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A Novel Coordination Strategy for Multi-Agent Control Using Overlapping Subnetworks with Application to Power Systems

R.R. Negenborn, G. Hug-Glanzmann, B. De Schutter, G. Andersson

Abstract Power networks are huge interconnected systems controlled by a large number of different control authorities. As the nature of power networks is evolving from a hierarchically structured system toward a much more decentralized system, the need to adequately control the power flows over the network using distributed control strategies increases. Currently available distributed control methods assume that the various subnetworks that individual control agents, i.e., the control authorities, control are usually touching, in the sense that the border of one subnetwork is at the same time also the border of a neighboring subnetwork. Such touching networks, however, do not necessarily capture the subnetwork that an agent can influence in the best way. To capture in the best way the subnetwork that an agent can influence overlapping subnetworks will usually have to be defined. In this chapter, we propose a strategy for coordinating multiple control agents that control overlapping subnetworks in a network. Simulations are carried out on an adjusted IEEE 57-bus power network in which the controlled entities are Flexible Alternating Current Transmission Systems (FACTS) and the objective is to improve system security.

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1 Introduction

Power networks [15, 24, 16] are one of the corner stones of our modern society. The dynamics of a power network as a whole are the result of the interactions between the millions of individual components. Conventionally, the power in power networks is generated using several large power generators. This power is then transported through the transmission and distribution network to the location where it is consumed, e.g., households and industry. Power flows are then relatively predictable, and the number of control agents is relatively low. Due to the ongoing deregulation in the power generation and distribution sector in the U.S. and Europe, the number of players involved in the generation and distribution of power has increased significantly. The number of source nodes of the power distribution network is increasing even further as also large-scale industrial suppliers and small-scale individual households start to feed electricity into the network [13].

As a consequence, the structure of the power network is changing from a hierarchical top-down structure into a much more decentralized system with many generating sources and distributing agencies. This causes that power flows become less predictable and may actually change their conventional directions. To still guarantee basic requirements and service levels, such as voltage magnitude and frequency levels, bounds on deviations, stability, elimination of transients, etc., and to meet the demands and requirements of the users, new infrastructure in the shape of transmission lines and so-called Flexible Alternating Current Transmission Systems (FACTS) [10] is installed. Transmission lines increase directly the capacity of the network on the one hand. FACTS devices can be used to actively change the way in which power flows over the network on the other hand. FACTS devices can change voltage magnitudes, line impedances, and phase angles, and therefore have the potential to improve the security of the network, to increase the dynamic and transient stability, to increase the quality of supply for sensitive industries, and to enable environmental benefits [10]. Two particular types of FACTS devices that frequently appear in practice and that also will be used later on in this chapter are Static Var Compensators (SVCs) and Thyristor Controlled Series Compensators (TCSCs) [5].

To optimally use and control such devices and to optimally use the existing infrastructure, new control techniques have to be developed and implemented [18]. A major challenge in this context is that the devices in the network, such as the various FACTS devices, are usually owned and operated by different authorities. Despite this, the operators of the various devices have as objective to determine their actions in such a way that the best overall network performance is obtained. Hence, multi-agent control, in which communication and cooperation between various control authorities is explicitly taken into account, has to be employed.

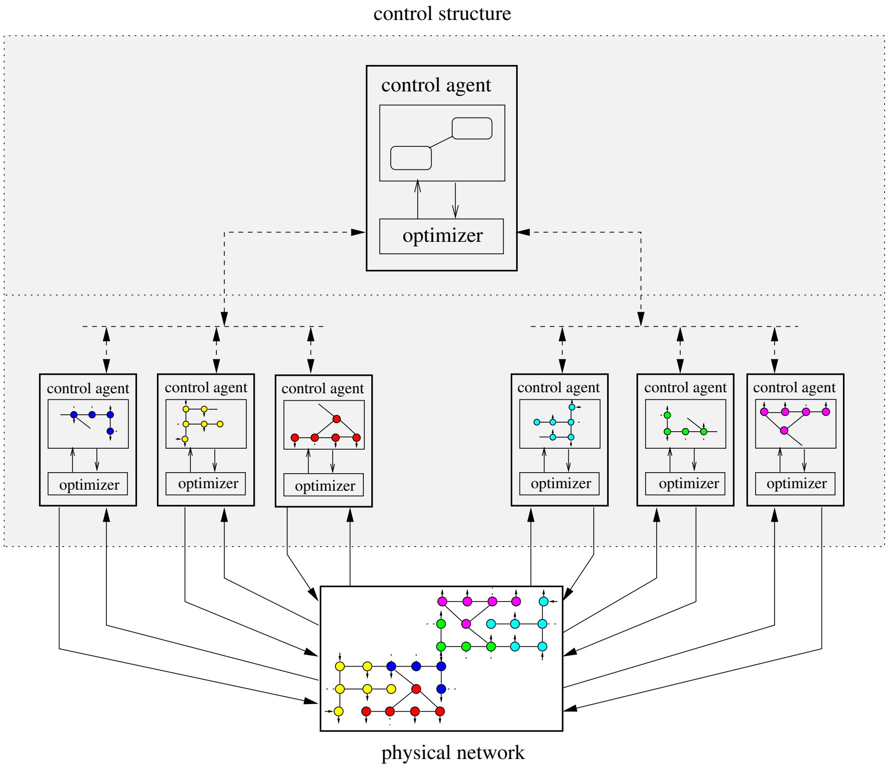


Fig. 1 Illustration of multi-layer control of large-scale networks (inspired by [18]). The control structure consists of several layers of control agents. The control agents make measurements of the state of the network and determine which actions to take.

1.1 Multi-agent control of power networks

The control structure of power networks can be represented as a multi-agent system [27, 25, 26, 18], in which the control agents are organized in several layers as illustrated in Figure 1. A control agent hereby is an entity, e.g., a human, a computer, or a hardware device, that on the one hand observes the state or situation of the network and on the other hand chooses actions to be taken in the network by changing settings of actuators, such as the reference for the power output of generators or the reference for settings of FACTS devices. A control agent has to choose its actions in such a way that the performance of the network in terms of safety, security, and stability, is the best possible, while respecting operational constraints and minimizing costs. High costs hereby indicate a bad performance of the network, whereas low costs indicate a good performance. In the control hierarchy that power networks are controlled by, at the lower layers control agents consider faster dynamics, more local information, smaller subnetworks, and shorter time spans. At the higher layers control agents consider slower dynamics, more global information, larger subnetworks, and longer time spans [20].

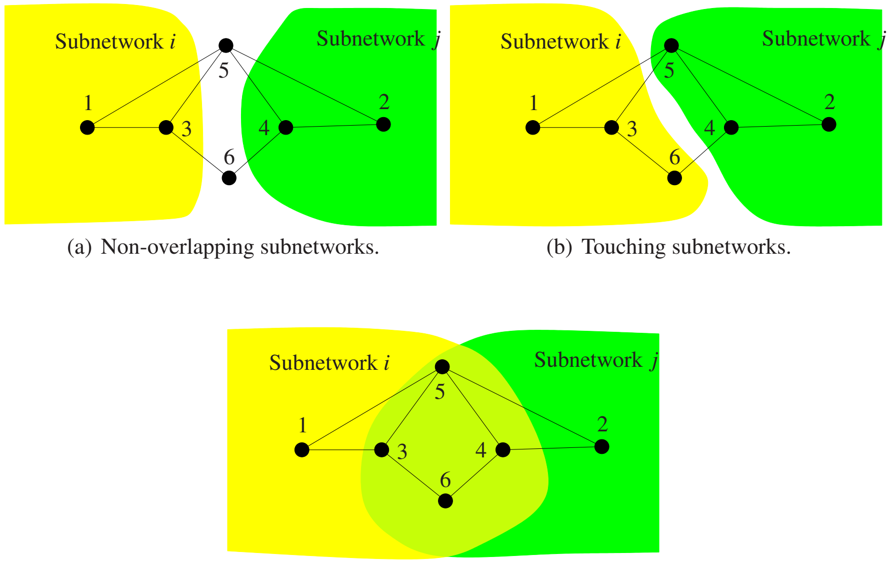
The control problem that an individual control agent in a control hierarchy faces can be cast as an optimization problem, based on a local objective function that encodes the control goals of the agent, subject to a model of the part of the network that the control agent controls, and additional constraints, e.g., on the range of the inputs. The model of the part of the network that the control agent controls is referred to as its prediction model. This prediction model describes how the values of variables of interest (such as voltage magnitudes, power flows, etc.) react to changes in inputs and can therefore be used to predict what the effect of certain input choices is going to be.

