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Anticipative Ramp Metering Control Using Dynamic Traffic Assignment

M. van den Berg, T. Bellemans, B. De Schutter, B. De Moor, J. Hellendoorn

***Abstract*— We develop an anticipative model-based traffic control approach for ramp metering in freeway networks. If ramp metering is implemented in a freeway network with alternative routes, traffic can spontaneously re-route due to the response of the drivers to the applied control actions. Although spontaneous re-routing can have a significant influence on the resulting traffic situation in the traffic network and on the performance of the traffic network as a whole, re-routing is usually not automatically included in current freeway traffic control frameworks. In this paper, we develop a new method to efficiently calculate and incorporate re-routing effects into a model-based predictive traffic control framework. In this way, anticipative model predictive control for ramp metering in freeway networks is realized.**

I. INTRODUCTION

It is becoming more and more important to find a solution for the increasing number of traffic jams. One feasible solution is to improve the existing traffic control systems. For freeways the control consists of, e.g., ramp metering, variable speed limits or in some lesser extent variable message signs. In a freeway network with multiple routes from origins to destinations, drivers have to select their route through the network. Traffic control measures may influence this route choice because they may modify the traffic flows in the network.

In this paper, which is an extension of [1], we develop a control strategy for ramp metering, taking these re-routing effects into account. The control strategy computes the traffic assignment based on the current flows, and uses a model to predict the future behavior of the traffic. In this way, the future traffic flows can be predicted for each setting of the control measures. Finally, numerical optimization is used to select the best settings.

This paper is organized as follows. A short description of the traffic model is given in Section II. In Section III the strategy for dynamic traffic assignment is explained. The control strategy is described in Section IV; note that although we focus on ramp metering, the presented technique can also be applied to other traffic control measures. Next we describe a simple network for a case study in Section V, and we present the simulation results for this case study.

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II. METANET TRAFFIC FLOW MODEL

We use the macroscopic METANET model [7], [14] developed by Papageorgiou and Messmer to describe the evolution of the traffic flows in the freeway network. In METANET the freeway network is represented as a graph with nodes and links, where the links correspond to freeway stretches with uniform characteristics; the nodes are placed at on-ramps and off-ramps, where two or more freeways connect, or where the characteristics change. Links are divided into one of more segments with a length of about 500 m.

The evolution of the traffic system is characterized by the average density $\rho_{m,i}(k_\mathrm{f})$, flow $q_{m,i}(k_\mathrm{f})$, and speed $v_{m,i}(k_\mathrm{f})$ for each segment i of each link m at time[1] $t = k_\mathrm{f} T_\mathrm{f}$:

$$\begin{aligned}
\rho_{m,i}(k_\mathrm{f}+1) &= \rho_{m,i}(k_\mathrm{f}) + \frac{T_\mathrm{f}}{L_m n_m}\left[q_{m,i-1}(k_\mathrm{f}) - q_{m,i}(k_\mathrm{f})\right] \\
q_{m,i}(k_\mathrm{f}) &= \rho_{m,i}(k_\mathrm{f}) v_{m,i}(k_\mathrm{f}) n_m \\
v_{m,i}(k_\mathrm{f}+1) &= v_{m,i}(k_\mathrm{f}) + \frac{T_\mathrm{f}}{\tau}\left(V(\rho_{m,i}(k_\mathrm{f})) - v_{m,i}(k_\mathrm{f})\right) + \\
&\quad \frac{T_\mathrm{f}}{L_m} v_{m,i}(k_\mathrm{f})\left[v_{m,i-1}(k_\mathrm{f}) - v_{m,i}(k_\mathrm{f})\right] - \\
&\quad \frac{\nu T_\mathrm{f}[\rho_{m,i+1}(k_\mathrm{f}) - \rho_{m,i}(k_\mathrm{f})]}{\tau L_m[\rho_{m,i}(k_\mathrm{f}) + \kappa]}
\end{aligned}$$

where T_f, L_m, $V(\rho_{m,i}(k_\mathrm{f}))$ and n_m respectively are the time step for freeway simulation, length of the segments of freeway link m, desired speed of the drivers on segment i of freeway link m, and the number of lanes of freeway link m, while τ, ν and κ are model parameters.

