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Modeling and Control of Legged Locomotion via Switching Max-Plus Systems

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Abstract:
We present a class of gait generation and control algorithms based on the Switching Max-Plus modeling framework that allow for the synchronization of multiple legs of walking robots. Transitions between stance and swing phases of each leg are modeled as discrete events in a system described by max-plus-linear state equations. Different gaits can be systematically generated and interleaved during motion by switching between different system matrices. We show that such gait switching can be done in an optimal way, minimizing the tip leg velocity variation for all legs simultaneously touching the ground.

Keywords: Max-Plus Algebra, Discrete Event Systems, Robotics, Legged Locomotion, Motion Control

1. INTRODUCTION

Due to their cyclic nature, legged locomotion systems can be modeled by limit cycles in cross products of circles in the phase space of the set of continuous time gaits. Such an abstraction, often denoted as “networks of phase oscillators” or “central pattern generators” (CPG), were introduced in the earlier works of Grillner (1985) and Cohen et al. (1988) and are now accepted by both Biology and Robotics communities as standard modeling tools (see Holmes et al. (2006) for an extensive review on the elements of dynamic legged locomotion). The biology community has explored these concepts to classify different gaits of various animals, such as horses (Hildebrand, 1965), insects (Wilson, 1966), and many other species (Alexander, 1984). In the robotics community different motion generation methods have been developed for legged robots: Klavins and Koditschek (2002) show how to systematically generate vector fields that reach piecewise constant velocity limit cycles, Erden and Leblebicioğlu (2008) use reinforcement learning tools on a hexapod robot, and Zhao et al. (2009) use CPG models for the control of a biomimetic fish. Recently Haynes et al. (2009) take a combinatorial approach to classify different gaits for multipedal robots.

An interesting analogy can be made between enforcing phase differences in continuous cycles and synchronization in timed discrete event processes. In a typical (continuous) walking motion of a biped robot, the left leg should only lift off the ground after the right leg has touched down, to make sure the robot does not fall from lack of support [1].

This synchronization requirement can be modeled by the evolution of a discrete event system (DES) by abstracting each limit cycle in the circle into two sequential events in a closed circuit: lift-off and touchdown. Each leg is then modeled by an event cycle and synchronization between legs is enforced by connections between each leg cycle. Synchronization of multiple legs is especially important for climbing robots (Guo et al., 1994; Autumn et al., 2005) where lack of support can have catastrophic consequences.

In this paper we continue our previous work (Lopes et al., 2009) to show how the max-plus framework can be naturally utilized to implement gaits for legged locomotion with synchronization guaranteed by design. This paper complements the previous work by showing that gaits can be generated systematically, and that gait switching can be done in an optimal way. Max-plus-linear discrete event systems (MPL-DES) are a subclass of timed DES (classes of discrete event systems where there exists an underlying time structure) that can be framed in sets of linear equations in the *max-plus algebra* (Cuninghame-Green, 1979; Baccelli et al., 1992; Heidergott et al., 2006). Systems that enforce synchronization and have no concurrency can be modeled in this framework. Systems that can be modeled as MPL-DES inherit a large set of analysis and control synthesis tools thanks to many parallels between the max-plus-linear systems theory and the traditional linear systems theory.

Some work has already been done on low-level walking gait generation from a discrete event systems point of view (Antoniotti and Mishra, 1995; Suzuki et al., 2002),

[1] In this paper we intentionally leave out locomotion gaits with aerial phases where robots/animals can spend most of their time with no ground support. We acknowledge their importance but defer their study to later publications.

and by using Petri nets (Guangtao et al., 2003). In those implementations the focus is put on generating each individual gait. In this paper we take advantage of the properties of *switching max-plus-linear models* (van den Boom and De Schutter, 2006) to not only generate locomotion gaits but more importantly to deal with the transitions between different gaits and recovering from large perturbations. In a switching max-plus-linear system one can interchange different modes of operation. In each mode the discrete event system is described by a max-plus-linear state space model with different system matrices. In the application to legged locomotion, a mode corresponds to a specific gait.

