Slip Estimation for Small-Scale Robotic Tracked Vehicles

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Abstract—A method is presented for using an extended Kalman filter with state noise compensation to estimate the trajectory, orientation, and slip variables for a small-scale robotic tracked vehicle. The principal goal of the method is to enable terrain property estimation. The methodology requires kinematic and dynamic models for skid-steering, as well as tractive force models parameterized by key soil parameters. Simulation studies initially used to verify the model basis are described, and results presented from application of the estimation method to both simulated and experimental study of a 60-kg robotic tracked vehicle. Preliminary results show the method can effectively estimate vehicle trajectory relying only on the model-based estimation and onboard sensor information. Estimates of slip on the left and right track as well as slip angle are essential for ongoing work in vehicle-based soil parameter estimation. The favorable comparison against motion capture data suggests this approach will be useful for laboratory and field-based application.

I. INTRODUCTION

Researchers have developed methods for estimating soil properties based on estimated slip parameters for wheeled robotic vehicles. For example, Iagnemma, et al. [1] estimated soil cohesion and internal friction angle for a wheeled planetary rover using an online least square method. More recently, Ray, et al. [2] applied an extended Kalman filter to estimate net traction in a differentially-wheeled robotic vehicle. It is yet to be determined whether a sensor-endowed tracked robotic vehicle like a wheeled robotic vehicle can be used to estimate the soil properties based on trajectory data or estimated slip parameters. Some work has been done in estimating the soil properties based on slip and trajectory data for full-scale tracked vehicles. For example, Schiller [3] established a method used later by Le [4] to study online terrain parameter estimation for a tracked vehicle. Le used trajectory data from a tracked vehicle and identified slip, and then used a skid-steered vehicle model and showed that these estimated parameters could be used to extend the speed and accuracy with which the vehicle can be skid-steered. Song [5] used an analytical model of a full-scale tracked vehicle and applied a Newton-Raphson and a least square estimation technique determining soil parameters.

This paper examines whether the detected mobility of small-scale tracked vehicles is sufficiently sensitive to estimate soil properties and variables used for mobility prediction models. Vehicle slip and slip angle are estimated for the purposes of this work using kinematic/dynamic equations of motion of the vehicle and established terramechanics equations [6]. Results obtained from simulation based studies are compared with preliminary data from experimental testing with an iRobot® PackBot (shown in Figure 1) on sand terrain at the Southwest Research Institute Small Robotic Vehicle Test Bed (San Antonio, Texas). The estimated slip variables will be essential in estimating soil parameters using a dynamic model of the tracked vehicle along with statistical estimation methods. It is desirable either during testing or for online operation to estimate terrain parameters, such as the angle of internal shearing friction, cohesion, shear deformation modulus, etc.. Knowledge about these parameters can aid development and/or use of prediction and control algorithms for tracked vehicles traversing various types of deformable terrains.

Fig. 1. The iRobot® PackBot robotic tracked-vehicle system is remotely-controlled. The flippers were not considered in this study.

Following the introduction, Section 2 reviews kinematic and dynamic models for basic tracked vehicles undergoing plane motion. Section 3 presents essential elements of a soil model and relationships between soil parameters and tractive forces. Section 4 presents an estimation methodology based on the, and Section 5 presents results using this approach to estimate track slip variables (left and right track, slip angle) from vehicle trajectory data. The results and conclusions are discussed in Section 6.

II. TRACkED VEHICLE PLATFORM AND MODEL

Methods for estimating the slip variables and soil properties for a small-scale tracked vehicle traversing unknown terrains require system model equations. Robotic vehicles, such as the iRobot® PackBot™, can be equipped with a number of state-of-the-art sensors that can track the vehicle trajectory and aid in estimating slip levels. It is important to be able to estimate slip, which underlies our ability to estimate tractive effort and possibly some key terrain parameters. The approach taken here is to estimate slip variables using a simple kinematic model of the vehicle, and a comparison can be made to laboratory data collected using a Vicon motion capture system.
capture system. For the purposes of this paper, data logged from the PackBot™ (Figure 1) included forward speed and turn-rate (yaw velocity), from which sprocket angular speeds were inferred, while the motion capture system provided absolute measures of vehicle location and orientation\(^1\). All test data was collected for operation within a small indoor sand pit. It should be emphasized that the motion capture data provide a basis for comparing the model-based estimation results.

