Delft University of Technology

Delft Center for Systems and Control

Technical report 06-038

Influencing long-term route choice by traffic control measures, a basic model study*

M. van den Berg, A. Hegyi, B. De Schutter, and H. Hellendoorn

If you want to cite this report, please use the following reference instead:
M. van den Berg, A. Hegyi, B. De Schutter, and H. Hellendoorn, "Influencing long-term route choice by traffic control measures, a basic model study," *Proceedings of the 9th TRAIL Congress 2006 – TRAIL in Motion – CD-ROM*, Rotterdam, The Netherlands, 16 pp., Nov. 2006.

Delft Center for Systems and Control
Delft University of Technology
Mekelweg 2, 2628 CD Delft
The Netherlands
phone: +31-15-278.51.19 (secretary)
fax: +31-15-278.66.79
URL: `http://www.dcsc.tudelft.nl`

Influencing long-term route choice by traffic control measures, a basic model study

TRAIL Research School, Delft, November 2006

Authors

Ir. Monique van den Berg, dr. ir. Andreas Hegyi, dr. ir. Bart De Schutter, and Prof. dr. ir. Hans Hellendoorn

Delft Center for Systems and Control, Delft University of Technology

Contents

Abstract

Currently used traffic control measures, such as traffic signals, ramp metering installations etc., are often not designed to influence the route choice of drivers. However, traffic control measures influence the travel times that are experienced in the network. Since route choice, at least for a part, is based on experienced travel times, the measures also influence the long-term route choice. This influence can be seen as a side-effect of the measures, but in this paper we will investigate the possibilities to explicitly use the influence of the traffic control measures to change the route choice. With basic traffic flow and route choice models we investigate possible equilibrium turning rates for a network with two routes. We use two different types of control: speed control and outflow control. The control method used is a simple controller which makes the analytical investigation of the effects of the controller possible, but the results can be extended to more sophisticated control methods.

Keywords

Traffic control, route choice, user equilibrium

1 Introduction

In road networks locations are nearly always connected by more than on route, which means that drivers who want to go from an origin to a destination have to choose which route to take. When all drivers make such a route choice this results in a traffic assignment that is preferred by the drivers. A traffic assignment gives the number of drivers that have selected each route. Such an assignment preferred by the drivers may lead to large traffic flows on narrow or dangerous roads, to socially undesired situations (e.g. a lot of vehicles in residential areas or near primary schools), or to too large flows near national parks causing pollution and noise. To prevent negative effects of large traffic flows, road administrators can try to influence the route choice of the drivers, to reach a traffic assignment with less traffic on some roads.

Different methods are available to influence the route choice. At this moment, much attention is payed to providing information, either pre-trip, en-route, or post-trip. Further, some experiments with traffic control measures have shown that these measures can influence the route choice (Haj-Salem & Papageorgiou (1995)). This has led to the theoretical development of methods to incorporate the effect of existing measures on route choice (Bellemans et al. (2003); Karimi et al. (2004); Wang & Papageorgiou (2002)). Descriptions of possible equilibria with the current measures are given in (Taale & van Zuylen (1999)). Taale and van Zuylen describe which equilibria can emerge with existing settings of the traffic control measures. In this paper however, we investigate how we can change the settings of the measures to steer towards different equilibria.

We investigate the possibilities of using traffic control measures like traffic signals, ramp metering installations, and dynamic speed limits to influence the route choice of drivers. The traffic control measures influence the travel time that the drivers experience, and in this way they influence the route choice. Note that the key assumption we make is that the experienced travel time is the most important factor in route choice, which is also argued for in Bogers et al. (2005).

To describe the properties of traffic that are useful for influencing route choice, two models are required: a route choice model and a travel time model. Since the main goal of this paper is to gain insight in the mechanisms regarding control of route choices, we select simple models and controllers, to be able to provide analytical descriptions of the behavior of the traffic flows, and to formulate intuitive explanations. Moreover, to obtain insight in what takes place when more realistic models are used, it is useful to get insight in the underlying principles by using simple models first. Therefore, we select models that are simple but have all relevant features.

Furthermore, to keep the analysis simple, we select basic controllers which can only change the parameters of route 1. Note that when however the parameters of the second route can also be changed, the actions on one of the routes can influence the traffic on the other route, which will lead to multi-variable behavior. This makes the analysis much more difficult, without giving more insight in the properties of the controllers.

In the next section we describe the small network with two routes that we use for our investigation. In Section 3 we select a model to describe the experienced travel times, and in Section 4 a model to describe the route choices. In Section 5 we look at the equilibrium turning rates that are the results of the selected models, network and traffic

demands. The next step is to apply outflow control, which is described in Section 6. Speed control has slightly other possibilities, which are given in Section 7. At last, some conclusions are drawn in Section 8.

2 Available routes and traffic demand

To obtain simple and intuitive results, we require a simple network. When working with route choice, a network with two routes between an origin and a destination is the easiest. The network contains all features that are required for route choice, but is small enough to make intuitive understanding possible. Such a network is shown in Figure 1. We will use this network during the remainder of the paper. Drivers enter this network at the origin and have to make their route choice immediately. Then they experience a travel time during their trip through the network and leave the network at the destination.