Gait transition has been studied in biology from an energetic point of view by Hoyt and Taylor (1981) and in the robotics field was approached informally by Raibert et al. (1989), and more recently by Haynes and Rizzi (2006); Haynes et al. (2009). Using the max-plus framework as presented in this paper, safe gait transitions occur naturally.

In Section 2 we review the tools behind switching max-plus systems, and we show in Section 3 how these can be used to model locomotion gaits for legged robots. In Section 4 we present a class of parameterizations for the gait space, and show how to optimally switch between various gaits in Section 5. We end in Section 6 by analyzing the eigenstructure of the system matrices to gain insight into the resulting robot velocity.

2. MAX-PLUS ALGEBRA

We start by revising the structure of the max-plus algebra. Let $\varepsilon := -\infty$, $e := 0$, and $\mathbb{R}_{\max} := \mathbb{R} \cup \{\varepsilon\}$. Define the operations $\oplus, \otimes : \mathbb{R}_{\max} \times \mathbb{R}_{\max} \to \mathbb{R}_{\max}$ by:

$$x \oplus y := \max(x, y)$$
$$x \otimes y := x + y$$

Definition 1. The set $\mathbb{R}_{\max}$ with the operations $\oplus$ and $\otimes$ is called the max-plus algebra, denoted by $\mathcal{R}_{\max} := (\mathbb{R}_{\max}, \oplus, \otimes, \varepsilon, e)$.

Theorem 2. (Baccelli et al. 1992). The max-plus algebra $\mathcal{R}_{\max}$ has the algebraic structure of a commutative idempotent semiring.

The max-plus algebra can be interpreted as the traditional linear algebra with the operations '+' and '×' replaced by the operators 'max' and '+', respectively, with the supplemental difference that the additive inverse does not exist, thus resulting in a semiring. Matrices can be defined by taking Cartesian products of $\mathbb{R}_{\max}$. Define the matrix sum $\oplus$, matrix product $\otimes$, and matrix power operations by:

$$[A \oplus B]_{ij} = a_{ij} \oplus b_{ij} := \max(a_{ij}, b_{ij})$$
$$[A \otimes C]_{ij} = \bigoplus_{k=1}^{m} a_{ik} \otimes c_{kj} := \max_{k=1,\ldots,m}(a_{ik} + c_{kj})$$
$$D^{\otimes k} := \underbrace{D \otimes D \otimes \ldots \otimes D}_{k\text{-times}},$$

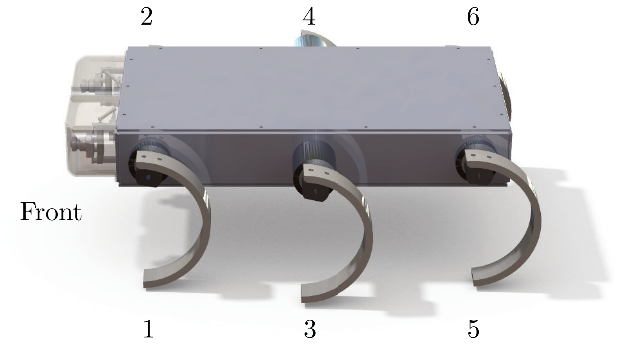


Fig. 1. Hexapod robot inspired by RHex (Saranli et al., 2001). The numbers represent the leg enumeration used throughout this paper.

where $A, B \in \mathbb{R}_{\max}^{n\times m}$, $C \in \mathbb{R}_{\max}^{m\times p}$, and $D \in \mathbb{R}_{\max}^{n\times n}$.The i, j element of A is denoted by $a_{ij} = [A]_{ij}$. In this context, the max-plus zero and identity matrices are defined by:

$$[\mathcal{E}]_{ij} := \varepsilon$$
$$[E]_{ij} := \begin{cases} e & \text{if } i = j \\ \varepsilon & \text{otherwise} \end{cases}$$

Let

$$A^* := \bigoplus_{k=0}^{\infty} A^{\otimes k}.$$

If A^* exists then the vector $x = A^* \otimes b$ solves the system of max-plus linear equations

$$x = A \otimes x \oplus b, \tag{1}$$

with $A \in \mathbb{R}_{\max}^{n\times n}$ and $b \in \mathbb{R}_{\max}^{n}$ (Baccelli et al. (1992), Theorem 3.17).