1.2 Control of subnetworks

In a multi-agent system, control is distributed over several control agents. Each of the control agents controls only its own part of the network, i.e., its own subnetwork. Let for now a network be modeled at an abstract level using a number of nodes with arcs interconnecting the nodes. The nodes represent characteristics of the components of the physical network, whereas the arcs model the direct interaction between the nodes. E.g., one node κ could model the characteristics of a power generator together with a bus and a transmission line, and another node ω could model the characteristics of a load and a bus. If the bus of this load is physically connected to the transmission line, then an arc is defined between the nodes κ and ω. The subnetwork of a control agent then constitutes a number of nodes together with the arcs connected to these nodes[1].

Usually subnetworks are defined through geographical or institutional borders, such as borders of cities, provinces, countries, the European Union, etc. Subnetworks can however also be defined differently, e.g., based on a fixed "radius" around input nodes. Nodes that are reachable within a certain number of arcs from a particular node with an actuator are then included in a particular subnetwork [9]. Or, subnetworks can be defined using an influence-based approach [8]. The idea of influence-based subnetworks is that the subnetworks are defined based on the nodes that a certain input and, hence, a control agent controlling that input, can influence. Sensitivities are then used to determine which variables an input can influence, and hence, which nodes should be considered part of a subnetwork. The fixed-radius and the influence-based approaches have as advantage that the subnetworks are defined taking a more actuator-centered perspective. Using the fixed-radius approach, this definition is somewhat ad hoc and heuristic. On the contrary, the influence-based approach is more flexible and allows for a structured determination of the subnetwork that a control agent has to consider.

[1] In Section 2 we define networks, nodes, arcs, and subnetworks more formally.



(a) Non-overlapping subnetworks. (b) Touching subnetworks.

(c) Overlapping subnetworks.

Fig. 2 Illustration of different types of subnetworks.

When using the mentioned approaches for defining subnetworks, any pair of two resulting subnetworks can be categorized as *non-overlapping*, *touching*, or *overlapping*, as illustrated in Figure 2. If for two subnetworks, the nodes belonging to one of these do not coincide with the nodes belonging to the other subnetwork, and if there are no arcs going from nodes in the one subnetwork into nodes of the other subnetwork, then the subnetworks are non-overlapping. If for two subnetworks, the nodes belonging to one of these do not coincide with the nodes of the other subnetwork, but if there are arcs between nodes of the one subnetwork and nodes of the other subnetwork, then the subnetworks are touching. If for two subnetworks, the nodes belonging to one of these partially coincide with the nodes belonging to the other subnetwork, then the subnetworks are overlapping. In that case, a *common* sub-subnetwork is defined consisting of those nodes and arcs that belong to both subnetworks.

If the subnetworks are non-overlapping, then the values of the variables of the nodes that control agents can influence significantly do not overlap, so no coordination among control agents is necessary. In that case, adequate control performance can be obtained, as illustrated in [8]. If the subnetworks are touching, coordination can be obtained by adapting the technique of [3], as will be discussed in Section 3. For subnetworks that are overlapping, no techniques have been proposed so far for obtaining coordination. For overlapping subnetworks, the control agents will have to find agreement on the values of variables involved in the characteristics of the common sub-subnetworks. This topic is addressed in this chapter.

1.3 Optimal power flow control

Optimal power flow control is a well known-method to optimize the operation of a power network at higher control layers [15]. Optimal power flow control is typically used to improve steady-state network security by improving the voltage profile, preventing lines from overloading, and minimizing active power losses. The optimal power flow control problem is usually stated as an optimization problem in which variables to be optimized consist of inputs or settings for generators, the objective function encodes the control goals (such as maintaining voltage magnitudes within desired bounds, preventing transmission lines from overloading, minimizing power losses, etc.), and the prediction model consists of the steady-state characteristics of the network.

To optimally make use of the FACTS devices installed in the power network we employ optimal power flow control to determine the settings for these devices. As mentioned, the devices in the network can be owned and controlled by different authorities. Traditional approaches for optimal power flow control in power networks using multiple control agents assume that control agents consider at most touching, and thus not overlapping, subnetworks [22, 14]. In these cases subnetworks are typically defined based on existing geographical borders of countries, states, provinces, cities, etc. However, when the subnetworks are overlapping, the traditional approaches may not be suitable. Therefore, a new coordination approach for control of overlapping subnetworks has to be developed. We have already made a first step in this with the proposal of the approach described in [12], of which the approach proposed in this chapter is a further elaboration and generalization.

1.4 Goal and outline of this chapter

In this chapter we propose a coordination scheme for control agents controlling overlapping subnetworks with the aim of obtaining the best overall network performance. This chapter is organized as follows. In Section 2, we formalize the modeling of networks, subnetworks, and control objectives used in this chapter. In Section 3, we first discuss a recently proposed approach that can be used for the multi-agent control of subnetworks that are *not* overlapping (i.e., non-overlapping or touching). We then propose an extension of this approach to multi-agent control of subnetworks that are overlapping in Section 4. In Section 5, we apply the proposed approach to an optimal power flow control problem from the domain of power networks. In particular, we employ the approach to control FACTS devices in an adjusted IEEE 57-bus power network, in which each FACTS is controlled by a different control agent. Section 7 contains conclusions and directions for future research.

2 Modeling of network characteristics and control objectives

In this section we formalize the way in which we describe the network characteristics, subnetworks, and control objectives in this paper. An example of the application of this formalization is given in Section 5.

2.1 Network characteristics

We consider the control of power networks by multiple control agents that operate in a higher control layer. At this layer, we are interested in controlling the very slow dynamics or the long-term behavior of the network, and therefore we can assume that dynamics of the lower control layers and physical network can be represented or approximated by instantaneous, steady-state characteristics.

Let a network be represented by a network model. Let the model consist of ν nodes, and let κ, for $\kappa \in \{1,\ldots,\nu\}$ denote a particular node. Each of the nodes in the network model is labeled with a set of variables (e.g., voltage magnitudes and angles) and constraints (e.g., power flow equations) used to compute the steady-state values for these variables, given values for inputs (e.g., amount of power to be generated) and disturbances (e.g., amount of power consumed). The constraints of a particular node κ involve variables of that particular node and variables of other nodes, referred to as the neighboring nodes $\mathscr{N}^{\kappa} = \{\omega_{\kappa,1},\ldots,\omega_{\kappa,n_\kappa}\}$. To indicate the interaction between node κ and its neighboring nodes in $\mathscr{N}^{\kappa}$, we define an arc between κ and each node $\omega \in \mathscr{N}^{\kappa}$.

Let for node $\kappa \in \{1,\ldots,\nu\}$, the variables $\mathbf{z}^{\kappa} \in \mathbb{R}^{n_{\mathbf{z}^{\kappa}}}$, $\mathbf{u}^{\kappa} \in \mathbb{R}^{n_{\mathbf{u}^{\kappa}}}$, and $\mathbf{d}^{\kappa} \in \mathbb{R}^{n_{\mathbf{d}^{\kappa}}}$, denote the (static) states[2], the input variables, and the disturbance variables associated with node κ, respectively, and let the constraints of node κ be given by

$$\mathbf{g}^{\kappa}(\mathbf{z}^{\kappa},\mathbf{u}^{\kappa},\mathbf{d}^{\kappa},\mathbf{z}^{\omega_{\kappa,1}},\ldots,\mathbf{z}^{\omega_{\kappa,n_\kappa}}) = 0, \tag{1}$$

where $\mathbf{z}^{\omega}$ are the variables of neighboring node $\omega \in \mathscr{N}^{\kappa}$, and $\mathbf{g}^{\kappa}$ are the constraint functions of node κ. These constraint function are assumed to be smooth. A steady-state model for the overall network is obtained by aggregating the constraints (1) for all nodes $\kappa \in \{1,\ldots,\nu\}$, and is compactly represented as

$$\mathbf{g}(\mathbf{z},\mathbf{u},\mathbf{d}) = 0, \tag{2}$$

where $\mathbf{z}$, $\mathbf{u}$, and $\mathbf{d}$ are the state, input, and disturbance variables of the overall network, and $\mathbf{g}$ defines the steady-state characteristics of the network. Given the inputs $\mathbf{u}$ and the disturbance variables $\mathbf{d}$, the steady state in which the network settles is determined by solving the system of equations (2).

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[2] Sometimes the static states are also referred to as algebraic variables.

2.2 Control objectives

With each node objective terms can be associated. These objective terms specify which behavior is desired by assigning costs to the values of the variables $\mathbf{z}^{\kappa}$ and $\mathbf{u}^{\kappa}$ of that node. The objective terms involve the variables of node κ and may in addition also involve the variables of the neighboring nodes $\omega \in \mathcal{N}^{\kappa}$. The summation of the objectives terms of all nodes in the network model gives the objective for the control of the overall network. E.g., if a node has assigned to it constraints representing the characteristics of a transmission line, then as objective term the costs on power losses of that transmission line may be associated to the node. In addition, if a node represents the characteristics of a bus, then an objective term representing costs on a voltage magnitude violation of that bus may be associated to this node.

