Parameter Estimation for Dynamic HVAC Models with Limited Sensor Information

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Abstract—This paper presents an approach for identifying critical model parameters in a HVAC system using limited sensor information. Both static and dynamic nonlinear models are addressed here. Two numerical search algorithms, nonlinear least squares and simplex search, are used to estimate the parameters. The parameter estimation algorithm developed is validated on two different experimental systems, to confirm the practicality of this approach. Knowing the model parameters accurately can lead to a better model for control and fault detection applications.

I. INTRODUCTION

The global demand for energy is ever increasing; a 44% increase in energy demand is expected in the next twenty years [1]. Improving the efficiency of the energy consuming devices will play a crucial role in meeting the future energy needs. Heating, Venting, Air Conditioning and Refrigeration (HVAC) systems account for 40% of the commercial energy consumed in the US [2]. The availability of control oriented dynamic models of these systems can greatly help in the design and analysis of better control strategies resulting in systems with higher efficiencies.

Vapor Compression Cycle is the most widely used method for HVAC applications. Dynamic models have been developed for Vapor Compression system components that can accurately predict the behavior of the system if the mass flow rates to and from the heat exchangers are known accurately. In particular the two-phase flow dynamics are extremely sensitive to small variations in mass flow rate. The prediction of mass flow rate relies heavily on the empirical expansion valve and compressor parameters.

Traditionally these empirical parameters have been estimated by employing expensive mass flow meters. But use of mass flow meters in every case is not possible which may lead to a badly tuned model. This research is motivated by the desire to find these empirical parameters on HVAC systems employing relatively low cost sensors like temperature and pressure sensors.

In this paper, the mathematical models of two commonly used expansion valves in HVAC systems, the Electronic Expansion Valve (EEV) and the Thermostatic Expansion Valve (TEV) are presented and analyzed. The parameters of the expansion valves and the compressor are estimated using nonlinear least squares and simplex search algorithms. Both of these algorithms are available in Matlab’s Simulink Response Optimizer Toolbox [3].

This parameter estimation approach can be used for all types of grey box model parameter estimation and in particular for cases where the analytical methods of parameter estimation fail.

II. BACKGROUND

In this section an overview of the different components in a HVAC system is given. The mathematical models of these components along with the unknown parameters are given.

A. Vapor Compression Cycle

There are four main components in a single-stage vapor compression system: a compressor, a condenser, an expansion valve and an evaporator. This system functions by transferring thermal energy from one heat exchanger to another through the circulation of a refrigerant.

B. Modeling of Expansion Valves

The Electronic Expansion Valve can be modeled by the orifice equation [4]. The dynamics of the heat exchangers in a Vapor compression system are much slower than the dynamics of the valve, hence a static algebraic expression is used to model the EEV.

\[
m_v = (v_1 + v_2 u_{eev}) \sqrt{(P_e - P_v)\rho_v}
\]  

A TEV regulates the amount of refrigerant entering the evaporator in a vapor compression system based on the superheat of the refrigerant at evaporator exit. Extensive literature are available on the different aspects of a TEV, mainly the mathematical model [5], [6], [7] and the hunting phenomenon [8], [9]. The mathematical model of the TEV mainly consists of the bulb model and the valve model.

One of the earliest works on vapor compression system modeling was done by [10]. The TEV model consisted of a differential equation relating the superheat to the mass flow.
rate through the expansion valve. It did not account for the different pressures acting on the diaphragm, hence was not able to predict rapid changes when encountered.

The TEV model was improved by representing the forces acting on the diaphragm in terms of temperature [5]. This can be done since the valve dynamics are much faster than the sensor dynamics. Sensor dynamics was modeled by a first order lag, and the time constant was assumed to be known. In [6] the mass flow rate through the expansion valve is linearly related to the net pressure acting on the diaphragm of the valve. This model was combined with the orifice equation in [7] and it assumes a constant pressure difference across the valve. For varying pressure difference, the valve model is given in [11]. This equation is,

\[ m_v = C_d (P_b - P_e - P_0) \sqrt{\rho_v (P_e - P_g)} \]  

(2)

The bulb model is found by applying the conservation of energy equation to the bulb and its contents. Since the bulb along with contents, which is a two phase substance is tough to model accurately; various assumptions can be made leading to models of varying complexity. In [7], the authors model the bulb with varying degrees of complexity. One of the most accurate ways to model a bulb will be to take a finite volume approach. The model can be simplified by assuming the entire bulb to be a single unit. The simplest model for the bulb will be to assume a first order lag for the bulb temperature. The authors [7] compare the results obtained by the different approaches and show that the model assuming a first order lag for the bulb temperature, behaves almost similarly to the most accurate model.