For the remainder of this paper we will consider implicit switching max-plus linear systems of the form:

$$x(k) = G_{m(k)} \otimes x(k) \oplus H_{m(k)} \otimes x(k-1), \tag{2}$$

where the event index dependent function $m(k)$ parameterizes the system matrices G and H (effectively acting as a gait switching mechanism). We will show that by carefully constructing G and H the system (2) can be rewritten as an explicit set of switching max-plus linear systems of the form

$$x(k) = A_{m(k)} \otimes x(k-1) \tag{3}$$

3. MODELING OF LEGGED LOCOMOTION

We model legged locomotion by abstracting the continuous time motion of the legs into discrete event cycles. Let $l_i(k)$ be the time instant leg i lifts off the ground and $t_i(k)$ be the time instant it touches the ground, both for the k-th event index. Here, k is considered to be a global event count. Figure 1 illustrates the leg numbering convention that we utilize for the examples. For a traditional alternating swing/stance [2] gait one can impose that the time instant when the leg touches the ground must equal the time

[2] The biology and robotics communities use the terms leg "swing" and "stance" to denote when a leg is in flight or is touching the

instant it lifted off the ground for the last time plus the time it stays in flight (denoted by τ_f):

$$t_i(k) = l_i(k) + \tau_f \tag{4}$$

Analogously, we get a similar relation for the lift-off time:

$$l_i(k) = t_i(k-1) + \tau_g, \tag{5}$$

where τ_g is the stance time and t_i uses the previous event index such that equations (4) and (5) can be used iteratively. Suppose now that one aims to synchronize leg i with leg j in such a way that leg i can only lift off τ_Δ seconds after leg j has touched the ground (τ_Δ is the double stance time). One can then write the relation:

$$\begin{aligned} l_i(k) &= \max\left(t_i(k-1) + \tau_g, t_j(k-1) + \tau_\Delta\right) \\ &= [\,\tau_g \;\; \tau_\Delta\,] \otimes \begin{bmatrix} t_i(k-1) \\ t_j(k-1) \end{bmatrix}. \end{aligned} \tag{6}$$

Equation (6) enforces simultaneously that both the leg i stays at least τ_g seconds in stance and will only lift off at least τ_Δ seconds after leg j has touched down. When both conditions are satisfied, lift-off takes place. Following this reasoning, one can efficiently represent motion gaits in terms of synchronization of timed events. For an n-legged robot, let the full discrete event state vector be defined by:

$$x(k) = [\underbrace{t_1(k) \; \cdots \; t_n(k)}_{t(k)} \; \underbrace{l_1(k) \; \cdots \; l_n(k)}_{l(k)}]^T.$$

The $2n$-dimensional system equations for the cycles represented by equations (4),(5) take the form:

$$\begin{bmatrix} t(k) \\ l(k) \end{bmatrix} = \left[\begin{array}{c|c} \mathcal{E} & \tau_f \otimes E \\ \hline \mathcal{E} & \mathcal{E} \end{array}\right] \otimes \begin{bmatrix} t(k) \\ l(k) \end{bmatrix} \oplus \left[\begin{array}{c|c} \mathcal{E} & \mathcal{E} \\ \hline \tau_g \otimes E & \mathcal{E} \end{array}\right] \otimes \begin{bmatrix} t(k-1) \\ l(k-1) \end{bmatrix} \tag{7}$$