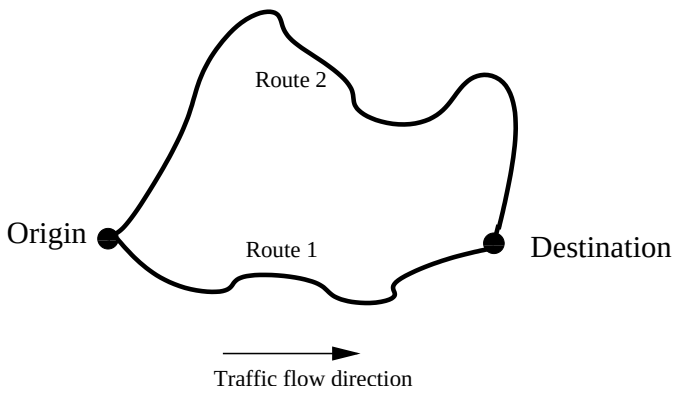

Figure 1: Network with two routes

Each route r ($r \in \{1, 2\}$) can be described by the following parameters, where d is the counter for the days. The outflow limit and speed limit can change each day:

l_r	length (km)
$Q_r(d)$	applied outflow limit at day d (veh/h)
$v_r(d)$	applied speed limit at day d (km/h)
$Q_r^{\min}$	minimum outflow limit (veh/h)
$Q_r^{\max}$	maximum outflow limit (veh/h)
C_r	capacity (veh/h)
$v_r^{\min}$	minimum speed limit (km/h)
$v_r^{\max}$	maximum speed limit (km/h)

The speed limit $v_r(d)$ gives the maximum speed that is allowed. We assume that this speed limit is enforced, so all drivers have a speed that is lower than or equal to the speed limit. The outflow limit $Q_r(d)$ gives the number of vehicles per hour that is allowed to leave the route. The outflow limit can be implemented via e.g. ramp metering installations or traffic signals. The maximum value of the outflow limit, $Q_r^{\max}$, must be equal to or lower than the actual capacity of the road: $Q_r^{\max} \leq C_r$. The minimum value $Q_r^{\min}$ can be selected to prevent total closure of the road when outflow control is

applied. $Q_r(d)$ gives the outflow limit that is applied at the current moment; without outflow control we have $Q_r(d) = C_r$. For the speed limits the same construction is used.

Another characteristic of the routes is the free-flow travel time, which describes the time that a vehicle needs to travel a route when there is no delay. The free-flow travel time is given by:

$$TT_r^{\text{free}} = \frac{l_r}{v_r^{\max}} \tag{1}$$

We look at one part of the day, for example the morning peak. During this peak we assume that the demand q (veh/h) in the network is constant. The demand is distributed over the two routes according to the turning rate $\beta(d)$, which gives the percentage of the traffic that selects route 1. The turning rate is computed with the route choice model described in Section 4. To obtain an equilibrium, a sequence of days with the same demand must be taken into account, to be able to eliminate transient effects.

3 Travel time model

We assume that travel time is the most important factor that influences route choice, based on Bogers et al. (2005). To describe the route choice, we thus need a model that describes the experienced travel time that is the result of a route choice.

Travel times can be computed in two different ways: performing a traffic flow simulation and determine the experienced travel times, or compute the travel times directly as a function of the number of vehicles on each route. Traffic flow simulations that can be used to compute travel times are described in Quadstone (2002); PTV (2003); Barceló & Ferrer (1997); Messmer & Papageorgiou (1990); Daganzo (1994); Lighthill & Whitham (1955). Travel time models were described in Fisk (1980), and an overview is given in Carey & Ge (2003).

In this paper we use a basic travel time model, which is as simple as possible to allow easy understanding and simple analysis. This results in a piecewise affine model, which describes the mean experienced travel time. We assume that the travel time on a route has two components: the time spent in the queue, which cannot be negative, and the free-flow travel time:

$$TT_1(d) = \max(0, TT_1^{\text{queue}}(d)) + TT_1^{\text{free}} \quad .$$

Note that the fact that the time in the queue cannot be negative, which makes the model piecewise affine instead of affine. In our model, the queues are vertical, and located at the end of the routes. We assume that the vehicles drive the whole route without delay, experiencing the free-flow travel time. At the end of the route, the vehicles enter the vertical queue and wait in this queue until they can leave the route. The free-flow travel time TT_r^{free} is already described in Equation 1. The time in the queue depends on the number of vehicles in the queue. During one peak period, the queue grows as shown in Figure 2. The length of the peak period is T, $N(t)$ gives the number of vehicles in the queue, and $\beta(d)q$ gives the flow on route 1. When the free-flow travel time has passed, the first vehicles reach the end of the route. In Figure 2 the queue length is plotted for different demands. When the demand is less than the outflow limit, $q \leq C_1$, no queue

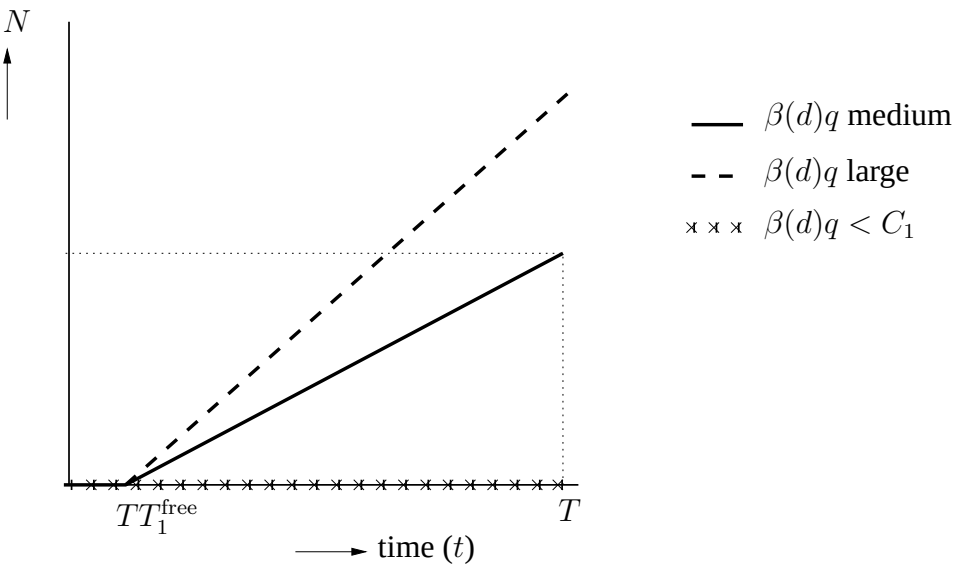

Figure 2: Queue length on a link during one day

appears. When the demand is larger than the outflow limit a queue starts to grow, with rate $\beta(d)q - C_1$. When the demand is larger, the queue length will increase faster.