![Free body diagram of a tracked vehicle](image)

**Fig. 2.** Free body diagram of a tracked vehicle

A. Dynamic Model

Figure 2 shows a free body diagram of a tracked vehicle moving on a horizontal plane, with the vehicle turning to the left and accelerating in the positive \(x\), \(y\) and \(\psi\) directions. The external forces acting on the vehicle consist of lateral resistive forces \(R_L\) and \(R_R\) and thrust forces, \(F_L\) and \(F_R\), on left and right tracks, respectively. The lateral friction force indicated by \(f_y\) results due to lateral soil shear and is distributed as shown in Figure 2.

Also shown in Figure 2 is an inertial frame of reference \(XYZ\), useful in indicating that the vehicle is turning about an instantaneous center, \(C\). The angle \(\alpha\) between velocity \(V\) and \(x\) is called the side slip angle. It is assumed that the normal pressure distribution along the track is uniform and coefficient of lateral resistance \(\mu_t\) is constant. To satisfy the equilibrium condition in the lateral direction the center of turn must lie at a distance \(d\) in front of the transverse centerline of the track-ground contact area as shown in Figure 2 [6], [3]. The equations of motion in the body reference frame are,

\[
\begin{align*}
    m\ddot{x} &= F_L + F_R - R_L - R_R \quad (1) \\
    m\ddot{y} &= f_l \quad (2) \\
    I_\psi &= (F_R - R_R) \frac{b}{2} - (F_L - R_L) \frac{b}{2} - M_r \quad (3)
\end{align*}
\]

where \(f_l\) is the net lateral friction force in (2), for lateral acceleration. This friction force is obtained in terms of vehicle speed by integrating the lateral shear force \(f_y\) and considering a condition on \(d\) above. Thus we have, \(f_l = 4\mu_y Wd\), where \(\mu_y\) is the lateral coefficient of friction, and the lateral shear force magnitude \(f_y\) is, \(f_y = \mu_y W\). Now, simplifying (2), \(\ddot{y} = 2\mu_y g \dot{y}/(\dot{\psi})\). \(M_r\) is the moment of turning resistance in (3) for angular acceleration. This \(M_r\) is calculated in a similar fashion as \(f_l\) and can be expressed in terms of vehicle speed,

\[
M_r = -\text{sgn}(\dot{\psi})\mu_y \frac{mg}{l} \left( \frac{L^2}{4} - d^2 \right). \quad (4)
\]

The effect of centrifugal force is included in the above equation for turning moment resistance in terms of distance \(d\). This shows that the moment of turning resistance decreases with an increase of lateral acceleration. It is thus safe to say that (4) is not for straight line motion. In order to obtain a simulated vehicle trajectory for various terrain conditions, the above equations of motion, (1) and (2), can be written in inertial frame of reference as,

\[
\begin{align*}
    m\ddot{X} &= (F_L + F_R - R_L - R_R) \cos \psi - f_l \sin \psi \quad (5) \\
    m\ddot{Y} &= (F_L + F_R - R_L - R_R) \sin \psi + f_l \cos \psi \quad (6)
\end{align*}
\]

The equation (3) for angular acceleration remains the same.

\[
I_\psi = (F_R - R_R - F_L + R_L) \frac{b}{2} - \text{sgn}(\dot{\psi})\mu_y \frac{mg}{l} \left( \frac{L^2}{4} - d^2 \right) \quad (7)
\]