According to system (7) all legs follow the same rhythm, i.e. all legs rotate with the same period of at least $\tau_f + \tau_g$ seconds. We assume that all leg synchronizations are achieved by enforcing a relation between the next lift-off time of a leg with the touchdown time of other legs (as in equation (6)). This assumption is expressed by the additional matrices P and Q (that we define in the next section) added to equation (7), resulting in the synchronized system:

$$\begin{bmatrix} t(k) \\ l(k) \end{bmatrix} = \left[\begin{array}{c|c} \mathcal{E} & \tau_f \otimes E \\ \hline P & \mathcal{E} \end{array}\right] \otimes \begin{bmatrix} t(k) \\ l(k) \end{bmatrix} \oplus \left[\begin{array}{c|c} \mathcal{E} & \mathcal{E} \\ \hline \tau_g \otimes E \oplus Q & \mathcal{E} \end{array}\right] \otimes \begin{bmatrix} t(k-1) \\ l(k-1) \end{bmatrix}, \tag{8}$$

which we can write using simplified notation as:

$$x(k) = G \otimes x(k) \oplus H \otimes x(k-1).$$

This poses the question whether equation (8) can be solved explicitly or not. We address this by looking at the solution of equation (1) to obtain:

ground supporting the body, respectively. "Double stance" represents two legs touching the ground. In this paper we use the term double stance to denote when more than one leg is in stance.

$$x(k) = G^* \otimes H \otimes x(k-1) \tag{9}$$

Lemma 3. A sufficient condition for G^* to exist is that the matrix P is nilpotent in the max-plus sense.

Proof. By direct computation, the repetitive products of G can be found to be

$$G^{\otimes k} = \begin{cases} \left[\begin{array}{c|c} \mathcal{E} & \tau_f^{\otimes k} \otimes P^{\otimes (k-1)} \\ \hline \tau_f^{\otimes (k-1)} \otimes P^{\otimes k} & \mathcal{E} \end{array}\right] & \text{if } k \text{ is odd} \\ \left[\begin{array}{c|c} \tau_f^{\otimes k} \otimes P^{\otimes k} & \mathcal{E} \\ \hline \mathcal{E} & \tau_f^{\otimes k} \otimes P^{\otimes k} \end{array}\right] & \text{if } k \text{ is even} \end{cases}$$

If P is nilpotent, then there exists a finite $p > 0$ such that $\forall k > p : P^{\otimes k} = \mathcal{E} \Rightarrow G^{\otimes k} = \mathcal{E}$, and therefore the sum for the computation of G^* is finite:

$$G^* = \bigoplus_{k=0}^{\infty} G^{\otimes k} = \bigoplus_{k=0}^{p} G^{\otimes k}$$

Remark 4. Note that in general P being nilpotent is not a necessary condition for the existence of G^*. According to Baccelli et al. (1992), Theorem 3.17, (please see notation within) if there are only circuits of non-positive weight in the graph of G then a solution for G^* can be found. This implies however, that some of the entries of G are negative, which we do not consider in this paper. Additionally, if all circuits have zero weight and G is not nilpotent, then G^* will also exist. Again, this "pathological" case cannot exist in our parameterization of G.

We conclude that in practice a necessary and sufficient requirement for achieving feasible leg synchronizations is that the matrix P be nilpotent.

4. PARAMETERIZATION OF THE GAIT SPACE

The matrices P and Q are used to encode all gaits, but nothing is said yet about how to fill such matrices. That process can be done systematically by introducing a new set of notation. For an n-legged system let

$$\{L_i\} \prec \{L_j\}$$

represent an ordering relation that enforces that leg $L_j \in \{1, \ldots, n\}$ is only allowed to lift-off after leg $L_i \in \{1, \ldots, n\}/\{L_j\}$ has touched down. This notation can be expanded to multiple ordered synchronizations between multiple legs by writing

$$\{L_{1,1}, \ldots, L_{1,j}, \} \prec \cdots \prec \{L_{r,1}, \ldots, L_{r,p}, \} \tag{10}$$