2.3 Definition of subnetworks

The values of the inputs $\mathbf{u}$ should be adjusted in such a way that the objectives associated with the nodes are achieved as well as possible. Let for a control agent i the nodes that it controls define its subnetwork. The prediction model that control agent i then uses consists of the union of the constraints of each node that is part of its subnetwork. Let the subnetwork and the control goals of a control agent be defined using one of the approaches mentioned in Section 1.2.

In the following we first discuss an approach that can be used in the case that the subnetworks are touching. Then we extend this approach to be able to deal with overlapping subnetworks. For the sake of simplicity we assume below that there are no nodes that do not belong to any subnetwork.

3 Multi-agent control of touching subnetworks

In this section we discuss a technique for coordinating control agents that use touching subnetworks. This technique is based on an adaptation of the ideas of the modified Lagrange technique proposed in [3] to our network and objective formalization. The technique requires that subnetworks of any two control agents are touching, i.e., the nodes in the subnetwork of one control agent are only taken into account (i.e., modeled and controlled) by that control agent and not by any other. In short, when the control agents have to determine actions, they perform a series of iterations, in each of which the control agents perform a local optimization step and communicate information. The local optimization problems are formulated using local objective functions, local prediction models of the subnetworks, and local constraints. After each local optimization the control agents exchange information, reformulate their local optimization, and perform a new optimization. This continues until a stopping condition is satisfied. Below we first introduce some terminology, then formulate the

Table 1 Overview of the localized constraint types of constraints associated with nodes in a subnetwork that touches other subnetworks. The location indicates the location of the node from the point of view of control agent *i*. The variables involved in the constraint indicate which variables are involved in the constraint, from the point of view of control agent *i*.

type	location	variables involved in constraint
$\mathscr{C}_{i,\text{int}}^{\text{int}}$	internal	internal
$\mathscr{C}_{i,\text{int}}^{\text{int+ext}}$	internal	internal+external
$\mathscr{C}_{i,\text{ext}}^{\text{ext}}$	external	external
$\mathscr{C}_{i,\text{ext}}^{\text{int+ext}}$	external	internal+external

control problem as considered by an individual control agent, and then we discuss the scheme used by multiple control agents for coordination and communication.

3.1 Internal and external nodes

We define the following concepts that will be frequently used in the remainder of this paper:

- We categorize the nodes that control agent i considers based on their location from the point of view of control agent i. For touching subnetworks, the nodes that control agent i considers can be *internal* nodes or *external* nodes. The internal nodes of control agent i are those nodes that belong exclusively to its subnetwork. The external nodes of control agent i are those nodes that do not belong to its subnetwork.
- Based on the distinction between internal and external nodes of control agent i, we make a distinction between internal and external variables of control agent i. The internal variables are those variables associated with the internal nodes of control agent i. The external variables are those variables associated with the external nodes of control agent i.
- For control agent i, the *localized constraint type* of a particular constraint associated with a node κ that control agent i considers is formed by the combination of the location and the types of variables involved in that constraint. The localized constraint type of a constraint associated with a node κ considered by control agent i is denoted by $\mathscr{C}_{i,\text{Loc}}^{\text{Vars}}$, where $\text{Loc} \in \{\text{int}, \text{ext}\}$ indicates the location of the node to which the constraint is associated, and $\text{Vars} \in \{\text{int}, \text{int+ext}\}$ indicates the variables involved in the constraint. Recall that a constraint associated with a particular node κ involves variables of that particular node and possibly variables of neighboring nodes. The constraints associated with the nodes considered by control agent i can therefore have the localized constraint types listed in Table 1. Figure 3 illustrates for some nodes the localized constraint types that can be found at these nodes.

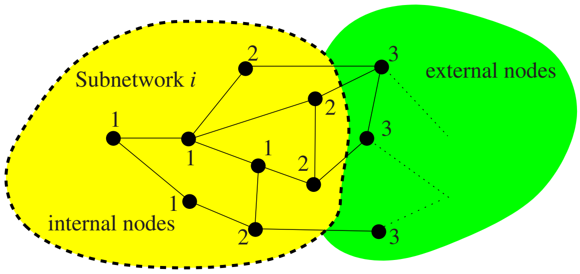


Fig. 3 Illustration of different localized constraint types that can be found at nodes considered by control agent i. The number next to a node in the figure corresponds as follows to the localized constraint types of the constraints that can be associated to that node: 1: $\mathscr{C}_{i,\mathrm{int}}^{\mathrm{int}}$; 2: $\mathscr{C}_{i,\mathrm{int}}^{\mathrm{int}}, \mathscr{C}_{i,\mathrm{int}}^{\mathrm{int+ext}}$; 3: $\mathscr{C}_{i,\mathrm{ext}}^{\mathrm{int+ext}}, \mathscr{C}_{i,\mathrm{ext}}^{\mathrm{ext}}$.

- In a similar way as we have defined localized constraint types $\mathscr{C}_{i,\mathrm{Loc}}^{\mathrm{Vars}}$, we also define localized objective term types $\mathscr{J}_{i,\mathrm{Loc}}^{\mathrm{Vars}}$, referring to the location of the node to which an objective term is associated and the variables that are involved in the objective function term.

3.2 *Control problem formulation for one agent*

The local optimization problem of control agent i consists of minimizing the local objective function J_i, subject to the prediction model of subnetwork i and additional constraints on inputs and outputs. Below we focus on the issues arising due to the presence of subnetworks that touch the subnetwork of control agent i. We discuss the issues arising with respect to the prediction model and the objective function of control agent i. For the sake of simplicity of explanation we consider two control agents, control agent i with neighboring agent j, that together control subnetworks that cover all nodes of the network model. The generalization to more than 2 control agents and not fully-covered networks is straightforward.

3.2.1 Prediction model

The prediction model of control agent i consists of the constraints associated with all its internal nodes. In order to make predictions, control agent i has to know accurate values for all variables involved in the constraints of these nodes. The internal nodes that do not have external neighboring nodes do not require special attention, since the variables involved in the constraints of these internal nodes are of localized constraint type $\mathscr{C}_{i,\mathrm{int}}^{\mathrm{int}}$ and thus only involve variables of the subnetwork of control agent i. However, the internal nodes that are connected to external nodes do require special attention, since the constraints associated with these internal nodes can be of localized constraint type $\mathscr{C}_{i,\text{int}}^{\text{int+ext}}$, and thus involve not only variables of the subnetwork of control agent i, but also variables of the subnetwork of neighboring agent j. For the external variables, control agent i has to coordinate with the neighboring agents which values these variables should have. To obtain coordination on the values of the external variables, we apply an idea that was first proposed in [3] as follows.

Below a distinction is made between constraints that are considered as *hard*, and constraints that are considered as *soft*. The hard constraints are constraints that have to be satisfied at all costs. The soft constraints are constraints for which it is desirable that they are satisfied, but for which this should not be done at any price. The hard constraints are included in the formulation of optimization problems as explicit equality constraints; the soft constraints are included in the objective function of optimization problems through a penalty term, weighted by a parameter specifying the costs for violation of the soft constraint.

Recall that the control agents perform a series of iterations and that in each iteration the control agents solve a local optimization problem followed by an exchange of information. Note that internal and external nodes of control agent i correspond to external and internal nodes, respectively, of control agent j. Control agent i considers in its local optimization problem the constraints that are associated with its internal nodes and that are of localized constraint type $\mathscr{C}_{i,\text{int}}^{\text{int+ext}}$ as hard constraints, using fixed values for the external variables. The values for these external variables are obtained from the neighboring agent j. Control agent i solves its local optimization problem using these values for the external variables. The optimization yields values for the internal variables of control agent i, and for the Lagrange multipliers that are associated with the constraints of localized constraint type $\mathscr{C}_{i,\text{int}}^{\text{int+ext}}$. The Lagrange multipliers of these constraints and the values of the internal variables involved in these constraints are sent to neighboring agent j.

Neighboring agent j considers the constraints of the internal nodes of control agent i that involve external variables of control agent i in its decision making by including the associated constraints as soft constraints in its local objective function. In the soft constraints of control agent j, the external variables, which correspond to internal variables of control agent i, are fixed to the values that control agent i has sent to control agent j. Also, the soft constraints are weighted by the Lagrange multipliers as given by control agent i. Neighboring agent j solves its optimization problem, yielding values for its internal variables. It sends the values of the internal variables that appear in the soft constraints to control agent i, such that control agent i can update its information about the corresponding external variables.

Based on this idea, Table 2 shows how control agent i deals with the different constraints when formulating its optimization problem.