At nodes with more than one leaving link the arriving flow is divided over the leaving links according to:

$$q_{m,0}(k_\mathrm{f}) = \beta_{n,m}(k_\mathrm{f}) q_{\mathrm{tot},n}(k_\mathrm{f})$$

where n gives the node index, and $q_{\mathrm{tot},n}$ the total flow at the node. The value of the splitting rates $\beta_{n,m}(k_\mathrm{f})$ will be computed by the assignment algorithm described in the next section.

III. DYNAMIC TRAFFIC ASSIGNMENT

When there are multiple routes from origins to destinations the traffic flows divide themselves over the network. It appears that every driver assigns a cost $C_r(k_\mathrm{f})$ to every route r, and selects the route with the lowest cost. This will

[1] The index k_f (with subscript f) denotes the freeway simulation step counter. This to make a clear difference with the controller sample step, for which we will introduce the index k_c later on.

result in a user equilibrium, where the costs of alternative routes have the same value [18]. In this paper we use the travel time to describe the cost assigned to each route, as is suggested in [5], [12]. The travel time is computed as follows:

$$C_r(k_\mathrm{f}) = \sum_{(m,i)\in M_r} \frac{L_m}{v_{m,i}(k_\mathrm{f})} \ ,$$

where M_r is the set of pairs of indexes (m, i) of all links and segments belonging to route r.

There exist several methods to compute the equilibrium traffic assignment, such as described in [12], [15], [17], [19]. In this paper we use the 'Method of the Successive Averages' (MSA) [11]. MSA is an iterative method that computes the cost of different routes according to the flows $q_{r,j}$ in iteration j. Then all traffic is assigned to the route with the lowest cost, resulting in the all-or-nothing assignment flows $q_{\mathrm{AON},j}(k_\mathrm{f})$. These flows are used to compute the flows for the next iteration:

$$q_{r,j+1}(k_\mathrm{f}) = \Big(1 - \frac{1}{j}\Big) q_{r,j}(k_\mathrm{f}) + \Big(\frac{1}{j}\Big) q_{\mathrm{AON},j}(k_\mathrm{f}) \ .$$

The stopping criterion is based on a maximum value for the difference between two successive iteration flows: when the difference is below this specified value the algorithm terminates. To prevent long computation times the algorithm will also exit when a maximum number of iterations is reached. The resulting flows are used to determine the equilibrium split rates:

$$\beta_{n,r}^{\mathrm{MSA}}(k_\mathrm{f}) = \frac{q_{r,j}}{D_{o,d}(k_\mathrm{f})}$$

with $D_{o,d}(k_\mathrm{f})$ the total demand from origin o to destination d of route r passing through node n.

In practice the equilibrium is never reached. The traffic is always changing, and therefore *dynamic* traffic assignment is needed. There are several methods to compute the dynamic traffic assignment in a network based on a specified cost function, (see [2], [4], [14], [16], [8]). A disadvantage of dynamic assignment models used in combination with model-based predictive control is that often the user equilibrium assignment has to be computed for every controller step. Since computing the equilibrium assignment is very time consuming, this method cannot be used in real-time controllers. In this paper we propose a method that requires less computation time and is therefore suitable to use in an on-line model-based control approach.

The dynamic traffic assignment method we describe in this paper first computes the static equilibrium as described earlier, and uses this to determine the dynamic assignment.

To compute this dynamic assignment we assume that the drivers have been gathering information about the traffic for some time τ_info. They use this information to determine their route choice, which will lead to an equilibrium traffic assignment during the time they travel in the network. The larger τ_info, the slower the response of the route choice behavior of the drivers to varying traffic demands and metering rates will be. The resulting presumed equilibrium is computed with the MSA, using the mean of the traffic information gathered during the interval $[t - \tau_\mathrm{info}, t]$ as an estimation of the future demands and using the mean of the traffic control measure settings in this period as estimation of the future traffic control settings.