Equation (10) states that legs $L_{1,1}, \ldots, L_{1,j}$ will swing simultaneously and will precede the (eventual) swing of legs $L_{r,1}, \ldots, L_{r,p}$. Using this notation, a tripod gait on a hexapod robot is written as

$$\{1, 4, 5\} \prec \{2, 3, 6\}, \tag{11}$$

and a quintuple-stance gait can be written as

$$\{1\} \prec \{2\} \prec \{3\} \prec \{4\} \prec \{5\} \prec \{6\}.$$

This notation[3] translates directly into a systematic way of generating the matrices P and Q. Consider the sequence of ordered pairs

$$\{L_1\} \prec \{L_2\} \prec \cdots \prec \{L_n\},$$

and assume that a double stance time τ_Δ is required between each leg touchdown and subsequent lift-off. Starting with the matrices $P = \mathcal{E}$ and $Q = \mathcal{E}$, for each pair $\{L_i\} \prec \{L_{i+1}\}$ with $i = 1, \ldots, n-1$ add an entry to the matrix P in row L_{i+1}, column L_i:

$$[P]_{L_{i+1},L_i} = \tau_\Delta.$$

To enforce the synchronization of the full cycle, the "boundary" pair of legs L_1 and L_n are added to the matrix Q such that

$$[Q]_{L_1,L_n} = \tau_\Delta.$$

When multiple legs are required to be synchronized simultaneously, as in equation (10), entries must be added to any combination of parameters of the pairs. The same is true for the boundary pairs and matrix Q:

$$\begin{array}{ll} [P]_{L_{2,1},L_{1,1}} = \tau_\Delta & [Q]_{L_{1,1},L_{r,1}} = \tau_\Delta \\ [P]_{L_{2,1},L_{1,2}} = \tau_\Delta & [Q]_{L_{1,1},L_{r,2}} = \tau_\Delta \\ \vdots & \vdots \end{array}$$

Figure 2 illustrates various examples of gaits generated by the leg ordering relations. Note that the present notation precludes gaits where legs have different cyclic periods. Such gaits can be obtained using a modified version of equation (8), but that goes beyond the scope of this paper.

5. QUASI-OPTIMAL GAIT SWITCHING

The notation presented in the previous section exposes the combinatorial nature of the gait space. Ordered pairs such as $\{1\} \prec \{2\} \prec \{3\} \prec \{4\} \prec \{5\} \prec \{6\}$ and $\{4\} \prec \{5\} \prec \{6\} \prec \{1\} \prec \{2\} \prec \{3\}$ result in different synchronization matrices P and Q but are in fact equal up to an "event shift" in the state variables. For these quintuple gaits for an hexapod, one can find $5! = 120$ different gaits and a total of $6! = 720$ different parameterizations of P and Q. When switching from different "structural" classes of gaits, e.g. tripod to quadruped, different transitions can occur depending on which particular elements from each classes are chosen. This can result in different stance times for each of the legs that translates into different velocities at the leg's ground contact point. From a practical point of view, different leg velocities induce turning moments that can take the legged platform off balance. This is specially true for climbing robots, such as in Autumn et al. (2005), where improper gait switching can result in a catastrophic failure by fall. It is then important to know how to optimally switch gaits in the sense of minimizing the stance velocity variation of all legs. One can solve this by extensively searching the gait space, since only a finite number of gaits can be generated using the notation presented in the beginning of Section 4. However, this can be avoided by observing that the leg stance velocity variation can be (informally) minimized by picking the gait whose event timings in steady state have the "biggest resemblance" with the final touchdown event times of the previous gait. For example, when switching from a quadruped gait that has reached a steady state represented by

$$\{1, 4\} \prec \{3, 6\} \prec \{2, 5\},$$

to a quintuple gait, the "optimal" transition will be

$$\begin{array}{l} \{1\} \prec \{4\} \prec \{3\} \prec \{6\} \prec \{2\} \prec \{5\}, \text{or} \\ \{4\} \prec \{1\} \prec \{6\} \prec \{3\} \prec \{2\} \prec \{5\}, \text{etc.} \end{array}$$

The term "optimal" is utilized here in an informal way since we present no formal proof that the proposed choice does indeed minimize the leg stance velocity variation. However, extensive simulation results corroborate this hypothesis. Figure 3 illustrates two sample simulations that use optimal and non-optimal transitions. The results suggest that classes of gaits should not be fixed a priory such as in the non-optimal case.