In Section 4 we will describe a route choice model that requires the mean travel time for each day. To obtain this mean travel time, we have to compute the average number of vehicles in the queue. The average number of vehicles is given by the area below the graph of Figure 2 divided by the period in which the queue exists:

$$N_1^{\text{mean}}(d) = \frac{(\beta(d)q - C_1)(T - TT_1^{\text{free}})}{2} \quad \text{if } \beta(d)q \leq C_1 \quad .$$

The travel time in the queue is given by the time that a vehicle needs to reach the downstream end of the mean queue when it enters at the upstream end. Since C_1 vehicles are leaving the queue each hour, the time that the last vehicle leaves the queue is given by:

$$TT_1^{\text{queue}}(d) = \frac{N_1^{\text{mean}}(d)}{C_1} = \frac{(\beta(d)q - C_1)(T - TT_1^{\text{free}})}{2C_1} \quad .$$

The same computations can be done for route 2. Note however that the traffic that enters route 2 is given by $(1 - \beta(d))q$, which results in:

$$TT_2^{\text{queue}}(d) = \frac{((1 - \beta(d))q - C_2)(T - TT_2^{\text{free}})}{2C_2} \quad .$$

The model describes vertical queues, without a maximum queue length. Since it is a piecewise affine model, non-linear effects such as the capacity drop or longitudinal waves are not modeled. The use of a piecewise affine travel time model with constant demands makes that the predictions are restricted to static traffic assignments, and thus unable to handle varying traffic demands.

4 Route choice model

Different models exist to describe how many drivers select each route. Traffic assignment models (Daganzo & Sheffi (1977); Bliemer (2000); Peeta & Mahmassani (1995))

compute an equilibrium traffic assignment for the whole network. Route choice models (Bogers et al. (2005); Mahmassani et al. (2003); Ben-Akiva et al. (1991)) describe the route choice of drivers at locations where a route must be selected. We develop a route choice model, that requires little computational effort and is very intuitive.

Route choice is in equilibrium when no driver can change its route without increasing its costs, as described by Wardrop (1952). Route choice models do not automatically result in such a user equilibrium, but the route choice model we develop here does. We assume that the route choice only depends on travel times experienced earlier (Bogers et al. (2005)). The mean experienced travel times are assumed to be known by all drivers, which leads to the assumption that all the drivers are informed drivers, meaning that they know the travel times on both routes, independent of their selected route. Since the historical mean travel times lead to a learned turning rate (which is the result of the route choice model) this turning rate is the only variable that should be updated when new mean travel times are available. This leads to the following model:

$$\beta^{\text{intended}}(d+1) = \beta(d) + \eta(TT_2(d) - TT_1(d))$$

where d is the counter for the day that the trip is made, and the parameter η is used to translate the time difference into a percentage of traffic that changes its route due to the travel time difference. Here we assume that η has a constant value, but note that in many studies a logit function (Dial (1971)) is used to describe the fact that for larger travel time differences exponentially more drivers change their route.

Drivers are often slow in changing their habitual route choice (Bogers et al. (2005)) so we include a learning factor ω, which describes that drivers update their route choice as a function of the previous route choice and the adapted route choice:

$$\beta(d+1) = (1-\omega)\beta(d) + \omega\beta^{\text{intended}}(d+1)$$

Merging the two equations, and setting $\kappa = \omega\eta$ gives:

$$\beta(d+1) = \beta(d) + \kappa(TT_2(d) - TT_1(d)) \quad .$$

5 Equilibrium turning rates

Although this paper focuses on investigating the possible influences of control measures, we first describe the situation where no control is applied. The travel time model and route choice model combined are used to describe the behavior of the traffic. The total model then looks as follows:

$$\begin{aligned}\beta(d+1) =& \beta(d) + \kappa\Big(\max(0, \frac{((1-\beta(d))q - C_2)(T - TT_2^{\text{free}})}{2C_2}) \\ &- \max(0, \frac{(\beta(d)q - C_1)(T - TT_1^{\text{free}})}{2C_1}) + TT_2^{\text{free}} - TT_1^{\text{free}}\Big)\end{aligned} \tag{2}$$

This model leads to an equilibrium traffic assignment when the parameter κ is within a certain range, outside this range oscillations occur. These oscillations can happen when for example drivers react too strongly on travel time differences or when the capacity

of one of the roads is relatively low. To compute the equilibrium turning rates we set $\beta(d+1) = \beta(d)$. The value of the equilibrium turning rate depends on the free-flow times $TT_1(d)$ and $TT_2(d)$ of the two routes, and on the traffic demand q. Note that we use a fixed demand, which together with the selected travel time model results in a static traffic assignment. This allows for the use of basic analytical methods to describe the resulting equilibria.

In the next sections, we investigate situations where route 1 is the shortest route, where route 2 is the shortest route, and where both routes are equally long. For each of these situations, we examine a low demand $q < C_r$, a demand that leads to a queue on one route $q > C_r \ \& \ q \leq C_1 + C_2$, and a demand that leads to a queue on both routes $q > C_1 + C_2$.

5.1 Shortest route is route 1: $TT_1^{\text{free}} < TT_2^{\text{free}}$

When the free-flow travel time of route 1 is shorter than the free-flow time of route 2, route 1 will in general be the most desired route. When the total demand is lower than the capacity of route 1, all the traffic will take it:

$$\beta^{\text{eq}} = 1 \quad \text{if} \ \ q \leq C_1$$

where β^{eq} is the turning rate during the equilibrium.

