \frac{\partial T_{ss}}{\partial z}
\]

with $u$ being the superficial velocity defined as a deviation variable.

The characteristic curves for the linearized model (8) are:

\[
\begin{align*}
C_1 &= \frac{dz}{dt} = \frac{\nu_{ss}}{\epsilon} \quad (9) \\
C_2 &= \frac{dz}{dt} = \nu_{ss} \quad (10)
\end{align*}
\]

and the characteristic equations become:

\[
\begin{align*}
\frac{dC_A}{dt} &= M_{11}C_A(t) + M_{12}T(t) + B_1u(t) \\
\frac{dT}{dt} &= M_{21}C_A(t) + M_{22}T(t) + B_2u(t)
\end{align*}
\]

The characteristic ODEs are coupled with respect to the two characteristic curves, and the future state variables at one spatial point should be determined by simultaneous integration of both characteristic ODEs along two nonparallel characteristic curves. In Fig. 3 the calculation of the future output variables using the method of characteristics is illustrated. This method for prediction of the future behavior is proposed in [12]. The idea is that at $t = t_k$ the measurements of the state variables are available at discretization points and these measurements are used to determine the value of the state variables at intersections of the characteristic curves. This algorithm provides us with the future values of the output variable. For example for point P we have:

\[
\begin{align*}
C_A(P) &= \int_{t(Q)}^{t(P)} f_1(Q) \quad (11) \\
T(P) &= \int_{t(R)}^{t(P)} f_2(R) \quad (12)
\end{align*}
\]

where:

\[
t(P) = \frac{a_1t(Q) - 2a_2t(Q) + a_2t(R) + Z(R) - Z(Q)}{a_1 - a_2} \quad (13)
\]

and $a_1$ and $a_2$ are $\frac{\nu_{ss}}{\epsilon}$ and $\nu_{ss}$ respectively. The position of the point P is calculated by:

\[
Z(P) = \frac{a_1Z(R) - a_2Z(Q) + a_1a_2[Z(R) - t(Q)]}{a_1 - a_2} \quad (14)
\]

Coefficients $a_1$ and $a_2$ have a significant impact on the dynamic of the system. In this work, it is assumed that the superficial velocity of the reactor is an uncertain variable, in addition to pre-exponential coefficient of the reaction kinetics. In other words, the slopes of the characteristic equations

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are uncertain, but their ratio is constant. The constant ratio is a result of constant porosity of the catalyst.

It should be mentioned that applying the method of characteristics does not cause deformation of upper and lower limit constraints, because this transformation keeps the physical variables as the original variables in any point in space and time.

A. State space model

In order to obtain the states space model of the system, characteristic equations should be integrated along the characteristic curves. Let us define the values of concentration and temperature at intersection points of characteristic curves as state variables, so that the state vector is defined as:

\[ x = [ C_{A_1}, C_{A_2}, \ldots, C_{A_N}, T_A, T_{A_2}, \ldots, T_{A_N}]^T \]

where \( N \) is a number of discretization points. The manipulated variable is the superficial velocity of the reactor and the controlled variable is the outlet temperature. Equations (12)-(14) should be used to compute positions of the intersection points and to integrate the characteristic ODEs along the characteristic curves. An important feature of this method is that position of the intersection points are not constant. This feature is shown in Fig.3. Variation in position of intersection points results in variable structure for state space representation of the system. Therefore, the resulting state space model is a linear time varying system which possesses certain dynamic features in its description and it is represented by

\[ x_{k+1} = A_k x_k + B_k u_k \]
\[ y_k = C_k x_k \]

Namely, one can observe that Eq.15 is time varying system since \( A_k, B_k \) and \( C_k \) are functions of time. However, there is a certain symmetry in the structure of the matrices \( A_1, A_2, \ldots \) which is a function of the ratio of the slopes of two characteristic curves. For the case study considered in this work this ratio is 0.4 and in Fig.3 it is illustrated that at time \( 3\Delta t \) intersections points are exactly in the same spatial points as intersection points at time 0. The same feature can be illustrated for time \( \Delta t \) and \( 4\Delta t \). Consequently, the structure of the state space matrices are repeated every 3 sample time. For this specific example \( A_k \) has the form of:

\[
A_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \alpha_{11} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \alpha_{11} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \alpha_{11} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \alpha_{21} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \alpha_{21} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \alpha_{21} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \alpha_{21} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \alpha_{21} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \alpha_{21} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \alpha_{21} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \alpha_{21} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \alpha_{21} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \alpha_{21} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \alpha_{21} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \alpha_{21} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 
\end{bmatrix}
\]

One can conclude that this repeated structure of the state space model is a result of periodic change of the position of the intersection points of two characteristic curves. Changes in the ratio of two slopes will change the period of the system.