The traffic flows will not divide themselves according to the equilibrium assignment immediately. Therefore, we assume that the current traffic assignment will change toward the equilibrium assignment in an exponential way. Here the parameter τ_reac influences how fast the current assignment converges toward the presumed equilibrium assignment. This swiftness depends on the time that is needed for the effects of a congestion to reach the drivers that still have to make their route choice. This results in an adaptation of the splitting rates according to:

$$\beta_{n,r}(k_\mathrm{f}{+}1) = \beta_{n,r}(k_\mathrm{f}) {+} (\beta_{n,r}^{\mathrm{MSA}}(k_\mathrm{f}) {-} \beta_{n,r}(k_\mathrm{f}))(1 {-} e^{-\frac{T_\mathrm{f}}{\tau_\mathrm{reac}}}) \ . \tag{1}$$

The equilibrium, and thus the resulting dynamic assignment computed with (1), will differ from the real situation, because traffic is a stochastic process or because incidents may occur. We will keep this difference small by updating the equilibrium assignment and computing new splitting rates every T_update seconds.

IV. Anticipative ramp metering using Model Predictive Control

Ramp metering is a control measure that is used to improve the traffic flow near an on-ramp. This is done by limiting the flow that leaves the on-ramp. The metering rate $p(k_\mathrm{f})$ gives the fraction of the maximum capacity flow of the on-ramp that is allowed to depart toward the freeway. The flow entering the freeway is then given by

$$\begin{aligned} q_{\mathrm{ramp},o}(k_\mathrm{f}) = \min \Big[& D_{\mathrm{ramp},o} + \frac{w_o(k_\mathrm{f})}{T_\mathrm{f}}, \\ & Q_{\mathrm{cap},o} \min \Big(p_o(k_\mathrm{f}), \frac{\rho_{\mathrm{jam},m_o} - \rho_{m_o,1}(k_\mathrm{f})}{\rho_{\mathrm{jam},m_o} - \rho_{\mathrm{crit},m_o}} \Big) \Big] \end{aligned} \tag{2}$$

with w_o the number of vehicles waiting at the on-ramp origin o, $Q_{\mathrm{cap},o}$ the maximum capacity flow of the on-ramp, m_o freeway link m connected to the on-ramp, ρ_{jam,m_o} the maximal density on freeway link m_o, and ρ_{crit,m_o} the density where congestion starts on segment m_o.

There are different methods to determine the metering rate, (see [9], [10] for an overview). We propose an on-line model-based predictive control design strategy that can handle constraints such as maximal on-ramp queue lengths, and minimal and maximal metering rates:

$$\begin{aligned} w_o(k_\mathrm{f}) &\le w_o^{\max} \\ p^{\min} &\le p(k_\mathrm{f}) \le p^{\max} \ . \end{aligned} \tag{3}$$

For ease of notation we first define the set of simulation steps k_f that correspond to a given interval $[k_\mathrm{c}^a, k_\mathrm{c}^b]$ of controller time steps as follows:

$$\mathcal{K}_{\mathrm{f}}(k_{\mathrm{c}}^{a},k_{\mathrm{c}}^{b})=\left[k_{\mathrm{c}}^{a}\frac{T_{\mathrm{c}}}{T_{\mathrm{f}}},k_{\mathrm{c}}^{a}\frac{T_{\mathrm{c}}+1}{T_{\mathrm{f}}},...,k_{\mathrm{c}}^{b}\frac{T_{\mathrm{c}}}{T_{\mathrm{f}}}-1\right]$$

where k_{c} is the control step and T_{c} is the control sample time.

The control strategy uses an indicator to determine the performance of the network. As performance indicator we will consider the total time spent (TTS) by all vehicles in network (but note that the proposed approach also works for other performance indicators). The TTS in the period $[k_{\mathrm{c}}^{a}T_{\mathrm{c}},k_{\mathrm{c}}^{b}T_{\mathrm{c}}]$ can be computed as:

$$\mathrm{TTS}(k_{\mathrm{c}})=T_{\mathrm{f}}\sum_{k_{\mathrm{f}}\in\mathcal{K}_{\mathrm{f}}}\left(\sum_{(m,i)\in M}L_{m}n_{m}\rho_{m,i}(k_{\mathrm{f}})+\sum_{o\in O}w_{o}(k_{\mathrm{f}})\right)$$

where M is the set of pairs of indexes (m,i) of all links in the network, and O the set of origins.