When the demand is larger than the capacity of route 1, but is smaller than the total capacity ($C_1 + C_2$), most of the traffic will take route 1, which will result in a queue on this route. When the travel time on route 1 exceeds the free-flow travel time on route 2, a part of the traffic will divert to route 2. In this case, Equation 2 can be reduced to:

$$\beta(d+1) = \beta(d) + \kappa\Big(- \frac{(\beta(d)q - C_1)(T - TT_1^{\text{free}})}{2C_1} + TT_2^{\text{free}} - TT_1^{\text{free}}\Big)$$

since there is only a queue on route 1 and not on route 2. Setting $\beta(d+1) = \beta(d) = \beta^{\text{eq}}$ gives:

$$\kappa\Big(- \frac{(\beta^{\text{eq}}q - C_1)(T - TT_1^{\text{free}})}{2C_1} + TT_2^{\text{free}} - TT_1^{\text{free}}\Big) = 0 \tag{3}$$

which leads to the following equilibrium turning rate:

$$\beta^{\text{eq}} = \frac{C_1}{q} + \frac{(TT_2^{\text{free}} - TT_1^{\text{free}})2C_1}{q(T - TT_1^{\text{free}})} \quad \text{if} \ \ q \geq C_1 \ \ \& \ \ q \leq C_1 + C_2 \quad . \tag{4}$$

The first term of this equation describes the turning rate that leads to a traffic flow that is equal to the capacity. The second term gives the extra traffic flow, which will lead to the formation of a queue. It can be seen that this amount of flow depends on the difference of free-flow times of the two routes, compared to the capacity of the first route. It corresponds to drivers that make a trade-off between driving an extra distance or waiting in a queue.

When the demand is larger than the total capacity, the traffic divides itself over the two routes. On both routes a queue is then formed. Equation 2 can then be rewritten as:

$$\begin{aligned}\beta(d+1) =& \beta(d) + \kappa\Big(\frac{((1-\beta(d))q - C_2)(T - TT_2^{\text{free}})}{2C_2} \\ & - \frac{(\beta(d)q - C_1)(T - TT_1^{\text{free}})}{2C_1} + TT_2^{\text{free}} - TT_1^{\text{free}}\Big)\end{aligned}$$

which at equilibrium, with $\beta(d+1) = \beta(d) = \beta^{\text{eq}}$ leads to:

$$\begin{aligned}\beta^{\text{eq}} =& \frac{C_1 q(T - TT_2^{\text{free}}) + C_1 C_2((T - TT_1^{\text{free}}) - (T - TT_2^{\text{free}}))}{q(C_1(T - TT_2^{\text{free}}) + C_2(t - TT_1^{\text{free}}))} \\ &+ \frac{2C_1 C_2(TT_2^{\text{free}} - TT_1^{\text{free}})}{q(C_1(T - TT_2^{\text{free}}) + C_2(T - TT_1^{\text{free}}))} \\ &\text{if } q > C_1 + C_2 \quad . \end{aligned} \tag{5}$$

5.2 Shortest route is route 2: $TT_1^{\text{free}} > TT_2^{\text{free}}$

In this case route 2 is the shortest and most desired route for the drivers. The possible equilibria are computed using the same equations as in Section 5.1, but with the two routes, and the turning rate, reversed. When the demand is larger than the capacity of the second route, this gives:

$$\beta^{\text{eq}} = \frac{q - C_2}{q} + \frac{2C_2(TT_2^{\text{free}} - TT_1^{\text{free}})}{q(T - TT_2^{\text{free}})} \quad \text{if } q \geq C_2 \;\&\; q \leq C_1 + C_2 \quad . \tag{6}$$

The other equations can be obtained in a similar way from the equations in Section 5.1.

5.3 The routes are equally long: $TT_1^{\text{free}} = TT_2^{\text{free}}$

When both routes have the same free-flow travel time, the equilibrium turning rates can depend on the turning rate at the beginning, β^0. This is because drivers in this case will only change their route when a queue is formed.

A queue on a route is formed when the turning rate at the beginning results in a flow larger than the capacity on this route: $\beta^0 q > C_1$ for route 1 or $(1-\beta^0)q > C_2$ for route 2. In these cases the turning rate changes until the flow equals the capacity flow:

$$\beta^{\text{eq}} = \frac{C_1}{q} \quad \text{if } \beta^{\text{eq}} \geq \frac{C_1}{q} \;\&\; q > C_1 \;\&\; q \leq C_1 + C_2$$

when a queue is formed on route 1 and

$$\beta^{\text{eq}} = \frac{q - C_2}{q} \quad \text{if } \beta^0 \leq 1 - \frac{C_2}{q} \;\&\; q \geq C_2 \;\&\; q \leq C_1 + C_2$$

when a queue is formed on route 2.

When no queue is formed, so when $\beta^0 q \leq C_1$ and $(1-\beta^0)q \leq C_2$ with $q \leq C_1 + C_2$, the turning rate does not change:

$$\beta^{\text{eq}} = \beta^0 \quad \text{if } \beta^0 \in [1 - \frac{C_2}{q}, \frac{C_1}{q}] \;\&\; q \leq C_1 + C_2 \quad .$$

This means that all values in the interval $[1 - \frac{C_2}{q}, \frac{C_1}{q}]$ can become the equilibrium turning rate when the demand is larger than either C_1 or C_2. When the demand is lower than both capacities, $q < C_1$ and $q < C_2$, the equilibrium turning rate is equal to the initial turning rate β^0, and can have all values in the interval $[0, 1]$.

When the demand is larger than the total capacity, the route choice depends on the ratio between the capacities of the two routes, since the free-flow travel times are equal and thus the capacities determine the queue lengths:

$$\beta^{\text{eq}} = \frac{C_1}{C_1 + C_2} \quad \text{if } q > C_1 + C_2 \quad .$$

Note that in general the equilibrium turning rate does not depend on the parameter κ. This means that the time constant (or learning rate) does not influence the equilibrium that is reached.

6 Outflow control

Traffic can be controlled in different ways, and the traffic control system can have many different goals, e.g. improve throughput, reduce travel time, decrease the delay, reduce queue lengths, etc. Goals directly related to route choice are for example reducing pollution in specified areas, creating traffic diversions around accidents or maintenance works, or reducing traffic flows in densely populated areas. Our controller focuses on steering the flow on route 1 to a desired flow q_1^{desired}. This q_1^{desired} can for example be the maximum flow that can safely drive through a residential area. Note that in most cases the goal should be to keep the flow below this desired flow. Our controller however, tries to reach this desired flow exactly, which means that the controller tries to increase the flow when it is lower than the desired flow. More advanced controllers can solve this problem but are less intuitive, so we decided to use a basic controller. When the desired flow is selected, it should be lower or equal to the capacity of the route, $q_1^{\text{desired}} \leq C_1$.