3.2.2 Objectives

The local objective function for control agent i consists of objective function terms that are associated with the nodes in its subnetwork. Objective terms associated with internal nodes that are only connected to internal nodes are simply included

Table 2 Overview of the constraints that control agent i can have and how it deals with these constraints. For the hard and soft constraints, the external variables are fixed to values obtained from neighboring agents. For the hard constraints with external variables Lagrange multipliers are determined. The soft constraints are weighted using the Lagrange multipliers received from neighboring agents.

localized constraint type	constraint
$\mathscr{C}_{i,\text{int}}^{\text{int}}$	hard
$\mathscr{C}_{i,\text{int}}^{\text{int+ext}}$	hard
$\mathscr{C}_{i,\text{ext}}^{\text{int+ext}}$	soft

Table 3 Overview of the localized objective term types that control agent i considers and how it deals with these terms. External variables are fixed to values obtained from neighboring agents.

localized objective term type	how deal with the objective term
$\mathscr{J}_{i,\text{int}}^{\text{int}}$	include as is
$\mathscr{J}_{i,\text{int}}^{\text{int+ext}}$	include as is

in the local objective function. However, objective terms associated with internal nodes that are also connected to external nodes cause problems for the same reason as constraints associated with such nodes. Coordination on the values of these variables is achieved by obtaining the desired values for the external variables from neighboring agents.

Table 3 summarizes how the different localized objective term types that control agent i are considered, and how the agent deals with these types, when formulating its optimization problem.

3.3 Control scheme for multiple agents

The outline of the scheme for coordination of control agents controlling touching subnetworks, based on the scheme proposed in [3], is as follows:

1. Each control agent i measures the current values for the state variables $\mathbf{z}_i$ and the input variables $\mathbf{u}_i$ that are associated with the nodes in its subnetwork. In addition, it obtains predictions of known disturbance variables $\mathbf{d}_i$. Furthermore, it obtains through communication from its neighbors values for the external variables and Lagrange multipliers associated with the external nodes that control agent i considers.
2. The iteration counter s is set to 1.
3. Let $\mathbf{w}_{\text{in},i}^{(s-1)}$ and $\lambda_{\text{soft},i}^{(s-1)}$ denote the external variables and Lagrange multipliers, respectively, of which control agent i has received the values from neighboring agents. Given $\mathbf{w}_{\text{in},i}^{(s-1)}$ and $\lambda_{\text{soft},i}^{(s-1)}$, each control agent $i \in \{1,\ldots,n\}$ performs concurrently with the other control agents the following steps:

a. Control agent i solves the local optimization problem:

$$\min_{\mathbf{z}_i,\mathbf{u}_i} J_i\left(\mathbf{z}_i,\mathbf{u}_i,\mathbf{w}_{\text{in},i}^{(s-1)}\right)+\left(\lambda_{\text{soft},i}^{(s-1)}\right)^{\mathrm{T}}\tilde{\mathbf{g}}_{\text{soft},i}\left(\mathbf{z}_i,\mathbf{u}_i,\mathbf{w}_{\text{in},i}^{(s-1)}\right) \tag{3}$$

subject to

$$\tilde{\mathbf{g}}_{\text{hard},i}\left(\mathbf{z}_i,\mathbf{u}_i,\mathbf{d}_i\right)=0 \tag{4}$$

$$\tilde{\mathbf{g}}_{\text{hard,ext},i}\left(\mathbf{z}_i,\mathbf{u}_i,\mathbf{d}_i,\mathbf{w}_{\text{in},i}^{(s-1)}\right)=0 \tag{5}$$

$$\mathbf{z}_{i,\min}\leq\mathbf{z}_i\leq\mathbf{z}_{i,\max} \tag{6}$$

$$\mathbf{u}_{i,\min}\leq\mathbf{u}_i\leq\mathbf{u}_{i,\max}, \tag{7}$$

where $\mathbf{z}_{i,\min}$ and $\mathbf{z}_{i,\max}$ are upper and lower bounds on $\mathbf{z}_i$, $\mathbf{u}_{i,\min}$ and $\mathbf{u}_{i,\max}$ are upper and lower bounds on $\mathbf{u}_i$, $\tilde{\mathbf{g}}_{\text{soft},i}$ are the constraints of localized constraint type $\mathscr{C}_{i,\text{ext}}^{\text{int+ext}}$, $\tilde{\mathbf{g}}_{\text{hard},i}$ are the constraints of localized constraint type $\mathscr{C}_{i,\text{int}}^{\text{int}}$, $\tilde{\mathbf{g}}_{\text{hard,ext},i}$ are the constraints of localized constraint type $\mathscr{C}_{i,\text{int}}^{\text{int+ext}}$. Solving this local optimization results in values for the variables $\mathbf{z}_i^{(s)}$ and $\mathbf{u}_i^{(s)}$, as well as Lagrange multipliers $\lambda_{\text{hard,ext},i}^{(s)}$ associated with the constraints (5) for current iteration s. After solving this optimization problem the variables $\mathbf{w}_{\text{out},i}^{(s)}$ can be determined as:

$$\mathbf{w}_{\text{out},i}^{(s)}=\tilde{\mathbf{K}}_i\left[\left(\mathbf{z}_i^{(s)}\right)^{\mathrm{T}}\ \left(\mathbf{u}_i^{(s)}\right)^{\mathrm{T}}\ \left(\mathbf{d}_i\right)^{\mathrm{T}}\right]^{\mathrm{T}}, \tag{8}$$

where $\mathbf{w}_{\text{out},i}$ are the so-called interconnecting output variables, selected using a selection matrix $\tilde{\mathbf{K}}_i$. These variables represent the variables that control agent i uses in its communication to neighboring agents. Selection matrix $\tilde{\mathbf{K}}_i$ has in each row only zeros, except for a single 1 in the column corresponding to the position of an element of $\left[\left(\mathbf{z}_i^{(s)}\right)^{\mathrm{T}},\left(\mathbf{u}_i^{(s)}\right)^{\mathrm{T}},\left(\mathbf{d}_i\right)^{\mathrm{T}}\right]^{\mathrm{T}}$ that is an interconnecting output variable.

b. Control agent i sends the values of the Lagrange multipliers $\lambda_{\text{hard,ext},i}^{(s)}$ of the hard constraints of localized constraint type $\mathscr{C}_{i,\text{int}}^{\text{int+ext}}$ and the values of $\mathbf{w}_{\text{out},i}$ corresponding to internal variables of these nodes to the neighboring agents that consider the involved external variables.

c. Control agent i receives from the neighboring agent j those Lagrange multipliers related to the localized constraint type $\mathscr{C}_{i,\text{ext}}^{\text{int+ext}}$ and those values of the internal variables of the neighboring agents that control agent i requires in order to fix its external variables. Control agent i uses this received information at the next iteration as $\lambda_{\text{soft},i}^{(s)}$ and $\mathbf{w}_{\text{in},i}^{(s)}$.

4. The next iteration is started by incrementing s and going back to step 3, unless a local stopping condition is satisfied for all control agents. The stopping condition is defined as the condition that the absolute changes in the Lagrange multipliers

from iteration $s-1$ to s are smaller than a pre-defined small positive constant $\gamma_{\varepsilon,\mathrm{term}}$.

A shortcoming of this method is that it requires that the subnetworks are touching, since it assumes that each node in the network model is assigned to only one of the subnetworks. However, in the case of control of overlapping subnetworks, some of the nodes are included in more than one subnetwork and the identification of internal and external nodes of a control agent is not straightforward any more. Therefore, the method is not directly applicable to overlapping subnetworks. In the following section we extend the method discussed above to control of overlapping subnetworks.

4 Multi-agent control for overlapping subnetworks

We first propose some new definitions, next we consider the issues appearing due to the overlap, and then we propose a way to deal with these issues. Again, for simplicity of explanation we consider two control agents, control agent i with neighboring control agent j, that together control the subnetworks, which are assumed to cover the full network model.

4.1 Common nodes

In addition to internal and external nodes as defined before, for control of overlapping subnetworks we make the following definitions:

- *Common* nodes are nodes that belong to the subnetwork of control agent i and that also belong to the subnetwork of the control agent j. A sub-subnetwork defined by the nodes common to several subnetworks is referred to as a common sub-subnetwork.
- The variables associated with the common nodes are referred to as the common variables.
- Given the definition of a common node, the number of possibilities for localized constraint types increases. Table 4 lists the localized constraint types that can be considered by a control agent when subnetworks can be overlapping. In total there are 12 different localized constraint types. Figure 4 illustrates some of the possible localized constraint types.
- In addition to the extension of the localized constraint types, the localized objective term types are extended as well, by also defining localized objective term types that are based on variables of common nodes.