Briefly, model predictive control (MPC) [3], [6] works as follows. At a given time $t=k_{\mathrm{c}}T_{\mathrm{c}}=k_{\mathrm{f}}T_{\mathrm{f}}$ the MPC controller uses the prediction model METANET and numerical optimization to determine the optimal ramp metering sequence $p^{*}(k_{\mathrm{c}}),\ldots,p^{*}(k_{\mathrm{c}}+N_{\mathrm{p}}-1)$ that minimizes the given performance indicator $\mathrm{TTS}(k_{\mathrm{c}})$ over the time horizon $[k_{\mathrm{c}}T_{\mathrm{c}},(k_{\mathrm{c}}+N_{\mathrm{p}})T_{\mathrm{c}})$ based on the current state of the traffic network and on the expected demands over this period, where N_{p} is called the prediction horizon. The prediction horizon should be long enough to show all the effects of a control action. This can be reached by choosing it larger or equal to the time that is needed by a vehicle to drive through the longest route of the network.

Furthermore, a receding horizon approach is used in which at each control step only the first control input sample $p^{*}(k_{\mathrm{c}})$ is applied to the system during the period $[k_{\mathrm{c}}T_{\mathrm{c}},(k_{\mathrm{c}}+1)T_{\mathrm{c}})$. When the first sample is applied the horizon is shifted, new measurements are made, and the process is repeated all over again.

Because the MPC controller makes a prediction of the traffic states during the period $[k_{\mathrm{c}}T_{\mathrm{c}},(k_{\mathrm{c}}+N_{\mathrm{p}})T_{\mathrm{c}})$, a prediction of the splitting rates over this period is required. This prediction is made using the dynamic traffic assignment algorithm described in Section III. The time step for updating the assignment, T_{update}, is chosen an order of magnitude smaller than the prediction period $N_{\mathrm{p}}T_{\mathrm{c}}$. The value of τ_{reac} depends on the re-routing dynamics in the network, which are typically much slower than the dynamics of the traffic system near the on-ramps and which depend on the topology of the network [13].

V. Results of a case study

We will illustrate the MPC-based anticipative ramp metering control using a simple network. The layout of the network is shown in Figure 1, where the arrow gives the direction of the traffic flows. The network consists of a freeway with four lanes that bifurcates into two branches of two lanes each. Downstream both branches join in a four-lane freeway. Both four-lane freeway links are 3 km long. Both two-lane links are 6 km long. The lower two-lane branch is the primary branch, with an on-ramp in the middle of the branch. Route 1 follows the primary branch, and route 2 the secondary. The traffic originating from the mainstream origin distributes over the two branches using the route choice mechanism described in Section III.

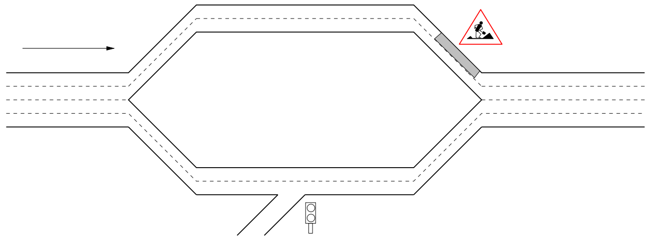


Fig. 1. Schematic representation of the freeway network of the case study.

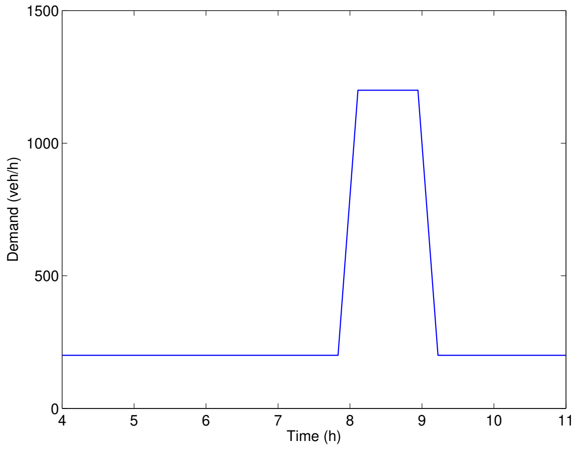


Fig. 2. Traffic demand on the on-ramp

The model and controller parameters are selected as follows: τ_{info} = 30 min, τ_{reac} = 20 min, T_{update} = 10 s, T_{f} = 10 s, T_{c} = 1 min, N_{p} = 15 min, $p^{\max}$ = 1, $p^{\min}$ = 0.05, $w_{o}^{\max}$ = 300 veh. The start of the simulation is at 4.00 a.m., and a period of seven hours is simulated.