The model described earlier allows us to describe two fundamentally different ways of control: outflow control and speed control. In this section we examine the possibilities of outflow control with respect to influencing route choice, and in the next section we investigate speed control. Outflow control means that the flow that is allowed to leave a link (or in our case a route) is limited below the capacity C_r determined by the road layout. Possible traffic control measures that can be used to limit the outflow are ramp metering installations, main stream metering installations, or traffic signals.

We apply a simple linear controller that is only able to influence the outflow limit of the first route, $Q_1(d)$. This outflow limit is allowed to vary between a minimum and maximum value $Q_1^{\min} \leq Q_1(d) \leq Q_1^{\max} = C_1$. The outflow limit for route 2 is kept constant, $Q_2(d) = C_2$. The control law is as follows:

$$Q_1(d+1) = Q_1(d) + P(q_1^{\text{desired}} - \beta(d)q) \tag{7}$$

where $P > 0$ is the proportional gain.

When a controller is applied, the equilibrium traffic assignments described in the previous section can change. We again look at the different combinations of free-flow times and demands, and determine the equilibrium turning rates β^{eq} and the corresponding values for the outflow limit $Q_1(d)$.

6.1 Shortest route is route 1: $TT_1^{\text{free}} < TT_2^{\text{free}}$

When the demand is lower than the desired flow, $q \leq q_1^{\text{desired}}$, the outflow limit $Q_1(d)$ has no influence on the route choice, and so the controller also has no influence. This

means that all the traffic enters route 1, so $\beta^{\text{eq}} = 1$. Since the desired flow is not reached the controller keeps increasing the outflow limit, which ends up at its maximum value:

$$Q_1^{\text{eq}} = Q_1^{\max} = C_1 \quad \text{if} \;\; q \leq q_1^{\text{desired}}$$

When the demand is larger than the desired flow, the capacity is lowered until the desired flow on route 1 is reached. This equilibrium value is reached when $Q_1(d+1) = Q_1(d) = Q_1^{\text{eq}}$. Together with Equation 7 this gives

$$P(q_1^{\text{desired}} - \beta(d)q) = 0 \quad \rightarrow \quad \beta(d)q = q_1^{\text{desired}}$$

which can be substituted in Equation 3:

$$\kappa\Big(- \frac{(q_1^{\text{desired}} - Q_1(d))(T - TT_1^{\text{free}})}{2Q_1(d)} + TT_2^{\text{free}} - TT_1^{\text{free}}\Big) = 0$$

and results in:

$$Q_1^{\text{eq}} = \frac{q_1^{\text{desired}}(T - TT_1^{\text{free}})}{(T - TT_1^{\text{free}}) + 2(TT_2^{\text{free}} - TT_1^{\text{free}})} \quad \text{if} \;\; q > q_1^{\text{desired}} \;\&\; q \leq q_1^{\text{desired}} + C_2 \quad .$$

The resulting flow on route 1 is exactly equal to q_1^{desired} which results the following equilibrium turning rate:

$$\beta^{\text{eq}} = \frac{q_1^{\text{desired}}}{q} \quad \text{if} \;\; q > q_1^{\text{desired}} \;\&\; q \leq q_1^{\text{desired}} + C_2 \quad .$$

When the demand exceeds the sum of the desired flow and C_2, the controller decreases $Q_1(d)$ in such a way that a long queue appears at this route, which results in most drivers taking route 2, and so the desired flow on route 1 can still be attained. This leads to the desired equilibrium turning rate $\beta^{\text{eq}} = q_1^{\text{desired}}/q$, so $q_1^{\text{desired}} = \beta^{\text{eq}} q$, which can be substituted in Equation 5. This gives the corresponding value for $Q_1(d)$:

$$Q_1^{\text{eq}} = \frac{q_1^{\text{desired}}(T - TT_1^{\text{free}})}{\frac{(q - q_1^{\text{desired}})(T - TT_2^{\text{free}})}{C_2} + (T - TT_1^{\text{free}}) - (T - TT_2^{\text{free}}) + 2(TT_2^{\text{free}} - TT_1^{\text{free}})} \quad \text{if} \;\; q > q_1^{\text{desired}} + C_2 \quad . \tag{8}$$

6.2 Shortest route is route 2: $TT_1^{\text{free}} > TT_2^{\text{free}}$

In this case all the drivers want to take route 2, and thus it does not seem very useful to limit the flow on route 1. For some demands this is indeed the case. When the demand is lower than C_2, the controller has no influence at all, leading to $\beta^{\text{eq}} = 1$ if $q \leq C_2$ and to a maximum value of $Q_1(d)$: $Q_1^{\text{eq}} = Q_1^{\max}$ because the controller tries to increase the flow on route 1 by increasing the outflow limit, but it is limited by $Q_1^{\max}$.

When the demand is slightly higher and exceeds C_2 but not the total capacity $q_1^{\text{desired}} + C_2$, the controller has also no influence, and the same turning rate of Equation 6 is reached. The desired flow is still not reached, and the controller increases the outflow limit on route 1 to its maximum value.

A demand that is larger than $q_1^{\text{desired}} + C_2$ leads to a turning rate of $\beta^{\text{eq}} = q_1^{\text{desired}}/q$, with an outflow limit as given in Equation 8.

6.3 The routes are equally long: $TT_1^{\text{free}} = TT_2^{\text{free}}$

If the routes are equally long the drivers will not change their route choice until the flow on a route reaches the outflow limit of this route. The controller can only lower the outflow limit on route 1, resulting in sending more traffic to route 2. The reverse action is not possible because we assume that $Q_2(d) = C_2$ is constant.