Table 4 Overview of the localized constraint types for overlapping subnetworks.

type	location	variables involved in constraint
$\mathscr{C}^{\text{int}}_{i,\text{int}}$	internal	internal
$\mathscr{C}^{\text{int+com}}_{i,\text{int}}$	internal	internal+common
$\mathscr{C}^{\text{int+ext}}_{i,\text{int}}$	internal	internal+external
$\mathscr{C}^{\text{int+com+ext}}_{i,\text{int}}$	internal	internal+common+external
$\mathscr{C}^{\text{int+com}}_{i,\text{com}}$	common	internal+common
$\mathscr{C}^{\text{int+com+ext}}_{i,\text{com}}$	common	internal+common+external
$\mathscr{C}^{\text{com}}_{i,\text{com}}$	common	common
$\mathscr{C}^{\text{com+ext}}_{i,\text{com}}$	common	common+external
$\mathscr{C}^{\text{ext}}_{i,\text{ext}}$	external	external
$\mathscr{C}^{\text{int+ext}}_{i,\text{ext}}$	external	internal+external
$\mathscr{C}^{\text{com+ext}}_{i,\text{ext}}$	external	common+external
$\mathscr{C}^{\text{int+com+ext}}_{i,\text{ext}}$	external	internal+common+external

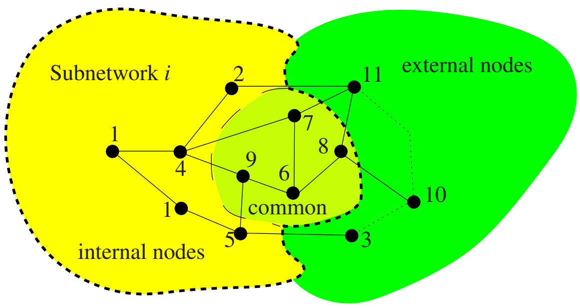


Fig. 4 Illustration of different localized constraint types that can be found at particular nodes considered by control agent i. The number next to a node in the figure corresponds as follows to the localized constraint types of the constraints that can be associated to that node: 1: $\mathscr{C}^{\text{int}}_{i,\text{int}}$; 2: $\mathscr{C}^{\text{int}}_{i,\text{int}}$, $\mathscr{C}^{\text{int+ext}}_{i,\text{int}}$; 3: $\mathscr{C}^{\text{int+ext}}_{i,\text{ext}}$, $\mathscr{C}^{\text{ext}}_{i,\text{ext}}$; 4: $\mathscr{C}^{\text{int}}_{i,\text{int}}$, $\mathscr{C}^{\text{int+com}}_{i,\text{int}}$; 5: $\mathscr{C}^{\text{int}}_{i,\text{int}}$, $\mathscr{C}^{\text{int+com}}_{i,\text{int}}$, $\mathscr{C}^{\text{int+ext}}_{i,\text{int}}$, $\mathscr{C}^{\text{int+com+ext}}_{i,\text{int}}$; 6: $\mathscr{C}^{\text{com}}_{i,\text{com}}$; 7: $\mathscr{C}^{\text{int+com}}_{i,\text{com}}$, $\mathscr{C}^{\text{com+ext}}_{i,\text{com}}$, $\mathscr{C}^{\text{com}}_{i,\text{com}}$, $\mathscr{C}^{\text{int+com+ext}}_{i,\text{com}}$; 8: $\mathscr{C}^{\text{com}}_{i,\text{com}}$, $\mathscr{C}^{\text{com+ext}}_{i,\text{com}}$; 9: $\mathscr{C}^{\text{com}}_{i,\text{com}}$, $\mathscr{C}^{\text{int+com}}_{i,\text{com}}$; 10: $\mathscr{C}^{\text{ext}}_{i,\text{ext}}$, $\mathscr{C}^{\text{ext+com}}_{i,\text{ext}}$; 11: $\mathscr{C}^{\text{int+ext}}_{i,\text{ext}}$, $\mathscr{C}^{\text{com+ext}}_{i,\text{ext}}$, $\mathscr{C}^{\text{ext}}_{i,\text{ext}}$, $\mathscr{C}^{\text{int+com+ext}}_{i,\text{ext}}$.

4.2 Control problem formulation for one agent

For multi-agent control of overlapping subnetworks an approach has to be found to deal with the common nodes. Since the common nodes are considered by several control agents, the constraints associated with these common nodes appear in the subnetwork models of multiple control agents. Even though the control agents have the same objective with respect to these nodes, combined with the objective for their internal nodes, conflicting values for the variables of the common nodes can be the result. Below we discuss how to extend the scheme of the previous section for control of overlapping subnetworks. Again, for the sake of simplicity of explanation we focus on two control agents: control agent i with neighboring agent j.

4.2.1 Prediction model

Similarly as for control of touching subnetworks, for control of overlapping subnetworks, internal nodes of control agent *i* that are connected to external nodes require special attention, since the constraints associated to these nodes may involve external variables. In addition to this, common nodes of control agent *i* that are connected to external nodes also require special attention. The extension of the approach for control of touching subnetworks to the control of overlapping subnetworks involves the following extension of the prediction model.

Control agent i considers as prediction model the constraints of all internal *and* common nodes. For the constraints of localized constraint types $\mathscr{C}_{i,\text{int}}^{\text{int+ext}}$, $\mathscr{C}_{i,\text{int}}^{\text{int+ext+com}}$, $\mathscr{C}_{i,\text{com}}^{\text{com+ext}}$, and $\mathscr{C}_{i,\text{com}}^{\text{int+com+ext}}$ the control agent takes for the external variables values that it has received from neighboring agent j. When control agent i has solved its optimization problem, it sends the values of the internal *and* the common variables of the constraints of these specialized constraint types to neighboring agents.

Neighboring agent j considers in its optimization problem the constraints of the internal and common nodes of control agent i that involve external variables of control agent i as soft constraints by including them in the objective function through a penalty term, weighted by the Lagrange multipliers provided by control agent i, and with fixed values for the external *and* common values in the soft constraints as received from control agent i. Note that although control agent j considers fixed values for the common variable in the soft constraints, it will not fix the values for the common variables in the hard constraints (similarly as control agent i). Hence, control agents i and j share the responsibility for the common variables. The result of solving the optimization problem of neighboring agent j therefore yields values for the internal, common, and external variables of control agent j. The internal variables of control agent j related to the soft constraints are sent to control agent i.

Table 5 summarizes how control agent i deals with the different localized constraint types.

4.2.2 Objectives

With the nodes that control agent i has in its subnetwork objective terms are associated. The objective function terms associated with each node can depend on the variables associated with that node and its neighboring nodes. As before, the objective terms involving only internal variables require no special attention. The objective terms involving both internal and external variables can be dealt with by fixing the external variables, as is also done for control of touching subnetworks. However, the common variables appearing in control of overlapping subnetworks do require special attention.

For control of overlapping subnetworks, multiple control agents will try to control the values of the common variables. To allow control agents to jointly achieve performance comparable to the performance that an overall centralized control agent

Table 5 Overview of the way in which control agent i considers the constraints of particular localized constraint types in its optimization problem. For the hard constraints all external variables are fixed to values obtained from neighboring agents. For the soft constraints all external and common variables are fixed. For the hard constraints with external variables Lagrange multipliers are determined. The soft constraints are weighted with Lagrange multipliers obtained from neighboring agents. Note that the soft constraint part of the inclusion of constraints of type $\mathscr{C}_{i,\text{com}}^{\text{int+com+ext}}$ involves fixed external and common variables and a Lagrange multiplier as obtained from neighboring agents, whereas the hard constraint part of the inclusion of constraints of type $\mathscr{C}_{i,\text{com}}^{\text{int+com+ext}}$ involves only fixed external variables.

localized constraint type	constraint
$\mathscr{C}_{i,\text{int}}^{\text{int}}$	hard
$\mathscr{C}_{i,\text{int}}^{\text{int+ext}}$, $\mathscr{C}_{i,\text{int}}^{\text{int+com}}$	hard
$\mathscr{C}_{i,\text{int}}^{\text{int+com+ext}}$	hard
$\mathscr{C}_{i,\text{com}}^{\text{int+com}}$	hard and soft
$\mathscr{C}_{i,\text{com}}^{\text{int+com+ext}}$	hard and soft
$\mathscr{C}_{i,\text{com}}^{\text{com}}$	hard
$\mathscr{C}_{i,\text{com}}^{\text{com+ext}}$	hard
$\mathscr{C}_{i,\text{ext}}^{\text{int+ext}}$	soft
$\mathscr{C}_{i,\text{ext}}^{\text{int+ext+com}}$	soft

Table 6 Overview of the localized objective term types that control agent i considers and how it deals with the associated objective terms. External variables are fixed. Variable N_κ is the number of control agents considering node κ as common node.

localized objective term type	how deal with the objective term
$\mathscr{J}_{i,\text{int}}^{\text{int}}$	include as is
$\mathscr{J}_{i,\text{int}}^{\text{int+ext}}$	include as is
$\mathscr{J}_{i,\text{int}}^{\text{int+com}}$	include as is
$\mathscr{J}_{i,\text{com}}^{\text{com}}$	include partially by weighting it with a factor $1/N_\kappa$
$\mathscr{J}_{i,\text{com}}^{\text{int+com}}$	include as is

can achieve, the responsibility for the objective terms involving only common variables, i.e., of localized objective term type $\mathscr{J}_{i,\text{com}}^{\text{com}}$, is shared equally by the control agents. Hence, each control agent i that considers a particular common node κ, includes in its objective function $1/N_\kappa$ times the objective function terms of such nodes of localized objective term type $\mathscr{J}_{i,\text{com}}^{\text{com}}$, where N_κ is the number of control agents considering node κ as common node. Control agent i in addition includes into its objective function the objective terms of all its internal nodes, and the objective terms of these common nodes that involve only internal and common variables, i.e., the objective terms of localized objective term types $\mathscr{J}_{i,\text{int}}^{\text{int}}$, $\mathscr{J}_{i,\text{int}}^{\text{int+ext}}$, $\mathscr{J}_{i,\text{int}}^{\text{int+com}}$ and $\mathscr{J}_{i,\text{com}}^{\text{int+com}}$.