We simulate a traffic scenario with road maintenance works on route 2 as shown in Figure 1. The maintenance works result in a reduction of the number of lanes from 2 to 1 in two segments (i.e., 1000 m) of the secondary branch. The maintenance work starts at 4.30 a.m. and persist during the remainder of the simulation.

The traffic demand on the mainstream is considered constant and equal to 6000 veh/h in this simulation. The traffic demand on the on-ramp is equal to 200 veh/h with a peak traffic demand of 1200 veh/h around 7 a.m., as given in Figure 2.

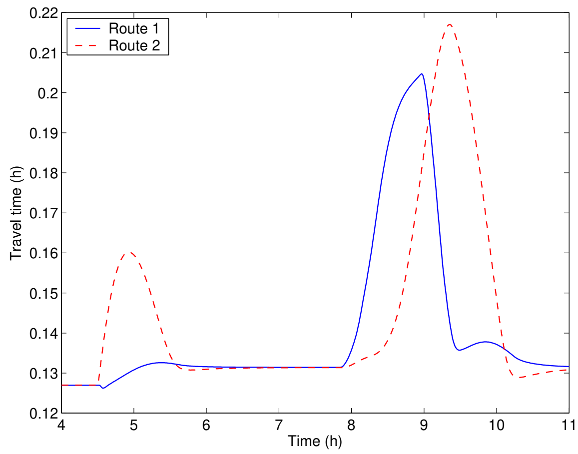
To show the effects of ramp metering we have done two simulations: one without ramp metering and one with ramp metering. The first is used to show the functioning of the dynamic traffic assignment, and the second shows the change in route choice and the improved travel times due to the ramp metering.

The results for the simulation without ramp metering are shown in Figure 3. At the beginning of the simulation an equilibrium situation exists: the two travel times have the same value. At 4.30 a.m. the maintenance work starts. The travel times become different, resulting in a change in the splitting rates. From 5.15 a.m. until 7.00 a.m. the exponential convergence to the equilibrium splitting rates can be seen. Until 8 a.m. the equilibrium is maintained. Then the traffic on the on-ramp increases, which causes a change in the travel time of route 1, and thus a change in the splitting rates. After 9.10 a.m. the peak demand on the on-ramp has ended. Due to the delay introduced by τ_{info} the travel times on the secondary route keep increasing for some time. After 9.30 a.m. the exponential behavior toward the equilibrium again can be seen.

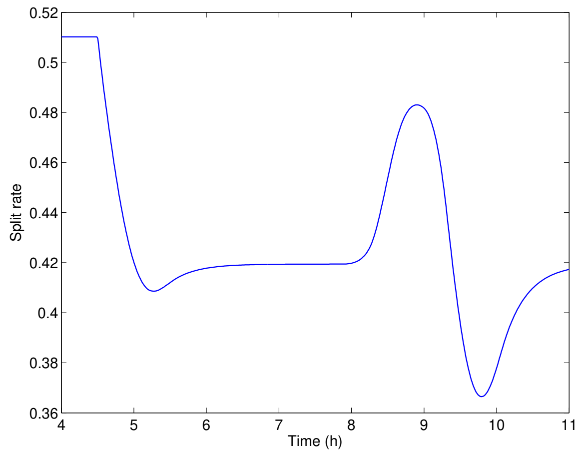
The bottom plot in Figure 3 shows the density on the segment downstream of the on-ramp. The horizontal line gives the critical density. The density is increasing when the maintenance works start. Due to the re-routing the density decreases until 5.15 a.m. Until 6.00 a.m. the exponential behavior can be seen, and the equilibrium is reached from 6.00 a.m. until 8.00 a.m. Then the traffic demand on the on-ramp increases, resulting in an increase of the density. At 9.10 a.m. the peak demand on the on-ramp ends, resulting in a lower density. After 9.30 a.m. more traffic starts taking the primary branch again, and the density increases. Afterward, the exponential behavior of the assignment leading to an equilibrium can be seen again.