6. EIGEN-STRUCTURE OF THE SYSTEM MATRIX

The above parameterization of the system matrices gives additional insight into the resulting steady state behavior of the system. For each class of gaits, the parameter τ_g does not represent the exact resulting stance time, but rather encodes the minimum possible time the legs spend on the ground. The true stance time is obtained by looking at the eigenvalues (in the max-plus sense) of the explicit system matrix $A = G^* \otimes H$ defined in equation (9). Due to the particular structure of A, one can find its (non-unique)[4] eigenvector, computed by

$$v = A \otimes \mathbf{0}, \text{ with } \mathbf{0} = [0\ 0\ \ldots\ 0]^T \tag{12}$$

and its associated (non-unique) eigenvalue:

$$\lambda = \max(A \otimes v - v).$$

Since the eigenvalue λ encodes the total cycle time, the true stance $\bar{\tau}_g$ time can be computed by $\bar{\tau}_g = \lambda - \tau_f$, and an approximation for the robot's true velocity V can be computed by

$$V \cong L \frac{\theta_l - \theta_t}{\bar{\tau}_g}$$

where θ_t, θ_l are the touchdown and lift-off angles respectively, and L is the leg length, assuming recirculating legs as in the morphology of RHex (Saranli et al., 2001) or the DCSC Quadruped (Lopes et al., 2009). For the classes of gaits presented in Figure 2 one can find the true stance times:

$$\begin{aligned} \text{Quintuple gait: } \bar{\tau}_g &= \max(6\tau_\Delta + 5\tau_f, \tau_g) \\ \text{Quadruped gait: } \bar{\tau}_g &= \max(3\tau_\Delta + 2\tau_f, \tau_g) \\ \text{Tripod gait: } \bar{\tau}_g &= \max(2\tau_\Delta + \tau_f, \tau_g) \end{aligned}$$

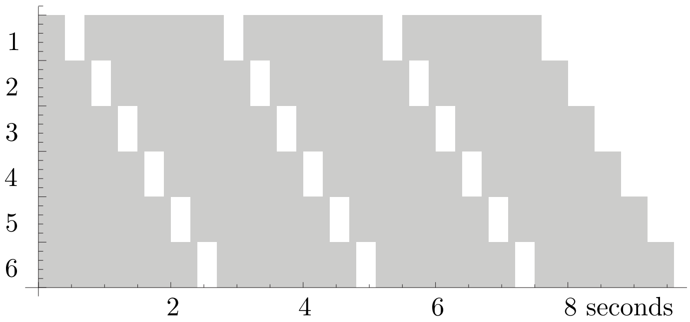
[3] Note that exotic gaits such as e.g. $\{3\} \prec \{1, 4, 5\} \prec \{2, 6\}$ are still valid in this framework.

[4] Due to the reducible structure of $A = [A_1\ \mathcal{E}]$ it additionally has a trivial eigenvalue $\lambda = \varepsilon$ and associated eigenvectors of the form $v = [\varepsilon \ldots \varepsilon\ v_i \ldots v_k]$. The computation of equation (12) was verified to be true by exhaustive search over all of the gait space up to 9 legs.

1. Quintuple gait

$\{1\} \prec \{2\} \prec \{3\} \prec \{4\} \prec \{5\} \prec \{6\}$ results into:

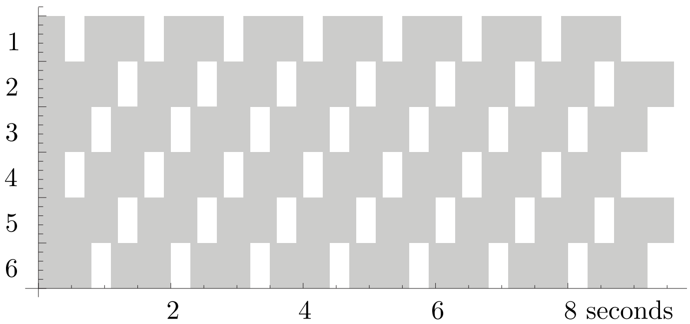
$$P = \begin{bmatrix} \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ \tau_\Delta & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & \tau_\Delta & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & \tau_\Delta & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & \tau_\Delta & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & \varepsilon & \tau_\Delta & \varepsilon \end{bmatrix} \quad Q = \begin{bmatrix} \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \tau_\Delta \\ \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \end{bmatrix}$$