B. Kinematic Model

Estimation of slip on the right and left track as well as slip angle relies on a kinematic model of the tracked vehicle in body-fixed coordinates,

\[
\begin{align*}
    \dot{x} &= \frac{r}{2} [\omega_L (1 - i_L) + \omega_R (1 - i_R)] \quad (8) \\
    \dot{y} &= -\dot{x} \tan \alpha \quad (9) \\
    \psi &= \frac{r}{2b} [\omega_R (1 - i_L) - \omega_L (1 - i_R)] \quad (10)
\end{align*}
\]

where \(i_L\) and \(i_R\) are the left and right side track slip, respectively. As shown, the vehicle is shown turning left with its left track as inner track, and \(\omega_L\) and \(\omega_R\) are the sprocket speeds for left and right side, respectively. Also in (9), \(\tan \alpha\) defines the slip angle, \(\alpha\),

\[
\tan \alpha = \frac{\dot{y}}{\dot{x}} \quad (11)
\]

---

\(^1\)Note: The GPS built in to this PackBot is not sufficiently accurate to aid estimation.
Tracks slip when the sprocket angular velocity exceeds the longitudinal velocity, and slip is calculated,
\[ i = \frac{r\omega - V}{\max(r\omega, V)} \]
\[ = \frac{V_j}{\max(r\omega, V)} \]  
(12)

(vehicle skid occurs if \( V > r\omega \)). In the equations above for slip and skid, \( V \) is the actual forward speed of the track, \( r \) is the pitch circle of the sprocket, and \( \omega \) is the angular speed of the sprocket. In (12), \( V_j \) is track slip speed with respect to ground. When a vehicle is slipping, \( V_j \) will be in a direction opposite to that of vehicle motion, and if a vehicle is skidding then it is in the same direction as that of vehicle [6]. It is common to define \( V_j \) positive when tractive effort assists the longitudinal motions. The kinematic equations can be written in inertial frame of reference as,
\[ \dot{X} = \frac{r}{2} \left[ \omega_L \left(1 - i_L \right) + \omega_L \left(1 - i_L \right) \right] \cos \psi + \left( \dot{x} \tan \alpha \right) \sin \psi \]  
\[ \dot{Y} = \frac{r}{2} \left[ \omega_L \left(1 - i_L \right) + \omega_R \left(1 - i_R \right) \right] \sin \psi - \left( \dot{x} \tan \alpha \right) \cos \psi \]  
(13)

(14)

Upon transformation into inertial frame of reference the equation (10) for yaw rate remains the same.

<table>
<thead>
<tr>
<th>Soil Type</th>
<th>c, N/m²</th>
<th>( \phi ), deg</th>
<th>k, meter</th>
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<tbody>
<tr>
<td>dry sand</td>
<td>1040</td>
<td>28</td>
<td>0.01</td>
</tr>
<tr>
<td>loose sand</td>
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<td>29</td>
<td>0.0254</td>
</tr>
<tr>
<td>clayey soil</td>
<td>4140</td>
<td>13</td>
<td>0.006</td>
</tr>
</tbody>
</table>

### III. MODEL SIMULATION VERIFICATION STUDIES

The diagram in Figure 3 represents a simulation model developed to study the sensitivity of a tracked vehicle to changes in soil properties and slip variables. Various simulation studies were conducted to verify this model for later use. For example, trajectories such as those shown in Figure 4 result from varying the soil properties in simulation of a simple U-turn maneuver. Keeping in mind the light weight of the small-scale tracked vehicle, it is assumed that the soil is homogenous and behaves as a perfect or ideal plastic material with failure occurring only when a stress condition is reached with no recoverable elastic deformation [6]. Nominal soil properties used in these simulations are summarized in Table I. The simulation uses a fixed-step integrator, and the sprocket speeds \( \omega_L(t) \) and \( \omega_R(t) \) are known inputs to the left and right track sprockets, respectively.