The controller is not able to change the turning rate when the traffic flow on route 1 is lower than the desired flow. This happens in three cases:

- The total demand is too low: $q \leq q_1^{\text{desired}}$
- The demand towards route 1 is too low:
  - $(1-\beta^0)q \leq C_2$ & $q > C_2$ & $q \leq q_1^{\text{desired}} + C_2$ thus $\beta^0 > 1 - \frac{C_2}{q}$
  - $\beta^0 q < q_1^{\text{desired}}$ & $q > q_1^{\text{desired}}$ &$q \leq q_1^{\text{desired}} + C_2$ thus $\beta^0 \leq q_1^{\text{desired}}/q$

In these cases, the turning rate does not change, and the controller reaches the maximum value:

$$\beta^{\text{eq}} = \beta^0 \quad \text{and} \quad Q_1^{\text{eq}} = Q_1^{\max}$$

with $\beta^{\text{eq}} \in [0,1]$ if $q < Q_1(d)$ & $q < C_2$, and with $\beta^{\text{eq}} \in [1 - \frac{C_2}{q}, \frac{Q_1(d)}{q}]$ if β^0 lies in this interval.

Problems arise when the total capacity is still sufficient, but when the first turning rate is such that the flow on route 1 is larger than the desired flow. This happens in two cases:

- $\beta^0 \leq 1 - \frac{C_2}{q}$ & $q > C_2$ & $q \leq q_1^{\text{desired}} + C_2$
- $\beta^0 > \frac{q_1^{\text{desired}}}{d}$ & $q > q_1^{\text{desired}}$ & $q \leq q_1^{\text{desired}} + C_2$

In these cases the controller tries to reach the equilibrium turning rate $\beta^{\text{eq}} = q_1^{\text{desired}}/q$. Figure 3 shows the turning rate as a function of the days when outflow control is applied in this situation. The turning rate corresponding to the desired flow is shown in Figure 3 with the dotted line.

When the flow on route 1 is larger than the desired flow, the controller decreases the outflow limit of route 1, until the flow on this route is lower than the desired flow. But a property of the kind of controller used is that it can have some undershoot. This means that, depending on the value of P, the turning rate first becomes lower than the desired turning rate. With a linear system, the turning rate will increase again, and reach the desired value. The dashed line in Figure 3 shows the evolution of the turning rate for a linear system. The number of days is plotted along the horizontal axis and the corresponding turning rate at the vertical axis. Our model however is a piecewise affine model. With our model, the controller can only lower the flow on route 1 and thus cannot correct for the undershoot, and the flow stays a too low. This means that the equilibrium turning rate will also be lower than q_1^{desired}/q. This is plotted with the solid line in Figure 3. The value of the outflow limit, shown in the second graph in Figure 3, first decreases until the undershoot is at its minimum. At this moment the controller

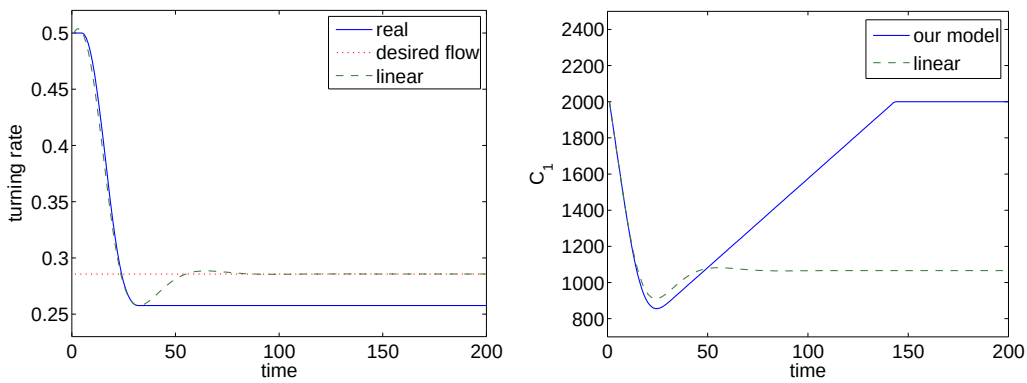

Figure 3: Turning rate and capacity on route 1 as function of days, where 'real' targets the turning rate with our model, 'desired flow' targets the turning rates that result in the desired flow, and 'linear' gives the turning rates when a linear model is used.

will try to increase the flow on route 1 again, and so the outflow limit starts to increase. Since the turning rate does not change as a reaction on this increase, the controller keeps increasing the outflow limit until the maximum value $Q_1^{\max}$ is reached.

For really high demands, the fact that the routes are equally long has no real influence, and the turning rate is given by $\beta^{\text{eq}} = q_1^{\text{desired}}/q$, and the corresponding outflow limit is given by Equation 8.

7 Speed control

The second way to influence traffic that we describe is speed control. When speed control is applied, the speed of the traffic is changed, for example via the use of speed limits displayed on variable message signs along the road.

Speed control changes the free-flow travel time for the routes, and in this way it changes the route choice. We again consider the different combinations of free-flow travel times and demands. For categorizing we use the real free-flow time TT_1^{free}, but due to the control method the actually experienced free-flow travel time $TT_1^{\text{free,control}}(d)$, which is required to compute the equilibrium turning rates, changes:

$$TT_1^{\text{free,control}}(d) = \frac{l_1}{v_1(d)}$$

where $TT_1^{\text{free,control}}(d) \geq TT_1^{\text{free}}$ because $v_1 \leq v_1^{\max}$.

Again we use a very simple control method:

$$v_1(d+1) = v_1(d) + K(q_1^{\text{desired}} - \beta(d)q)$$

where K is the gain of the controller. The goal of the controller is the same as with the outflow control: reaching a desired flow at route 1.

7.1 Shortest route is route 1: $TT_1^{\text{free}} < TT_2^{\text{free}}$

Since route 1 is the shortest, all the traffic wants to take route 1. When the traffic demand is lower than the desired flow, this leads to a turning rate of 1. In this case, the

desired flow stays higher than the realized flow, $q_1^{\text{desired}} > \beta(d)q$, so the speed limit will increase until it reaches its maximum value $v_1^{\max}$.