2. Quadruped gait

$\{1,4\} \prec \{3,6\} \prec \{2,5\}$ results into:

$$P = \begin{bmatrix} \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & \tau_\Delta & \varepsilon & \varepsilon & \tau_\Delta \\ \tau_\Delta & \varepsilon & \varepsilon & \tau_\Delta & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & \tau_\Delta & \varepsilon & \varepsilon & \tau_\Delta \\ \tau_\Delta & \varepsilon & \varepsilon & \tau_\Delta & \varepsilon & \varepsilon \end{bmatrix} \quad Q = \begin{bmatrix} \varepsilon & \tau_\Delta & \varepsilon & \varepsilon & \tau_\Delta & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & \tau_\Delta & \varepsilon & \varepsilon & \tau_\Delta & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \end{bmatrix}$$



3. Tripod gait

$\{1,4,5\} \prec \{2,3,6\}$ results into:

$$P = \begin{bmatrix} \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ \tau_\Delta & \varepsilon & \varepsilon & \tau_\Delta & \tau_\Delta & \varepsilon \\ \tau_\Delta & \varepsilon & \varepsilon & \tau_\Delta & \tau_\Delta & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ \tau_\Delta & \varepsilon & \varepsilon & \tau_\Delta & \tau_\Delta & \varepsilon \end{bmatrix} \quad Q = \begin{bmatrix} \varepsilon & \tau_\Delta & \tau_\Delta & \varepsilon & \varepsilon & \tau_\Delta \\ \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & \tau_\Delta & \tau_\Delta & \varepsilon & \varepsilon & \tau_\Delta \\ \varepsilon & \tau_\Delta & \tau_\Delta & \varepsilon & \varepsilon & \tau_\Delta \\ \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \end{bmatrix}$$

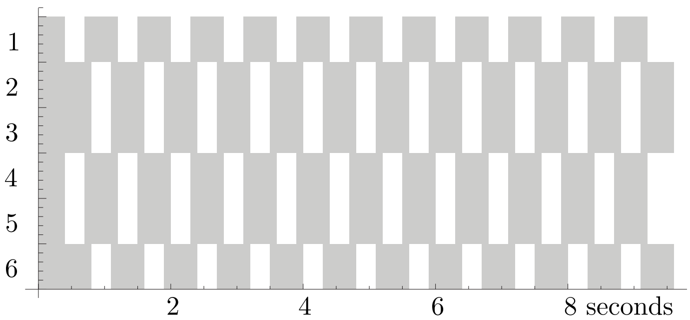


Fig. 2. Various examples of gaits for an hexapod robot, including the leg ordering relations, the synchronization matrices P, Q, and simulations of the evolution of the gaits in time. Solid bars represent leg stance, and white bars represent leg swing. For the presented simulations we use $\tau_g = 0.4$s, $\tau_f = 0.3$s, and $\tau_\Delta = 0.1$s.

7. CONCLUSIONS AND FUTURE WORK

We have presented a modeling tool for locomotion of multi-legged robots based on the max-plus algebra. Gait design and implementation can be efficiently accomplished by connecting circuits of discrete events. Leg synchronization is guaranteed by design when the max-plus framework is used. We have shown how to systematically generate gaits via a simple parameterization of a set of max-plus matrices. Gait switching is efficiently implemented by simply switching system matrices in the evolution of a max-plus-linear system. We have also shown how to effectively choose which gaits to pick in order to minimize the velocity variation of all the legs touching the ground. We aim to proceed this research by exploiting the rich set of tools available for the max-plus framework, and by establishing parallels with the combinatorial view of the recent work of Haynes et al. (2009).