Table 6 summarizes how control agent i deals with the different localized objective term types.

4.3 Control scheme for multiple agents

We have discussed how each control agent formulates its prediction model and objective function. The scheme that we propose for multi-agent control for overlapping subnetworks consists of the scheme proposed in Section 3 for touching subnetworks, with the following changes:

- Control agent i receives from the neighboring agents the following information at initialization and after each iteration:
  - Lagrange multipliers with respect to the constraints of localized constraint type $\mathscr{C}_{i,\mathrm{com}}^{\mathrm{int+com}}$, $\mathscr{C}_{i,\mathrm{com}}^{\mathrm{int+com+ext}}$, $\mathscr{C}_{i,\mathrm{ext}}^{\mathrm{int+ext}}$, and $\mathscr{C}_{i,\mathrm{ext}}^{\mathrm{int+ext+com}}$.
  - Values for the external variables *and* the common variables involved in these constraints.
- The optimization problem that each agent solves is changed accordingly to reflect the extensions discussed in this section, i.e., to take into account the constraints as given in Table 5 and the objective terms as given in Table 6.

The result is a control scheme that can be used by higher-layer control agents that control subnetworks that are overlapping. The control agents hereby share the responsibility for the common variables. In the next section we apply this scheme on an optimal flow control problem in power networks.

5 Application: Optimal flow control in power networks

In this section apply the scheme for multi-agent control of overlapping subnetworks, as discussed in Section 4, to the problem of optimal power flow control in power networks. A case study is carried out on the IEEE 57-bus power network [2], comprising as components generators, loads, transmission lines, and buses, with in addition FACTS devices installed at various locations, as illustrated in Figure 5. Two configurations are considered: in the first configuration only SVCs are included; in the second configuration only TCSCs are present. Each of the FACTS devices is controlled by an individual control agent.

6 Parameters of the power network

The parameters of the IEEE 57-bus base network can be obtained from the Power Systems Test Case Archive [2]. Line limits on the apparent power flows have been assigned to all transmission lines in such a way that no lines are overloaded. In order to find an interesting and meaningful situation for FACTS control, the grid was adapted by placing an additional generator at bus 30 leading to increased power flows in the center of the grid. The parameters of this generator are as follows:

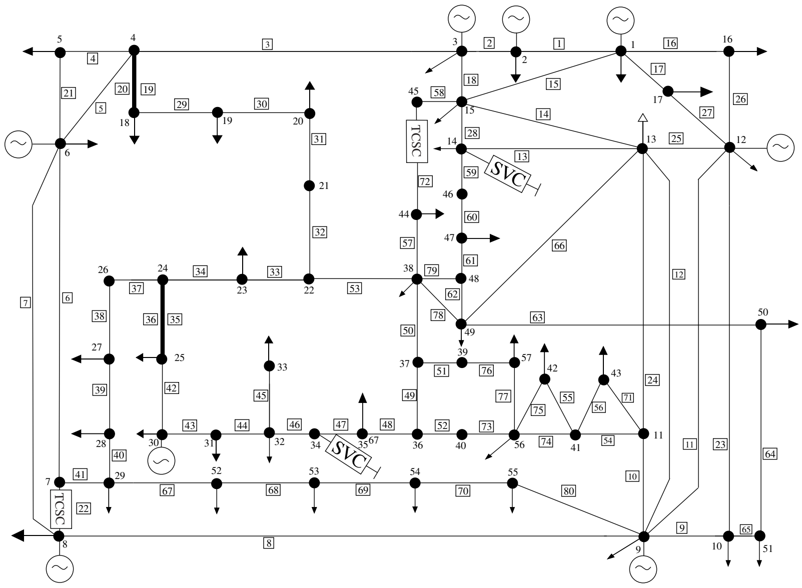


Fig. 5 IEEE 57-bus network extended with either SVCs installed at buses 14 and 34, or with TCSCs in lines 22 and 72.

Table 7 Line limits on the apparent power flows.

line no.	limit (p.u.)	line no.	limit (p.u.)	line no.	limit (p.u.)	line no.	limit (p.u.)
1	1.800	21	0.550	41	1.100	61	0.350
2	1.650	22	2.160	42	0.500	62	0.300
3	0.721	23	0.700	43	0.500	63	0.500
4	0.412	24	0.700	44	0.400	64	0.300
5	0.800	25	0.900	45	0.300	65	0.450
6	0.750	26	0.600	46	0.400	66	0.600
7	1.200	27	0.750	47	0.400	67	0.550
8	2.200	28	1.000	48	0.400	68	0.500
9	0.600	29	0.300	49	0.700	69	0.350
10	0.450	30	0.300	50	0.800	70	0.400
11	0.300	31	0.300	51	0.300	71	0.300
12	0.500	32	0.300	52	0.300	72	0.700
13	0.700	33	0.400	53	0.400	73	0.300
14	0.936	34	0.450	54	0.300	74	0.300
15	1.900	35	0.300	55	0.300	75	0.300
16	1.050	36	0.300	56	0.300	76	0.300
17	1.200	37	0.350	57	0.500	77	0.300
18	1.200	38	0.350	58	0.600	78	0.550
19	0.300	39	0.350	59	0.636	79	0.550
20	0.300	40	0.400	60	0.650	80	0.500

$d_{\mathrm{V,gen},m} = 1.03$ p.u., $d_{\mathrm{P,gen},m} = 0.5$ p.u., for $m = 30$. The parameters of the base IEEE 57 network used in this paper can be found in [2].The limits on the apparent power flows are set as listed in Table 7.

In the following we make a distinction between a *bus* and a *node*. A bus refers to an element of the physical power network, whereas a node refers to an element of the model of the physical power network. Since for each physical bus a corresponding node is included in the model, references to a bus or its corresponding node can be interchanged, except for when assigning constraints relating to two buses, such as constraints imposed due to transmission lines, to a single node, as we will see next.

Below we formulate the steady-state models used to describe the network behavior, we assign the constraints to nodes, we set up the objective terms associated with the nodes, we discuss the way in which the subnetworks can be determined using the influence-based approach, and we illustrate the workings of the proposed approach.

6.1 Steady-state characteristics of power networks

As the focus lies on improving the steady-state network security, the power network is modeled using equations describing the steady-state characteristics of the power network. As we will see, the aspects of the steady-state security that we are interested in can be determined from the voltage magnitude and voltage angle at each of the 57 (physical) buses in the network. We therefore define 57 nodes to model the network, and assign to each node m the voltage magnitude $z_{\mathrm{V},m}$ per unit (p.u.) and the voltage angle $z_{\theta,m}$ (degrees) as variables. In order to determine the values for these variables under different disturbance variables and actuator values, models for the components and their influence on the voltage magnitude and angle are defined. We model the transmission lines, the generators, the loads, and the FACTS devices.

6.1.1 Transmission lines

For the transmission lines the well-known π-model is used [15]. The active power $z_{\mathrm{P},mn}$ (p.u.) and the reactive power $z_{\mathrm{Q},mn}$ (p.u.) flowing from bus m over the transmission line to bus n are then given by:

$$
\begin{aligned}
z_{\mathrm{P},mn} = (z_{\mathrm{V},m})^2 &\left(\frac{\eta_{\mathrm{R},mn}}{(\eta_{\mathrm{R},mn})^2 + (\eta_{\mathrm{X},mn})^2} \right) \\
- z_{\mathrm{V},m} z_{\mathrm{V},n} &\left(\frac{\eta_{\mathrm{R},mn}}{(\eta_{\mathrm{R},mn})^2 + (\eta_{\mathrm{X},mn})^2} \cos(z_{\theta,m} - z_{\theta,n}) \right) \\
+ z_{\mathrm{V},m} z_{\mathrm{V},n} &\left(\frac{\eta_{\mathrm{X},mn}}{(\eta_{\mathrm{R},mn})^2 + (\eta_{\mathrm{X},mn})^2} \sin(z_{\theta,m} - z_{\theta,n}) \right)
\end{aligned} \tag{9}
$$

$$
\begin{aligned}
z_{\mathrm{Q},mn} = (z_{\mathrm{V},m})^2 &\left(\frac{\eta_{\mathrm{X},mn}}{(\eta_{\mathrm{R},mn})^2 + (\eta_{\mathrm{X},mn})^2} \right) \\
- z_{\mathrm{V},m} z_{\mathrm{V},n} &\left(\frac{\eta_{\mathrm{R},mn}}{(\eta_{\mathrm{R},mn})^2 + (\eta_{\mathrm{X},mn})^2} \sin(z_{\theta,m} - z_{\theta,n}) \right) \\
- (z_{\mathrm{V},m})^2 &\left(\frac{\eta_{\mathrm{B},mn}}{2} \right) - z_{\mathrm{V},m} z_{\mathrm{V},n} \left(\frac{\eta_{\mathrm{X},mn}}{(\eta_{\mathrm{R},mn})^2 + (\eta_{\mathrm{X},mn})^2} \cos(z_{\theta,m} - z_{\theta,n}) \right),
\end{aligned} \tag{10}
$$

where $\eta_{\mathrm{B},mn}$ (p.u.) is the shunt susceptance, $\eta_{\mathrm{R},mn}$ (p.u.) is the resistance, and $\eta_{\mathrm{X},mn}$ (p.u.) is the reactance of the line between buses m and n.