The results for the simulation with ramp metering are shown in Figure 4, and the effects on the on-ramp traffic are shown in Figure 5. The simulation with control starts with the same equilibrium traffic assignment as the simulation without control. Until 8.00 a.m. it behaves the same as the simulation without control, because the density on the first route does not rise above the critical density, and limiting the on-ramp flow is not useful. At 8.00 a.m. the demand on the on-ramp increases. As a result the travel time on the first route increases, just as the density on the segment after the on-ramp. The ramp metering is not directly activated when the critical density is reached. This is due to the fact that the controller takes rerouting into account. By allowing a higher density, and thus longer travel times, on the first route more vehicles are selecting the second route where they are not influenced by the traffic on the on-ramp. But after 8.30 a.m. too many vehicles select the second one, which will give longer travel times on that route. At this moment the ramp metering is activated, lowering the travel time of the first route. The metering signal is chosen so that when the queue clears the equilibrium is reached as soon as possible, at 10.00 a.m.

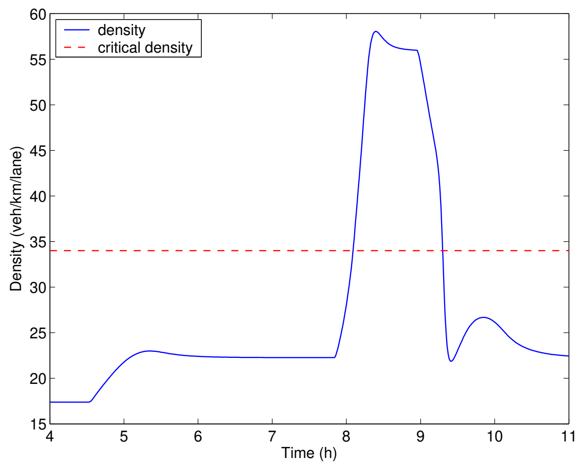
Figure 5 shows the effects of ramp metering on the on-



(a) Travel time on the two different routes

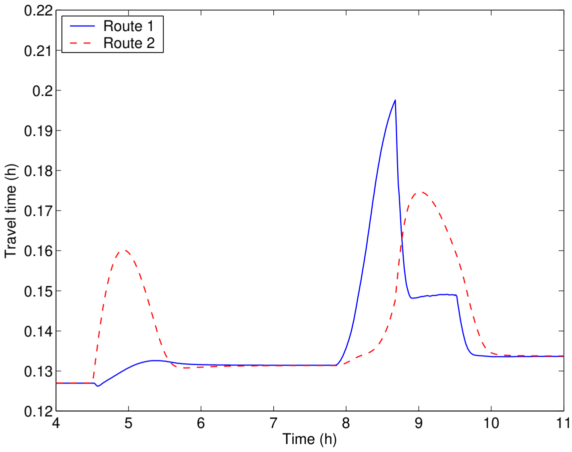


(b) Split rate toward route 2

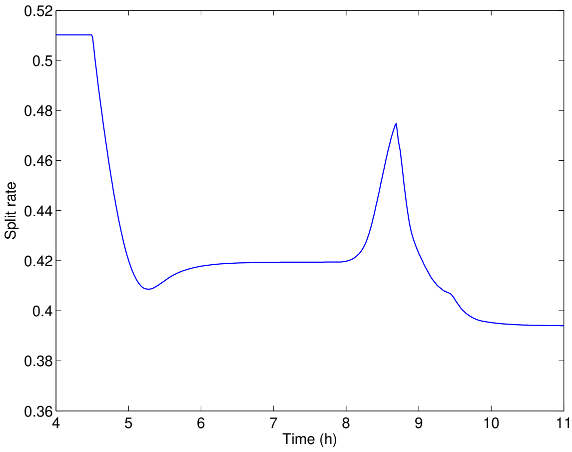


(c) Density on segment downstream of the on-ramp

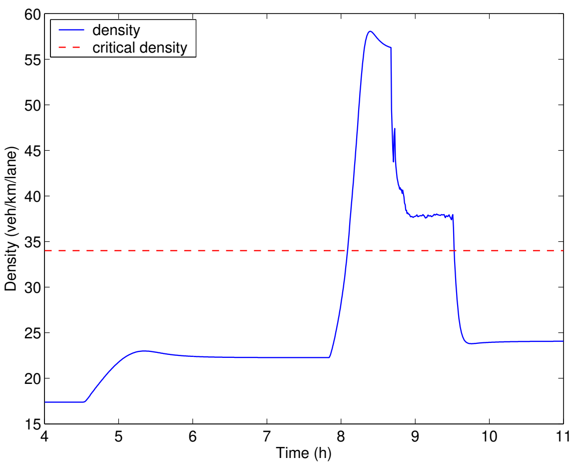
Fig. 3. Simulation results without ramp metering