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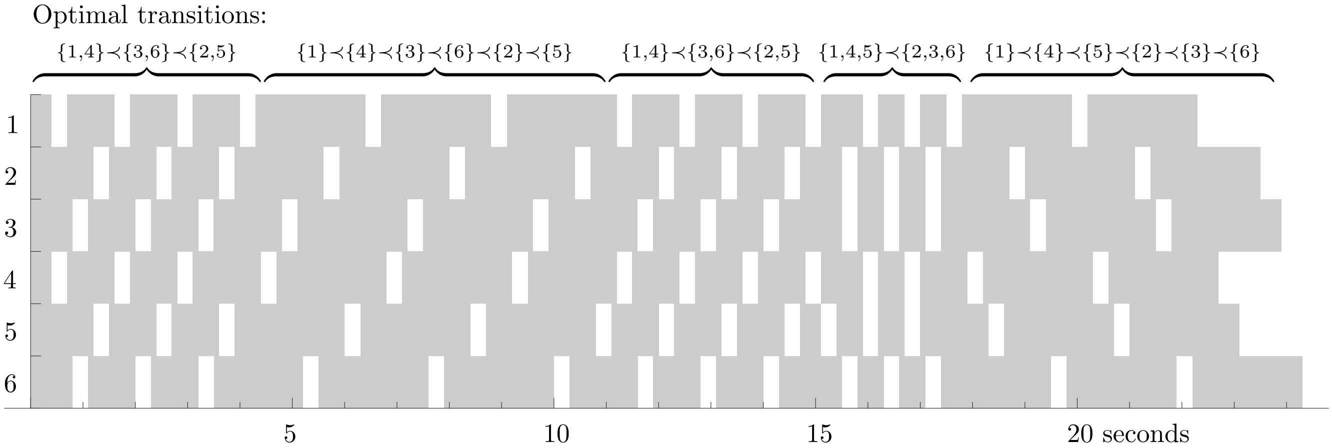

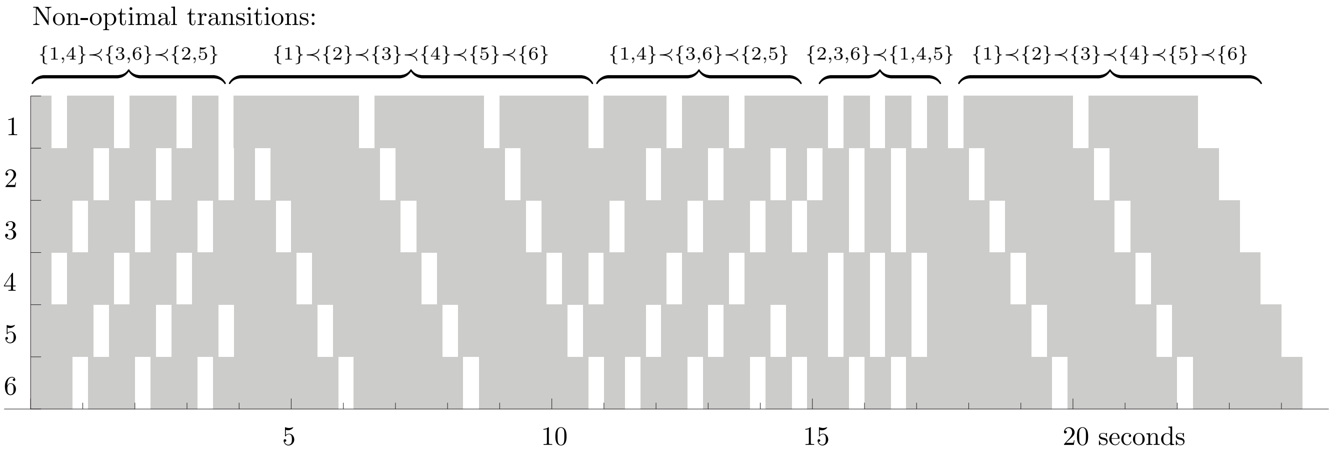

Fig. 3. Examples of sequences of gaits with and without optimal transitions. Only two tripod gaits are considered: $\{2,3,6\} \prec \{1,4,5\}$ and its "reverse" $\{1,4,5\} \prec \{2,3,6\}$, since in real robots these are typically the only stable tripod gaits. For the presented simulations we use $\tau_g = 0.4$s, $\tau_f = 0.3$s, and $\tau_\Delta = 0.1$s.

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