Besides soil parameters, the other critical parameters are the longitudinal and lateral coefficients of friction. The resistive forces on the tracks are modeled, \( R = \mu_x W/2 \), where the longitudinal coefficient of friction, \( \mu_x \), is considered independent of velocity. During a turning or yawing maneuver, for example, the lateral forces play a dominant role in controlling the yaw angle of the vehicle. As per Wong, the lateral force per unit length is \( f_l = \mu_l W/l \), with \( \mu_l \) the lateral coefficient of friction [6]. The lateral resistance of a track also depends on lateral track skid and turning radius. However, the lateral coefficient of friction is assumed to be in the range from 0.45 to 0.9 for a rubber track on various soil types [6]. The effect of changes in lateral friction on a U-turn trajectory in dry sand is shown in Figure 5. For estimation of soil properties, these coefficients of friction can be estimated using a dynamic tracked vehicle model. However, for the sake of brevity, these coefficients are not estimated in the current study.

Computed left and right track slip values for the different soil types are shown in Figure 6, corresponding to the trajectories in Figure 4. The negative force or braking force is essential for the left track whenever a tracked vehicle turns left. This braking force is related to induced negative slip (skid). For any maneuver or turn, the force developed on the outer track, or in this case the right track, is more critical as
The simplest method for estimating slip is by using the kinematic model equations. The trajectory data is either stored to be used later as truth table or used in real-time to estimate slip using statistical estimation techniques. For this particular study, an Extended Kalman Filter (EKF) has been selected. The advantage of using an EKF is that the reference trajectory is updated after each observation to reflect the best estimate of the true trajectory. Also, the state transition matrix developed to evaluate the nominal trajectory in this case is nonlinear, so the EKF is required and a simple Kalman filter or batch estimation methods would not be able to give accurate estimates.

Through the kinematic model, it is desired to estimate the left and right track slip values, slip angle, and the orientation of the tracked vehicle. These states and parameters to be estimated are defined through the system state vector as: 

\[ x(k) = [X(k); Y(k); \Psi(k); i_L(k); i_R; \alpha(k)] \]

In the kinematic equations, it will be assumed that the left and right sprocket speeds, \( \omega_L \) and \( \omega_R \), are known inputs.

The state transition matrix is re-initialized for each updated observation of the reference trajectory during an integration routine. The kinematic equations of motion are integrated from \( t_{k-1} \) to \( t_k \) and can be expressed in the form:

\[ \dot{X}^* = F(X^*, t) \quad \forall, \quad X^*(t_{k-1}) = \tilde{X}_{k-1} \]

\[ \phi(t, t_{k-1}) = A(t)\phi(t, t_{k-1}) \quad \forall, \quad \phi(t_{k-1}, t_{k-1}) = I \]

In the state transition matrix, \( A(t) \) is linearized and evaluated on a nominal trajectory. Based on the preceding discussion, the process model can be expressed in discrete form in the presence of process noise \( v(k) \) as,

\[ x(k) = f(x(k-1), u(k)) + v(k). \]

The observation model is computed, \( Z_k = Z_k - G(X^*_k, t_k) \).
where \( Z_k = [X_{obs}; Y_{obs}; \Psi_{obs}] \) are the observations to be read online or recorded earlier. Similarly, the observation state matrix is also linearized and computed at nominal trajectory before the computation of the Kalman gain matrix.

To curtail the divergence in the estimate due to some unknown error in the model and to compensate for the effects of nonlinearities, it is assumed that linearized dynamics can be approximated by process noise using State Noise Compensation (SNC) [7]. SNC will improve estimation performance, especially of slip values, through partial compensation for uncertain slip variations. It has been assumed for this algorithm that state dynamics are influenced by random slip variations characterized as white noise. Thus, the process noise covariance integral needed for the time update of the estimation error covariance matrix at time \( t_k \) is computed as:

\[
\tilde{P}_{k+1} = \Phi(t_{k+1}, t_k)P_k\Phi^T(t_{k+1}, t_k) + 
\Gamma(t_{k+1}, t_k)Q_k\Gamma^T(t_{k+1}, t_k)
\]

where,

\[
\Gamma(t_{k+1}, t_k) = \int_{t_k}^{t_{k+1}} \Phi(t_{k+1}, \tau)B(\tau)d\tau 
\]

\( Q \) is process noise covariance matrix and \( B \) is process noise mapping matrix. \( Q(t) \) is a simple diagonal matrix and its elements are determined by trial and error. The advantage of using process noise lies in the fact that value of \( P(t) \) will approach a non-zero value determined by \( Q(t) \).