(a) Travel time on the two different routes

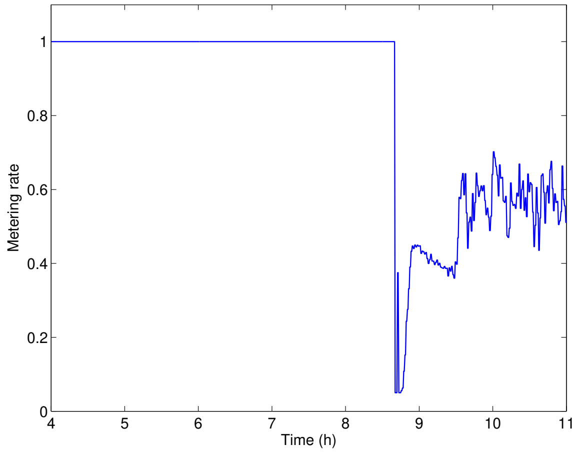


(b) Split rate toward route 2

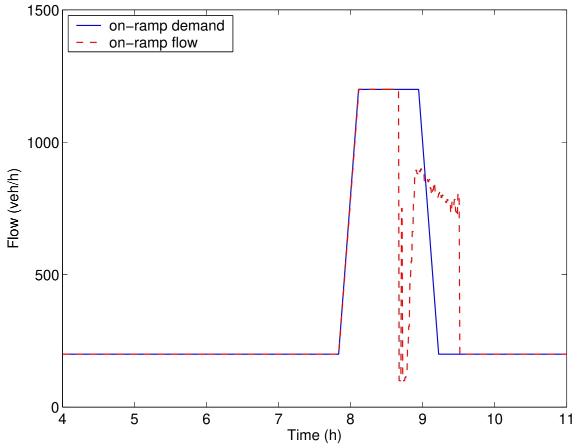


(c) Density on the segment downstream of the on-ramp

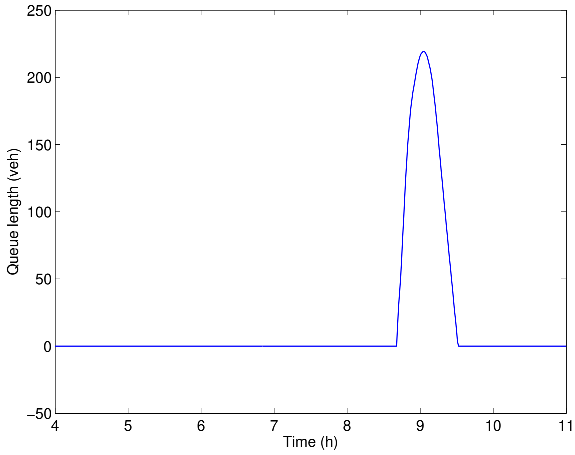
Fig. 4. Simulation results with ramp metering



(a) Metering rate



(b) Traffic demand on the on-ramp, and flow allowed to enter the freeway



(c) Queue on the on-ramp

Fig. 5. Effects of ramp metering on the on-ramp traffic

ramp traffic. First the ramp metering signal is shown. It starts at 1, which means that all the traffic is allowed to enter the freeway. At 8.30 a.m. it limits the on-ramp flow to decrease the travel time at the first route. Until 9.30 a.m. the traffic is still limited, and after 9.30 a.m. the ramp metering signal varies, but it does not limit the flow anymore.

The next figure shows the flow on the on-ramp. The solid line gives the demand, and the dashed line the flow that is allowed to enter the freeway. At 8.30 a.m. there is a difference between these two, resulting in a queue shown in the last figure. When the demand on the on-ramp decreases at 9.00 a.m. the vehicles in the queue can start to enter the freeway, resulting in an empty queue at 9.30 a.m.