Due to this reason, the TEV bulb is modeled assuming first order dynamics. The following assumptions are made for the TEV model:

1. The refrigerant present in the bulb of the TEV as well as its thermodynamic properties are known.
2. The spring is linear in the operating range. A valid assumption considering the very minute net displacement of the spring during operation.

The TEV bulb is modeled using the lumped capacitance method [7]. Here the heat transfer to the outside environment is neglected

\[ h_{rb} A_{rb}(T_b - T_{evo}) = m_b C_b \left( \frac{dT_b}{dt} \right) \]  

(3)

The Laplace transform of the above equation gives,

\[ \frac{T_b(s)}{T_{evo}(s)} = \frac{1}{1 + Ts} \]  

(4)

The bulb pressure is the saturation pressure of the refrigerant in the bulb, \( P_b = P_{sat}(T_b) \). The force balance on the diaphragm of the expansion valve is given by,

\[ P_b A_1 = P_e A_2 + K_s (x_0 + \delta x) \]  

(5)

Where, \( x_0 \), is the initial compression of the spring and \( \delta x \), is the net axial movement of the valve head. Let us define, \( P_0 = \frac{K_s x_0}{A_2} \). Equation (6) can be written as,

\[ \delta x = \frac{(P_b A_1 - (P_e - P_0) A_2)}{K_s} \]  

(6)

Near a particular operating condition the area of the valve opening is directly proportional to the displacement of the valve head. Hence,

\[ A_v = \alpha_0 P_b - \alpha_1 (P_e + P_0) \]  

(7)
Combining the above equations with the equation of flow through an orifice one can obtain the equation for the mass flow rate with respect to the bulb pressures and other parameters.

\[ \dot{m}_v = (v_1 + v_2P_b + v_3P_e)\sqrt{\rho_v(P_t - P_e)} \]  

(8)

If \( A_1 = A_2 \), then the above equation reduces to,

\[ \dot{m}_v = (v_1 + v_2(P_b - P_e))\sqrt{\rho_v(P_t - P_e)} \]  

(9)

Equations (4) and (9) represent the mathematical model of a TEV. The parameters that need to be identified in this model are \( \tau, v_1, \) and \( v_2 \). Once these parameters have been estimated the mass flow rate of the refrigerant through the expansion valve can be known.

C. Modeling of a variable speed compressor

The variable speed compressor is modeled by the following equation:

\[ \dot{m}_k = \eta_k \omega V_k \rho_k \]  

(10)

\[ \eta_k = k_1 + k_2(P_t/P_e) \]  

(11)

The volumetric efficiency of a compressor is a function of the pressure ratios. In case of a constant speed compressor, or when the volume of the compression chamber is not known, these terms could be combined with the unknowns.

D. Modeling of the Evaporator

Moving boundary approach is used to model the evaporator. This approach was chosen over the finite control volume approach due to its better computational speed and less complexity of the model. In this approach the heat exchanger is split into different regions according to the fluid phases existing in it as shown in Figure 3. Average refrigerant properties are assumed over this entire region. Conservation of mass and energy equations are solved over these regions to obtain the model of the heat exchange.

![Fig. 3. MB Evaporator Model Diagram](image)

The moving boundary model has been exhaustively explored \([12], [13], [14]\) and hence the details of this approach are not mentioned here. For the purposes of parameter estimation, linearized moving boundary model is preferred over the nonlinear evaporator model due to its higher computational speed.

The evaporator model has five inputs and two outputs; the inputs are the mass flow rate of the refrigerant at evaporator inlet, mass flow rate of the refrigerant at evaporator outlet, enthalpy of refrigerant at evaporator inlet, mass flow rate of the secondary coolant over the evaporator coils, temperature of the secondary coolant at the inlet of the evaporator. The output of the evaporator model is the pressure in the evaporator and temperature of the refrigerant at evaporator exit.

Of all the inputs, the evaporator model is most sensitive to the mass flow rate of the refrigerant at the inlet and exit of the evaporator. This high sensitivity makes it impossible to run the evaporator model alone in all practical cases.