The constraints for each transmission line going from bus m to bus n, for $n \in \mathcal{N}^m$ (where $\mathcal{N}^m$ is the set of neighboring buses of bus m, i.e., the buses that are physically connected to bus m through a transmission line), are assigned to node m, if $m < n$, and to node n otherwise.

6.1.2 Generators

Generators are assumed to have constant active power injection and constant voltage magnitude, and therefore

$$z_{\mathrm{P,gen},m} = d_{\mathrm{P,gen},m} \tag{11}$$

$$z_{\mathrm{V},m} = d_{\mathrm{V,gen},m}, \tag{12}$$

where $d_{\mathrm{P,gen},m}$ is the given active power that the generator produces, and $d_{\mathrm{V,gen},m}$ is the given voltage magnitude that the generator maintains. At most one generator can be connected to a bus, since a generator directly controls the voltage magnitude of that bus.

The generator connected to bus 1 is considered as a slack generator, i.e., a generator with infinite active and reactive power capacity, with fixed voltage magnitude and angle [15]. So, for this generator we have with $m = 1$

$$z_{\mathrm{V},m} = d_{\mathrm{V,gen},m} \tag{13}$$

$$z_{\theta,m} = d_{\theta,\mathrm{gen},m}, \tag{14}$$

where $d_{\theta,\text{gen},m}$ is the given voltage angle ensured by the generator.

The constraints of a generator at bus m are assigned to node m.

6.1.3 Loads

The loads are constant active and constant reactive power injections, i.e.,

$$z_{\text{P,load},m} = d_{\text{P,load},m} \tag{15}$$

$$z_{\text{Q,load},m} = d_{\text{Q,load},m}, \tag{16}$$

where $d_{\text{P,load},m}$ and $d_{\text{Q,load},m}$ are the given active and reactive power consumption, respectively, of the load connected to bus m. For simplicity, only one load can be connected to a bus. Multiple loads can easily be aggregated to obtain a single load.

The constraints of the loads at bus m are assigned to node m.

6.1.4 FACTS devices

SVC

An SVC is a FACTS device that is shunt-connected to a bus m and that injects or absorbs reactive power $z_{\text{Q,SVC},m}$ to control the voltage $z_{\text{V},m}$ at that bus [10]. The SVC connected to bus m accepts as control input the effective susceptance $u_{\text{B,SVC},m}$. The injected reactive power $z_{\text{Q,SVC},m}$ of the SVC is:

$$z_{\text{Q,SVC},m} = (z_{\text{V},m})^2 u_{\text{B,SVC},m}. \tag{17}$$

The control input $u_{\text{B,SVC},m}$ is limited to the domain:

$$u_{\text{B,SVC,min},m} \leq u_{\text{B,SVC},m} \leq u_{\text{B,SVC,max},m}, \tag{18}$$

where the values of $u_{\text{B,SVC,min},m}$ and $u_{\text{B,SVC,max},m}$ are determined by the size of the device [7].

The constraints of an SVC at bus m are assigned to the node m.

TCSC

A TCSC is a FACTS device that can control the active power flowing over a line [10]. It can change the line reactance $z_{\text{X,line},mn}$. The TCSC is therefore considered as a variable reactance $u_{\text{X,TCSC},mn}$ connected in series with the line. If a TCSC is connected in series with a transmission line between buses m and n, the total reactance $z_{\text{X,line},mn}$ of the line including the TCSC is given by:

$$z_{\mathrm{X,line},mn} = \eta_{\mathrm{X},mn} + u_{\mathrm{X,TCSC},mn}, \tag{19}$$

where $\eta_{\mathrm{X},mn}$ is the reactance of the line without the TCSC installed. The reactance $u_{\mathrm{X,TCSC},mn}$ is limited to the domain:

$$u_{\mathrm{X,TCSC,min},mn} \leq u_{\mathrm{X,TCSC},mn} \leq u_{\mathrm{X,TCSC,max},mn}, \tag{20}$$

where the values of $u_{\mathrm{X,TCSC,min},mn}$ and $u_{\mathrm{X,TCSC,max},mn}$ are determined by the size of the TCSC and the characteristics of the line in which it is placed, since due to the physics the allowed compensation rate of the line $u_{\mathrm{X,TCSC},mn}/\eta_{\mathrm{X},mn}$ is limited [7].

The constraints of the TCSC at the line between bus m and n are assigned to node m, if $m < n$, and to node n otherwise.

6.1.5 Power balance

By Kirchhoff's laws, at each bus the total incoming power and the total outgoing power has to be equal. This yields the following additional constraints for bus m:

$$z_{\mathrm{P,load},m} - z_{\mathrm{P,gen},m} + \sum_{n \in \mathcal{N}^m} z_{\mathrm{P},mn} = 0 \tag{21}$$

$$z_{\mathrm{Q,load},m} - z_{\mathrm{Q,gen},m} - z_{\mathrm{Q,SVC},m} + \sum_{n \in \mathcal{N}^m} z_{\mathrm{Q},mn} = 0. \tag{22}$$

If no generator is connected to bus m, then $z_{\mathrm{P,gen},m}$ and $z_{\mathrm{Q,gen},m}$ are zero. If no load is connected to bus m, then $z_{\mathrm{P,load},m}$ and $z_{\mathrm{Q,load},m}$ are zero. If no SVC is connected to bus m, then $z_{\mathrm{Q,SVC},m}$ is zero.

The constraints resulting from Kirchhoff's laws for bus m are assigned to node m.

6.2 Control objectives

The objectives of the control are to improve the system security through minimization of the deviations of the bus voltages from given references to improve the voltage profile, minimization of active power losses, and preventing lines from overloading, by choosing appropriate settings for the FACTS devices. These objectives are translated into objective terms associated with the buses as follows:

- To minimize the deviations of the bus voltage magnitude $z_{\mathrm{V},m}$ of bus m from a given reference $d_{\mathrm{V,ref},m}$, an objective term $p_{\mathrm{V}}\left(z_{\mathrm{V},m} - d_{\mathrm{V,ref},m}\right)^2$ is associated with node m, where p_{V} is a weighting coefficient.
- To minimize the active power losses over a line between bus m and bus n, an objective term $p_{\mathrm{loss}}(z_{\mathrm{P},mn} + z_{\mathrm{P},nm})$, where p_{loss} is a weighting coefficient, is associated to node m, if $m < n$, and to node n otherwise. Note that the term $z_{\mathrm{P},mn} + z_{\mathrm{P},nm}$, which represents the power losses, is always nonnegative.

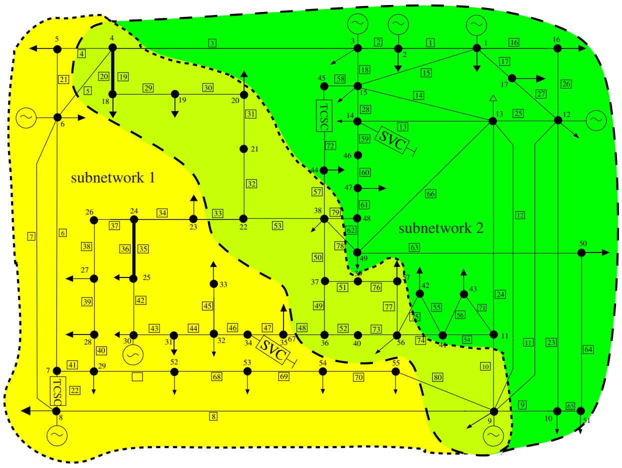


Fig. 6 IEEE 57-bus system with decomposition into 2 subnetworks. Scenario 1: SVCs at buses 14 and 34, scenario 2: TCSCs in lines 22 and 72. The dotted line indicates the borders of subnetwork 1 (light shaded); the dashed line indicates the borders of subnetwork 2 (dark shaded). The region encapsulated both by subnetwork 1 and subnetwork 2 is the common region (medium shaded).