VI. CONCLUSIONS

When there are multiple routes from origins to destinations, drivers tend to take the cheapest route available. We have developed a control strategy based on model predictive control (MPC) that takes the re-routing of traffic into account. As control measure we have selected ramp metering, and as model we have selected the METANET model. For the traffic assignment we have developed an algorithm based on the Method of Successive Averages. We have done a synthetic case study on a small network with two possible routes, with maintenance works on one of them. As control measure we used ramp metering.

Topics for future research include: inclusion of other static and dynamic traffic assignment methods and investigation of their effect on the trade-off between accuracy or performance and computational complexity; comparison with existing methods for ramp metering; investigation of other scenarios and larger networks; and inclusion of other traffic control measures.

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REFERENCES

[1] T. Bellemans, B. De Schutter, and B. De Moor. Anticipative model predictive control for ramp metering in freeway networks. In *Proceedings of the 2003 American Control Conference*, pages 4077–4082, Denver, Colorado, June 2003.

[2] M.C.J. Bliemer. A quasi-variational inequality approach to multi-user class dynamic traffic assignment. In *Proceedings of the 79th Annual Meeting of the Transportation Research Board*, Washington, D.C, 2000.

[3] E.F. Camacho and C. Bordons. *Model Predictive Control in the Process Industry*. Springer-Verlag, Berlin, Germany, 1995.

[4] I. Chabibi and Y. He. Dynamic traffic assignment: An analytical tool and its application. In *Proceedings of the International Conference on Intelligent Transportation Systems*, pages 32–38, Washington, D.C, 2000.

[5] C.F. Daganzo and Y. Sheffi. On stochastic models of traffic assignment. *Transportation Science*, 11:253–274, 1977.

[6] J.M. Maciejowski. *Predictive Control with Constraints*. Prentice Hall, Harlow, England, 2002.

[7] A. Messmer and M. Papageorgiou. METANET: A macroscopic simulation program for motorway networks. *Traffic Engineering and Control*, 31(9):466–470, 1990.

[8] M. Papageorgiou. Dynamic modeling, assignment, and route guidance in traffic networks. *Transportation Research Part B*, 24(6):471–495, December 1990.

[9] M. Papageorgiou, H. Hadj-Salem, and J.M. Blosseville. ALINEA: A local feedback control law for on-ramp meetering. *Transportation Research Record*, (1320):58–64, 1991.

[10] M. Papageorgiou and A. Kotsialos. Freeway ramp metering: An overview. In *Proceedings of the International Conference on Intelligent Transportation Systems*, pages 228–239, 2000.

[11] W.B. Powell and Y. Sheffi. The convergence of equilibrium algorithms with predetermined step sizes. *Transportation Science*, 16(1):45–55, January 1982.

[12] Y. Sheffi. *Urban Transportation Networks*. Prentice-Hall, Englewood Cliffs, N.J., 1985.

[13] H. Taale and H.J. van Zuylen. Traffic control and route choice: Occurrence of instabilities. In *TRAIL 5th Annual Congress 1999: Five Years Crossroads of Theory and Practice*, volume 2, pages 1–19. Trail, 1999.

[14] Technical University of Crete, Dynamic Systems and Simulation Laboratory and A. Messmer. *METANET – A simulation program for motorway networks*, July 2000.

[15] D. Van Vliet. The Frank-Wolfe algorithm for equilibrium traffic asignment viewed as a variational inequality. *Transportation Research Part B*, 21(1):87–89, 1987.

[16] Y. Wang and M. Papageorgiou. A predictive feedback routing control strategy for freeway network traffic. In *Proceedings of the American Control Conference*, pages 3606–3611, Anchorage, May 2002.

[17] Y. Wang, M. Papageorgiou, and A. Messmer. A predictive feedback routing control strategy for freeway network traffic. In *Proceedings of the 82nd Annual Meeting of the Transportation Research Board*, Washington, D.C, January 2003.

[18] J.G. Wardrop. Some theoretical aspects of road traffic research. *Proceedings of the Institute of Civil Engineers, Part II*, 1(2):325–378, 1952.

[19] M.B. Wisten and M. J. Smith. Distributed computation of dynamic traffic equilibria. *Transportation Research Part C*, 5:77–93, 1997.