The evaporator model is augmented with the valve and compressor model and the pressure calculated by the evaporator model is fed to the valve and compressor models. The valve and compressor models calculate the mass flow rates to and out of the evaporator respectively. Thus augmenting the models gives an internal feedback loop and this keeps the evaporator model stable. Figure 6 has a graphical representation of this model augmentation.

III. PARAMETER ESTIMATION

In case of static models, e.g. EEV and compressor model, the parameters can be estimated using mass flow rate measurements if a mass flow meter is present. But in case of a dynamic model like the TEV model, even with mass flow rate measurements one cannot estimate the parameters of the model.

In \([15]\), the TEV bulb’s time constant has been estimated by attaching the bulb to an independent tube where water of varying temperature is allowed to flow. This method of estimation is not possible in many cases; but this parameter plays a major role in hunting of the evaporator and regulating superheat \([8]\).

A parameter estimation algorithm is required to identify the valve and compressor parameters so that the vapor compression cycle model can mimic the actual HVAC plant. A parameter estimation algorithm identifies the unknown parameters of a given grey box model knowing the actual plant input and output \([16]\). Figure 4 mentions some of the most common parameter estimation methods and their type.

![Fig. 4. Common parameter estimation methods and their types](image)

A common algorithm used for parameter estimation in grey box models is the nonlinear least squares \([17]\). It is a type of gradient based numerical search method that makes
use of the model information while computing the estimate. The use of nonlinear least squares for parameter estimation can be found in [18] and [19].

Another common algorithm used is the simplex search. It is a non-gradient based numerical search method [20] and [21], and has used simplex search for parameter estimation [22]. The advantage of simplex search is that since it does not compute gradients it is a comparatively faster than nonlinear least squares algorithm, but is less robust, that is, it is more prone to settle at a local minima. Both these algorithms minimize the squares of the prediction error. A graphical representation of the numerical search process is given in Figure 5.

\[
\theta[k] = \min \left\{ \sum_{i=0}^{n} \left( Pe_i - \hat{Pe}_i \right)^2 + s_2 \left( T_{evo1} - \hat{T}_{evo1} \right)^2 \right\}
\] (12)

In both the algorithms discussed here, initial values of the parameters need to be provided and it is important that these values are not way too far from the actual values. The initial estimates for the EEV, TEV and the compressor parameters used are given in Table 2 and 3. If the initial operating conditions are known, then this information can be used to reduce the total number of parameters estimated by two using (1) and (10). Doing so increases the speed of parameter estimation.

IV. EXPERIMENTAL SET UP

Two experimental set ups are used to test the parameter estimation algorithms. One test rig is a custom instrumented air conditioning unit from Trane. This system is shown in Figure 11. The expansion valve used in this system is an EEV. The compressor in this set up is a constant speed scroll compressor. The mass flow rate of the air over the evaporator coils can be independently adjusted by varying the evaporator fan speed. The refrigerant used is R410A.

The second test rig is a custom designed refrigeration system with water as the secondary coolant. TEV is used as the expansion valve in this system. It has a variable speed scroll compressor. The secondary coolants flow rate over the evaporator coils is controlled by using a variable flow rate valve. The refrigerant used is R134a.

V. RESULTS

A. Parameter estimation of the EEV installed on a residential air-conditioner

The system is excited by stepping the EEV opening as seen in Figure 7. In Figure 8 it can be seen that the parameter estimation algorithm (simplex search) is able to find the parameters such that the predicted mass flow rate is exactly the same as the measured mass flow rate. Figure 9 and 10 show that the parameter estimation algorithm was successful in reducing the error as defined in Equation (12).
The time taken for Simplex search and nonlinear least squares given the same initial estimate as mentioned in Table II is 360 and 700 seconds respectively. A comparison of both these methods with respect to error and speed is given in Table II.

### Table II
EEV parameter estimation using different approaches for a dataset with 4200 samples. Linearized evaporator model used.