When the demand is larger than the desired flow, the speed limit will decrease, until the desired flow is reached. In this case, Equation 2 can be simplified to

$$\beta(d{+}1) = \beta(d){+}\kappa\Big(-\frac{(\beta(d)q - C_1)(T - TT_1^{\text{free,control}}(d))}{2C_1}{+}TT_2^{\text{free}}{-}TT_1^{\text{free,control}}(d)\Big) \quad .$$

In order to compute the equilibrium value for $v_1(d)$, v_1^{eq}, we can assume that the desired flow is reached, so $\beta(d)q = q_1^{\text{desired}}$. Substituting this, and $TT_1^{\text{free,control}}(d) = l_1/v_1(d)$ into the equation, results in

$$v_1^{\text{eq}} = \frac{l_1(q_1^{\text{desired}} - 3C_1)}{(q_1^{\text{desired}} - C_1)T - 2C_1TT_2^{\text{free}}} \quad .$$

The corresponding turning rate is $\beta^{\text{eq}} = q_1^{\text{desired}}/q$.

A demand that exceeds the total capacity, $q > q_1^{\text{desired}} + C_2$, leads to queues on both routes. This leads to an equilibrium speed of

$$v_1^{\text{eq}} = \frac{l_1(q_1^{\text{desired}} - 3C_1)}{(q_1^{\text{desired}} - C_1)T - 2C_1TT_2^{\text{free}} - \frac{C_1}{C_2}((q - q_1^{\text{desired}}) - C_2)(T - TT_2^{\text{free}})} \tag{9}$$

with the corresponding turning rate $\beta^{\text{eq}} = q_1^{\text{desired}}/q$.

7.2 Shortest route is route 2: $TT_1^{\text{free}} > TT_2^{\text{free}}$

All the traffic wants to enter route 2, which will lead to a turning rate of $\beta^{\text{eq}} = 0$, as long as the demand is lower than C_2. The speed limit on route 1 will reach its maximum, $v_1^{\text{eq}} = v_1^{\max}$.

When the demand is larger than C_2, but still less than $q_1^{\text{desired}} + C_2$, the controller has no influence. The equilibrium turning rate of Equation 6 is reached, and the speed limit increases to $v_1^{\text{eq}} = v_1^{\max}$.

The largest demand leads to a speed limit according to Equation 9, with the equilibrium turning rate $\beta^{\text{eq}} = q_1^{\text{desired}}/q$.

7.3 The routes are equally long: $TT_1^{\text{free}} = TT_2^{\text{free}}$

The controlled free-flow times of the two routes can be equally long in this case. The controller can only increase the free-flow time of route 1, which will make route 2 the shortest route. The difference with the case where route 2 always is the shortest route ($TT_1^{\text{free}} > TT_2^{\text{free}}$) lies in the fact that the turning rate stops changing when $v_1^{\max}$ is reached.

If the total demand is lower than the desired flow, the controller tries to increase the flow on route 1, by increasing the maximum speed. This can be done until $v_1^{\max}$ is reached. At this moment the turning rate does not change anymore. The actual value of the turning rate thus depends on β^0, v_1^0 and in general also on the rate of change of the turning rate, κ.

When the demand is larger than C_2 but smaller than $q_1^{\text{desired}} + C_2$, the actual turning rate depends on the start value for the turning rate β^0. If β^0 is smaller than the equilibrium turning rate of Equation 6, the flow at route 1 is lower than the desired flow. As a result, the controller will increase the speed limit, until $v_1(d) = v_1^{\max}$. Since the speed limit cannot increase further, the equilibrium turning rate that is reached will be equal to the turning rate computed with Equation 6. If β^0 lies between the turning rate computed with Equation 6, $\beta^{(6)}$, and q_1^{desired}/q:

$$\beta^{(6)} < \beta^0 < \frac{q_1^{\text{desired}}}{q} \quad ,$$

the desired flow is still not reached. The controller keeps increasing the speed, until $v_1^{\max}$. The turning rate also increases, but stops at the moment that $v_1(d) = v_1^{\max}$. When $\beta^0 > q_1^{\text{desired}}/q$, the flow on route 1 is too high. The controller lowers the speed limit to decrease the flow. When the flow comes below the desired flow (due to undershoot, see Section 6.3), the speed limit starts to increase again until it has reached $v_1^{\max}$. The turning rate first decreases. When the minimum flow is reached, the turning rate starts to increase. From the moment that $v_1(d) = v_1^{\max}$ the turning rate stays constant.

With a large demand, $q > q_1^{\text{desired}} + C_2$, the resulting speed limit is given by Equation 9, and $\beta^{\text{eq}} = q_1^{\text{desired}}/q$.

8 Conclusions

In traffic networks origins and destinations are often connected with more than one route, and drivers select a route based on for example travel times on the different routes. These travel times are influenced by the control measures at these routes. In this paper we have investigated how control of these measures can influence the route choice of the drivers.

The main goal of the paper was to get insight in the phenomena that take place when conventional control methods are used to influence route choice. To obtain this insight, we first have selected simple models to describe the travel times on two routes and the route choice behavior of the drivers. We have determined the equilibrium route choices that are reached when no control is applied. These equilibria give reason to believe that: drivers select the shortest route when both routes are free; drivers make a trade-off between waiting in a queue on the shortest route and driving the longer route without a queue when there is a queue at one route; and drivers select a route as a function of the capacities and free-flow travel times when both routes contain a queue.

Next, we have investigated two types of control: outflow control and speed control. In both cases we used basic linear controllers, which can only influence route 1, with as goal to reach a desired flow at the first route. When outflow control was applied (with traffic signals, ramp metering or mainstream metering) the equilibrium turning rate could only be influenced when route 1 was shorter than route 1, and the demand was larger than the desired flow at route 1 or when the demand was larger than the total capacity. In these cases the resulting turning rate gave the desired flow at route 1. A special case was when both routes have the same free-flow travel time. The effect of the controller then depended on the turning rate at the beginning of the simulation. When the turning rate was too low, the controller had no influence.

Speed control influences the free-flow travel time of the routes. This control method also influenced only the cases where route 1 was the shortest, and either the demand was larger than the desired flow or larger than the total capacity. When the routes are equally long, the possible equilibrium turning rates lay in an interval and depended on the start value. As a result, flows that are lower than the desired flow occurred.