![Fig. 8. Estimated and actual Packbot parameters for dry sand using simulated data](image)

V. ESTIMATION RESULTS

The estimation method described in the previous section has been evaluated using both simulation studies and against experimental data for a small-scale tracked vehicle having parameters equivalent to those of the PackBot. In these studies, the EKF estimates the slip values using vehicle trajectory data. The EKF reads \((X, Y, \Psi)\) from a truth table saved earlier as observations (from motion capture) and estimates left and right track slip, \( \alpha_L \) and \( \alpha_R \), and slip angle, \( \alpha \), as well as Packbot orientation.

Measured input angular velocities, \( \omega_L \) and \( \omega_R \), were used to drive the simulation model, which was parameterized for a dry sand terrain\(^2\). For this case, the simulated results are shown in Figure 8. As might be expected, the trajectory is estimated with negligible error, and the track and slip values are estimated with good accuracy and negligible time lag. Similar results have been obtained for testing with other terrain types.

During testing, the Packbot was manually driven in U-turns in the sand pit, and onboard data was logged and synchronized later with full vehicle motion measurement from a Vicon motion capture system. The sprocket angular velocities measured by onboard encoders during a U-turn maneuver are shown in Figure 9.

![Fig. 9. Control input to left and right track sprockets measured on an iRobot PackBot during a U-turn maneuver in dry sand.](image)

The observed and estimated trajectory as well as estimated slip variables are summarized in Figure 10. It can be seen that the EKF is sufficiently optimized to enable tracking the vehicle very closely. It should be clarified that the trajectory and yaw for the PackBot are measured using the motion capture system, while the EKF estimations are derived from onboard data. Estimated slip values are accurate and have trends similar to those observed in simulation results for a nominal dry sand (soil properties were not measured in these tests). Another notable comparison is made in Figure 10(c). This graph overlays the estimated inner (left) and outer (right) track slip values against values calculated using a speed estimate for the vehicle derived using motion capture data. Note also that coefficients of friction in these models are nominal values tuned during the simulation studies.

Finally, the effect of adding SNC in the EKF algorithm is examined in Figure 11. It was found that estimation of the trajectory improved as the slip and slip angle estimations improved through addition of process noise. The vehicle orientation estimates in Figure 11 should be contrasted with those given in Figures 10(a) and (b).

It is pertinent to mention that during the U-turn trajectory

\(^2\)Known conditions during testing or by sensors in field operation.
the outer slip on right track plays a dominant role in maneuvering and turning the PackBot with a specific turning radius. The force developed on the outer track must overcome the turning moment resistance due to lateral friction. The estimated trajectory and yaw angle orientation of the PackBot has been developed after updating and computing the process noise covariance matrix. If the process noise mapping matrix is not introduced in the algorithm error crops up as shown in Figure 11.

VI. CONCLUSION AND FUTURE WORK

EKF results indicate it is possible to estimate track slip values for a small-scale tracked vehicle with relatively good accuracy based only on trajectory/orientation observed/measured data. It was confirmed that a skid-steer simulation model for a small-scale tracked vehicle is sensitive enough to distinguish between various forms of unknown terrains. The work presented is a first step toward enabling certain long term objectives. In ongoing work, both slip values and coefficients of friction are estimated to aid in determining soil properties. Estimates of soil parameters are needed for developing and evaluating statistical mobility prediction simulations for small-scale tracked vehicles. The results can also aid vehicles with onboard sensing, enabling improved path planning as well as traction control algorithms. Finally, these new and improved slip value estimations methods should also find applicability for larger-scale tracked vehicles.

ACKNOWLEDGMENTS

The experimental data was collected as part of a collaboration with Southwest Research Institute, San Antonio, TX, and Griffin Technologies, Wynnewood, PA, funded by a STTR contract through the ERDC, U.S. Army Corps of Engineers, Vicksburg, MS.

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