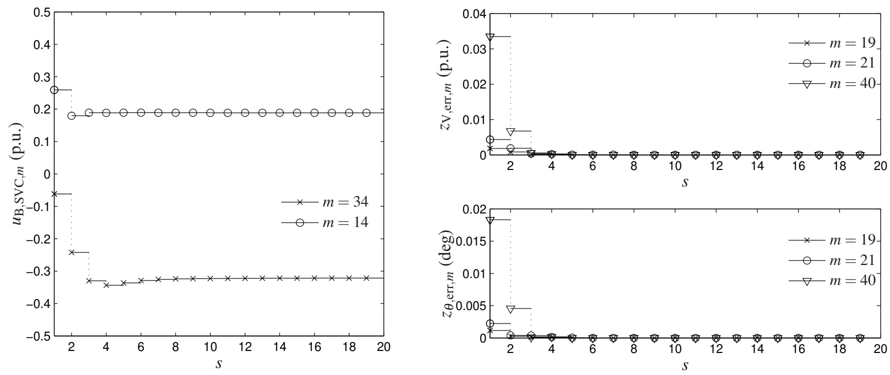
- To minimize the loading of the line between buses m and n, an objective term is associated to node m, if $m < n$, and to node n otherwise, as $p_{\text{load}}\left(\frac{z_{\text{S},mn}}{z_{\text{S,max},mn}}\right)^2$, where p_{load} is a weighting coefficient, and where $z_{\text{S},mn}$ is the apparent power flowing over the line from bus m to bus n, defined as $z_{\text{S},mn} = \sqrt{(z_{\text{P},mn})^2 + (z_{\text{Q},mn})^2}$. The relative line loading is penalized in a quadratic way such that an overloaded line is penalized more severely than a line that is not overloaded.

The weighting coefficients p_{V}, p_{loss}, and p_{load} allow to change the weight given to each objective. In the following we take $p_{\text{V}} = 1000$, $p_{\text{loss}} = 100$, and $p_{\text{load}} = 1$.

6.3 Setting up the control problems

Each FACTS device is controlled by a different control agent. The influence-based subnetworks of the control agents controlling the FACTS devices can be overlapping, and therefore the control problems of the control agents are set up using the approach discussed in Section 4. To solve their subproblems at each iteration the control agents use the nonlinear problem solver SNOPT v5.8 [6], as implemented in Tomlab v5.7 [11], and accessed from Matlab v7.3 [17].

In the following we illustrate how the approach works for a particular assignment of nodes to subnetworks in two representative scenarios.



(a) (b)

Fig. 7 (a) Convergence of the settings of the SVCs at buses 14 and 34, as a function of the iteration, for scenario 1. (b) Convergence of the difference between the values of the voltage magnitudes (top) and the voltage angles (bottom) as considered by both control agents for buses 19, 21, 40, as a function of the iteration, for scenario 1.

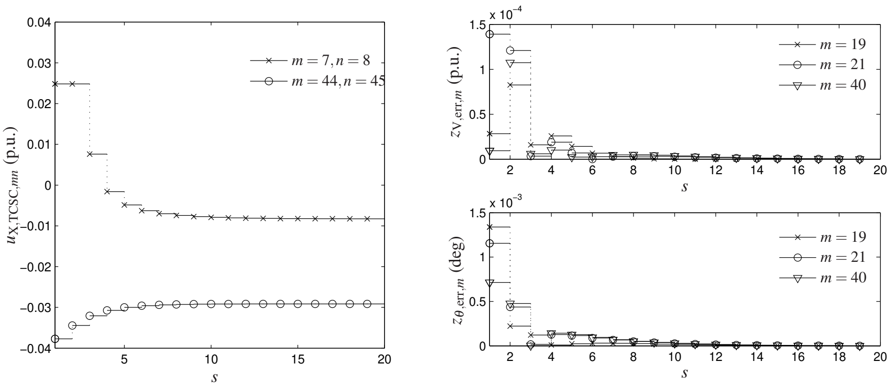
6.4 Simulations

Various test scenarios with different FACTS devices and subnetworks have been examined. Here we present two representative scenarios. The subnetworks used in these scenarios are shown in Figure 6. It can be seen that these subnetworks are overlapping, since there are several nodes that are included in both subnetworks.

6.4.1 Scenario 1: Control of SVCs

In the first scenario, SVCs are placed at buses 14 and 34. As the SVCs are mainly used to influence the voltage profile, the line limits are chosen such that no line is at the risk of being overloaded.

Figure 7(a) shows the convergence of the SVC settings over the iterations. As can be seen, the settings of the SVCs converge within only a few iterations to the final values, which in this case are equal to the values obtained from a centralized optimization. Figure 7(b) shows the evolution of the deviations between the values determined by both subnetworks for the voltage magnitudes and angles at some common buses. In the figure the error $z_{\mathrm{V,err},m}$ is defined as the absolute difference between the values that control agents 1 and 2 want to give to the voltage magnitude $z_{\mathrm{V},m}$. Similarly, the error $z_{\theta,\mathrm{err},m}$ is defined as the absolute difference between the values that control agents 1 and 2 want to give to the voltage angles. As can be seen fast convergence is obtained.



(a) (b)

Fig. 8 (a) Convergence of the settings of the TCSCs in lines 22 and 72 (i.e., the lines between buses 7 and 8, and buses 44 and 45, respectively), as a function of the iteration number, for scenario 2. (b) Convergence of the difference between the values of the voltage magnitudes (top) and the voltage angles (bottom) as considered by both control agents for buses 19, 21, 40, as a function of the iteration number, for scenario 2

6.4.2 Scenario 2: Control of TCSCs

In the second scenario, TCSCs are installed on lines 72 and 22. Since TCSCs are mainly used to influence active power flows and to resolve congestion, the line limits are chosen such that lines 7 and 60 are overloaded if the FACTS devices are not being used.

The results for the TCSC settings and the difference between the voltage magnitudes and angles for some common buses over the iterations are given in Figures 8(a) and 8(b), respectively. The control agent of subnetwork 1 sets the TCSC to its upper limit at the first few iterations. But after some additional iterations, the values that the control agents choose converge to their final values, which are again equal to the values obtained by a centralized control agent.

In Figure 9 the line loadings of lines 7 and 60, i.e., the lines which are overloaded without FACTS devices in operation, are shown. Line 7 is immediately brought below its limit whereas for line 60, the loading approaches 100% in the course of the optimization process.

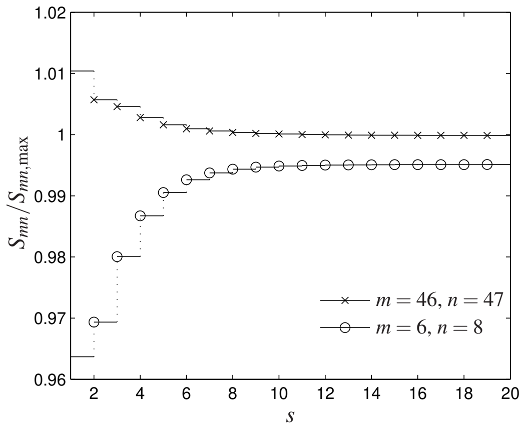


Fig. 9 Convergence of the relative line loadings of lines 7 and 60 (i.e., the lines between buses 6 and 8, and 46 and 47, respectively), as a function of the iteration number, for scenario 2.

7 Conclusions and future research

In this chapter we have focused on an alternative way to define subnetworks for higher-layer multi-agent control. The higher control layer uses steady-state characteristics only. We have discussed how subnetworks can be defined based on the influence of inputs on the variables of nodes. When such an approach is used to define subnetworks, some subnetworks could be overlapping, resulting in constraints and objectives in common sub-subnetworks. We have proposed a method for higher-layer multi-agent control that can be used by control agents that control such overlapping subnetworks.

To illustrate the topics discussed and the proposed approach, we have defined overlapping subnetworks for Flexible Alternating Current Transmission Systems (FACTS) in an adjusted version of the IEEE 57-bus power network. Using the proposed control approach, we have then solved an optimal power flow control problem. The simulations illustrate that in the considered cases the proposed approach can achieve fast convergence to actuator values that are globally optimal.

Further research will address the following issues and topics. It will be determined formally when the approach converges and what the quality of the obtained solutions is, in particular when compared to an overall single-agent, centralized, control scheme. This will provide more insight into the quality of the solutions and the time required to obtain these solutions. Also, power networks are just a particular network from the general class of transportation networks. Other examples from the class of transportation networks to which the approach discussed in this paper could be successfully applied in future work are traffic and transportation systems [4], natural gas networks [23], combined electricity and gas networks [1], water networks [21], etc. The approach will also be extended to deal with dynamics using ideas from [19].

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