<table>
<thead>
<tr>
<th>Estimation method</th>
<th>EEV Parameters</th>
<th>Compressor Parameters</th>
<th>RMS error in mass flow rate at valve (grams/second)</th>
<th>Time taken (seconds)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial Parameters</td>
<td>$v_1$</td>
<td>$v_2$</td>
<td>$k_1$ $k_2$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$-1.70$</td>
<td>$1.00$</td>
<td>$-2.46$ $1.00$</td>
<td></td>
</tr>
<tr>
<td>Mass flow measurements</td>
<td>$-7.67 \times 10^{-3}$</td>
<td>$1.93$</td>
<td>$9.16$ $1.54$</td>
<td>$0.152$</td>
</tr>
<tr>
<td>Simplex search</td>
<td>$-1.34 \times 10^{-2}$</td>
<td>$1.94$</td>
<td>$9.05$ $1.59$</td>
<td>$0.151$</td>
</tr>
<tr>
<td>Nonlinear least squares</td>
<td>$-7.63 \times 10^{-2}$</td>
<td>$1.97$</td>
<td>$9.05$ $1.59$</td>
<td>$0.156$</td>
</tr>
</tbody>
</table>

### Table III
TEV parameter estimation using different approaches for a dataset with 2000 samples. Linearized evaporator model used.

<table>
<thead>
<tr>
<th>Estimation method</th>
<th>TEV Parameters</th>
<th>Compressor Parameters</th>
<th>RMS error in mass flow rate at valve (grams/second)</th>
<th>Time taken (seconds)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$v_1$</td>
<td>$v_2$ $\tau (s)$ $k_1$ $k_2$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Initial Parameters</td>
<td>$1.00$</td>
<td>$1.00$</td>
<td>$1.00$ $1.00$</td>
<td></td>
</tr>
<tr>
<td>Simplex search</td>
<td>$5.60$</td>
<td>$3.01$</td>
<td>$0.94$ $0.20$</td>
<td>$0.6766$</td>
</tr>
<tr>
<td>Nonlinear least squares</td>
<td>$5.31$</td>
<td>$3.12$</td>
<td>$0.94$ $0.20$</td>
<td>$0.5365$</td>
</tr>
</tbody>
</table>

Table II is 360 and 700 seconds respectively. A comparison of both these methods with respect to error and speed is given in Table II.

### B. Parameter estimation of a TEV on a Chiller unit

The water flow rate over the evaporator is varied as shown in Figure 12. While estimating the parameters of the EEV, it was seen that the Simplex search algorithm was faster but for the TEV case, which is a more complex problem, it took a longer time. The reason for this is that the algorithm settles at a local minima (the simplex shrinks to a point) and the optimization routine needs to be restarted. For this case, the nonlinear least squares provides a better estimate both in terms of speed and accuracy as can be seen by comparing the RMS error. With the estimated parameters, the augmented model is simulated and is compared with experimental data in Figures 13, 14 and 15. From the close match seen between the experimental data and the model outputs it is safe to say that the parameter estimation approach has worked well in this case.

Similar results were obtained with nonlinear least squares. The time taken for Simplex search and nonlinear least squares given the same initial estimate as mentioned in

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**Fig. 11.** 3-Ton residential air conditioner from Trane with an EEV

**Fig. 12.** Water flow rate.

**Fig. 13.** Comparison of experimental and simulated mass flow rate at the expansion valve
The above experimental results indicate that the parameter estimation method is successfully able to identify the valve and compressor parameters and is robust enough to overcome the problems of measurement noise, unmodeled dynamics. Also the time consumed for parameter estimation using both the optimization algorithms are low and can be used in real life applications.

VI. CONCLUSIONS

A novel approach to estimate the empirical parameters of a HVAC model has been provided. Easy to use models have been presented for the EEV, TEV and the compressor (both fixed speed and variable speed type) used in HVAC systems. The estimation problem was approached using Simplex search and nonlinear least squares. An important advantage of the Simplex search algorithm is that it is a faster algorithm since it does not compute the gradients which are computationally intensive owing to the complexity of the model. The ability to identify critical model parameters without expensive mass flow meters is a significant contribution in the area of HVAC dynamic modeling. This paper also offers a validated TEV model that was previously lacking in the literature. The technique proposed here can be used with other types of grey box identification problems that are difficult to solve using the identification methods like linear least squares approach or the maximum likelihood method.

This work is an enabling technique for HVAC system simulation and control. Knowing the valve and compressor parameters one can get accurate models which can be used for control applications, fault detection purposes. Another important application is the virtual sensors. All these applications have a potential economic impact.

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REFERENCES