In order to use common MPC algorithms, the periodic/repeated system should be converted to a linear time invariant system. This can be done by assembling 3 sample times to one larger sample time and defining:

\[
\tilde{A} = A_3 A_2 A_1 \]
\[
\tilde{B} = \begin{bmatrix} A_3 A_2 B_1 & A_3 B_2 & B_3 \end{bmatrix} \]
\[
\tilde{C} = [C_1, C_2 A_1, C_3 A_2 A_1]^T
\]
Finally, the LTI system representation suitable for the design of MPC controller is given as:

\[ x_{k+1} = \bar{A}x_k + \bar{B}u_k, \quad y_k = \bar{C}x_k. \]  

**Remark 1:** For co-current hyperbolic systems the slopes of two characteristics is always positive, and if the porosity of the bed remains constant, their ratio will be constant. Changes in the slopes of the curves affect the sampling time of the system if the number of discretization points is constant.

**B. Robust characteristic-based MPC**

In order to include the parametric uncertainty of the plant, we assume that the small variations of the uncertain parameters of the system dynamics generate a family of the linear system given in Eq.20 which can lie in a polytopic set \(\{[\bar{A}_i \ \bar{B}_i]\}\). Thus the system can be described by a polytopic uncertain system. Polytope \(\Omega\) is defined by

\[
\Omega = \text{Co}\{[\bar{A}_1 \ \bar{B}_1],[\bar{A}_2 \ \bar{B}_2], \ldots, [\bar{A}_L \ \bar{B}_L]\} \quad (21)
\]

So if \([\bar{A}, \bar{B}] \in \Omega\) then, for some nonnegative \(\lambda_1, \lambda_2, \ldots, \lambda_L\) summing to one, we have

\[ [\bar{A} \ \bar{B}] = \sum_{i=1}^{L} \lambda_i [\bar{A}_i \ \bar{B}_i] \]

which means that there is a convex hull of uncertain plants which is generated by considering all possible combinations of uncertain parameters. In this work we utilize the model predictive control algorithm proposed by Muske and Rawlings [19]. Matrix \(\bar{A}\) in Eq. 20 is a nilpotent matrix, which implies that the system has FIR property. This FIR property is a natural feature of co-current hyperbolic system. The reason is that each species is processed in a limited time in the reactor and leaves out of the reactor. So we can conclude that the system is always stable and all eigenvalues of \(\bar{A}\) are inside the unit circle.

The quadratic performance index is constructed based on the nominal model of the system, while the construction of constraints captures the dynamics of all possible parametric uncertain models. Therefore the characteristic based MPC formulation is:

\[
\min_{u^N} \quad [y_{k+N}^TQU_{k+N} + \sum_{j=0}^{N} (y_{k+j}^TQy_{k+j} + u_{k+j}^TRu_{k+j} + \Delta u_{k+j}^TAS\Delta u_{k+j})] \quad (22)
\]

Subject to:

\[
\begin{align*}
& x(0) = x_0, \quad y(0) = y_0, \\
& x(k+1) = A_i x(k) + B_i u(k), \quad i = 1, 2, \ldots, L \\
& y(k) = C_i x(k)
\end{align*}
\]

\[ u_{\min} \leq u_{k+j} \leq u_{\max}, \quad y_{\min} \leq y_{k+j} \leq y_{\max}, \quad \Delta u_{\min} \leq \Delta u_{k+j} \leq \Delta u_{\max} \]

This problem can be converted to the following quadratic problem:

\[
\min_{u^N} \Phi_k = (u^N)^T H u^N + 2(u^N)^T (G x_k - F u_{k-1}) \quad (23)
\]

Subject to:

\[
\begin{bmatrix}
A_1 \\
A_1 \\
\vdots \\
A_L
\end{bmatrix} u^N \leq 
\begin{bmatrix}
B_1 \\
B_1 \\
\vdots \\
B_L
\end{bmatrix}
\]

**IV. Numerical Simulations**

The parameters of the system given by Eqs. 2-3 are given in Table I. The objective is to control the reactor’s outlet temperature at specified setpoint. It is assumed that the parameters \(k\) and \(v_{ss}\) are uncertain variables and there are four polytopic system models. It is assumed that the pre-exponential factor, \(k\), can have 40% uncertainty and \(v_{ss}\) can have 20% uncertainty. 20 discretization points is used for modelling and 60 discretization points for simulation. The prediction horizon is assumed to be 20 sample times. The initial conditions are: \(T_0 = 0.95T_{in}, C_{Ao} = C_{A_{in}}\). Weighting matrices are: \(Q = I\) and \(S = 100L\).

It should be mentioned that the resulting state space system is a FIR system, so increasing the prediction horizon to more than resident time of the reactor will not improve the performance of the controller. In order to evaluate the performance of the controller, it is assumed the plant model changes randomly every two sample time. Fig. 4 and 5 illustrate the profile of temperature and concentration respectively. Fig. 6 shows the profile of the manipulated variable and Fig. 7 is the trajectory of the control variable. As Figs. 6-7 show the constraints on input and output trajectories are satisfied.
V. Conclusion

In this work a robust model predictive control algorithm is developed for hyperbolic partial differential equations. In order to specify the uncertainty of the plant we assumed that the parameters of the system lie in a polytope $\Omega$. Two uncertain parameters have been considered, and the MPC problem is defined by nominal performance index and constraint satisfaction for all polytopic models. Simulation results indicate that this algorithm is able to satisfy input and output constraints under parameter uncertainty.

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