The fact that the controllers could only influence the traffic in specific situations did not really limit the use of the controllers. This was because the two situations in which the controllers were effective were the situations where control was most useful, because the large demands that were present in these situations led to the need of improvements obtained by the controllers.

The results obtained for the simple control methods show that the traffic assignment can be influenced by available control methods. When the demand exceeds the total capacity the controllers can have the most influence on the route choice. In most cases the turning rates that can be reached with outflow control are equal to the turning rates with speed control. Only when the routes have the same free-flow travel time some differences occur. But for both control methods an interval of equilibria is possible in this case, depending on the initial value of the turning rate.

The insight obtained with the simple models and controllers can be useful to interpret results of more advanced controllers. These advanced controllers could be able to solve the problems where the basic controllers are ineffective. The ineffectiveness of the basic controllers can clearly be seen. First, the decision that only the parameters of route 1 can be influenced reduces the influence of the controller. Using a control method that can also influence route 2 will be more effective. This will change the whole system in a multi-variable system. Second, the controllers are only looking at the current state of the traffic, and react on it. This means that the undershoot as described in Section 6.3 cannot be prevented. A solution for the problems with the controllers can be the introduction of predictive controllers. With more sophisticated controllers the goals of the control can also be changed. Possible goals are e.g. minimizing the flow on one route, minimizing the total travel time, minimizing the queue length, etc.

In our future work we will investigate the use of more advanced, model-based predictive control methods. We will also look at traffic control using more realistic traffic models, multi class route choice models and more advanced control measures, e.g. information panels, dedicated lanes, in-car devices, etc. Some attention should also be paid to investigating stability regions, and on introducing larger traffic networks with more available routes.

Acknowledgments

Research supported by the NWO-CONNEKT project “Advanced Multi-Agent Control and Information for Integrated Multi-Class Traffic Networks (AMICI)” (014-34-523), the Transport Research Centre Delft program “Towards Reliable Mobility”, and the BSIK project “Next Generation Infrastructures (NGI)”.

References

Barceló, J., J. L. Ferrer (1997) AIMSUN2: Advanced interactive microscopic simulator for urban networks, user's manual, Tech. rep., Universitat Politècnica de Catalunya.

Bellemans, T., B. De Schutter, B. De Moor (2003) Anticipative model predictive control for ramp metering in freeway networks, in: *Proceedings of the 2003 American Control Conference*, Denver, Colorado, pp. 4077–4082.

Ben-Akiva, M., A. D. Palma, I. Kaysi (1991) Dynamic network models and driver information systems, *Transportation Research A*, 25(5), pp. 251–266.

Bliemer, M. (2000) A quasi-variational inequality approach to multi-user class dynamic traffic assignment, in: *Proceedings of the 79th Annual Meeting of the Transportation Research Board*, Washington, D.C, USA.

Bogers, E., F. Viti, S. Hoogendoorn (2005) Joint modeling of ATIS, habit and learning impacts on route choice by laboratory simulator experiments, in: *Proceedings of the 84th Annual Meeting of the Transportation Research Board*, Washington DC, USA.

Carey, M., Y. Ge (2003) Comparing whole-link travel time models, *Transportation Research Part B*, 37, pp. 905–926.

Daganzo, C. (1994) The cell transmission model: A dynamic representation of highway traffic consistent with the hydrodynamic theory, *Transportation Research B*, 28B(4), pp. 269–287.

Daganzo, C., Y. Sheffi (1977) On stochastic models of traffic assignment, *Transportation Science*, 11, pp. 253–274.

Dial, R. (1971) A probabilistic multipath traffic assignment model, *Transportation Research*, 5, pp. 83–111.

Fisk, C. (1980) Some developments in equilibrium traffic assignment, *Transportation Research B*, 14B, pp. 243–255.

Haj-Salem, H., M. Papageorgiou (1995) Ramp metering impact on urban corridor traffic: Field results, *Transportation Research A*, 29(4), pp. 303–319.

Karimi, A., A. Hegyi, B. De Schutter, J. Hellendoorn, F. Middelham (2004) Integration of dynamic route guidance and freeway ramp metering using model predictive control, in: *Proceedings of the 2004 American Control Conference (ACC 2004)*, Boston, MA, USA, pp. 5533–5538.

Lighthill, M. J., G. B. Whitham (1955) On kinematic waves, II. a theory of traffic flow on long crowded roads, *Proceedings of the Royal Society*, 229A.

Mahmassani, H., N. Huynha, K. Srinivasan, M. Kraanc (2003) Tripmaker choice behavior for shopping trips under real-time information: model formulation and results of stated-preference internet-based interactive experiments, *Journal of Retailing and Consumer Services*, 10(6), pp. 311–321.

Messmer, A., M. Papageorgiou (1990) METANET: A macroscopic simulation program for motorway networks, *Traffic Engineering and Control*, 31, pp. 466–470.

Peeta, S., H. Mahmassani (1995) Multiple user classes real-time traffic assignment for online operations: a rolling horizon solution framework, *Transportation Research C*, 3(2), pp. 83–98.

PTV (2003) Vissim – traffic flow simulation, Tech. rep., PTV, Germany, http//:www.ptv.de.

Quadstone (2002) Paramics system overview, Tech. rep., Quadstone limited, Edinburgh, Schotland, www.paramics-online.com.

Taale, H., H. van Zuylen (1999) Traffic control and route choice: Occurrence of instabilities, in: *TRAIL 5th Annual Congress 1999: Five Years Crossroads of Theory and Practice*, vol. 2, TRAIL, pp. 1–19.

Wang, Y., M. Papageorgiou (2002) A predictive feedback routing control strategy for freeway network traffic, in: *Proceedings of the American Control Conference*, Anchorage, Alaska, USA, pp. 3606–3611.

Wardrop, J. (1952) Some theoretical aspects of road traffic research, *Proceedings of the Institute of Civil Engineers, Part II*, 1(2), pp. 325–378.