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Technical report 10-045

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If you want to cite this report, please use the following reference instead:
K. Staňková and B. De Schutter, "On freeway traffic density estimation for a jump Markov linear model based on Daganzo's cell transmission model," *Proceedings of the 13th International IEEE Conference on Intelligent Transportation Systems (ITSC 2010)*, Madeira Island, Portugal, pp. 13–18, Sept. 2010.

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On freeway traffic density estimation for a jump Markov linear model based on Daganzo's cell transmission model

Kateřina Staňková, Bart De Schutter

***Abstract*— This paper deals with problem of the real-time freeway traffic density estimation/prediction for a jump Markov linear model based on Daganzo's cell transmission variant of the Lighthill-Whitham-Richards continuous macroscopic freeway model. To solve the problem we propose a particle-filtering-based estimation/prediction method. Its performance is illustrated on case studies involving a four-cell freeway segment. The case studies suggest that the proposed methodology can be used for real-time traffic density estimation/prediction. Possible pitfalls of our approach are also discussed.**

Keywords: traffic density estimation, jump Markov linear models, particle filtering, Lighthill-Whitham-Richards equation, cell transmission model.

I. Introduction & Literature Overview

Freeway traffic congestion, caused by the rapid increase of traffic demand in the past decades has become a big problem, due to its high social and economical costs [1]–[11]. The control measures that are typically employed in freeway networks in order to eliminate congestion are ramp metering (with traffic signals at on-ramps or freeway interchanges) [6], [10], variable speed limits [2], road pricing [11], or driver information and guidance systems [9]. One of the prerequisites for efficient traffic control is efficient (real-time) estimation of the traffic densities [12], [13], knowing only a limited amount of (possibly biased) data.

In this paper we introduce a jump Markov linear model (JMLM) [14]–[16] for freeway traffic density estimation. This model is based on the linear switching model formulation [17] of the Daganzo's cell transmission model (CTM) introduced in [18]. The CTM is derived from the Lighthill-Whitham-Richards (LWR) model [19], [20], when assuming that the fundamental diagram is piecewise-affine (triangular or trapezoidal). The CTM simplifies the LWR model while keeping its fundamental physical properties [6], [21]. The linear switching model can be derived from the CTM with additional assumptions on the congestion wavefront behavior.

The estimation problem dealt with in this paper is NP-hard, and, therefore, efficient approximation methods are needed to make the solution process tractable. We apply particle filtering [22]–[26], also known as bootstrap filtering [27], [28], as an efficient and scalable method of this type. In this approach all information about the states of interest is obtained from the conditional distribution of the states given the past observations and the dynamics of the system. It approximates the posterior density function of the state by an empirical histogram obtained from the samples generated by a Monte Carlo simulation. In the past, particle filtering was used mainly for general JMLM estimation problems and for navigation, tracking, and maneuvering problems [25], [29]. In [8] a particle filtering method was applied for traffic density estimation with a second-order nonlinear model (the so-called compositional stochastic macroscopic traffic model developed in [5]), as opposed to the first-order jump Markov linear model introduced in this paper. In [30] and [31] the so-called piecewise-linearized cell transmission model, which is also a linear switching model, for traffic density estimation using mixture Kalman filters was introduced. It was assumed that all freeway segments are either free or congested. In our approach, we use the sampling importance resampling method [24]–[26], known as well as adaptive importance sampling [32], as a selection step. To the best of the authors' knowledge, there is no research focusing on particle filtering methods for the state estimation/prediction for CTM-based jump Markov models.

This paper is organized as follows: After a presentation of the model and a discussion of its properties in Section II, the estimation problem is stated in Section III. The particle filtering-based method for the traffic state estimation is proposed in Section IV. In Section V we present case studies with the model and method introduced in the previous sections. In Section VI we discuss advantages and disadvantages of the proposed model as well as those of the proposed method. The conclusions, possible extensions of the currently proposed model and method, and the future research possibilities are be the subject of Section VII.

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II. The model

A. Preliminaries: The LWR equation & the Godunov scheme

The classical Lighthill-Whitham-Richards (LWR) model [19], [20] is based on the assumption (motivated by experimental data) that vehicles tend to travel at the equilibrium speed $v = V(\rho)$ [km/h], where $\rho(x,t)$ [veh/km] is the vehicle density along the freeway section, and on the scalar conservation law principle

$$\partial_t \rho + \partial_x \Phi(\rho) = 0, \tag{1}$$

where $\Phi(\cdot)$ [veh/h] is the traffic flow function. These assumptions lead to an equilibrium flow function $\Phi(\rho) = \rho V(\rho)$ known as the fundamental diagram in traffic engineering [19], [20]. The problem is to find ρ satisfying (1) with initial condition $\rho(x,0) = \rho_0(x)$ and known boundary densities. The solution ρ of this problem may develop discontinuities satisfying the so-called entropy condition [33].

As conservation laws generate irregular flows, they cannot be integrated numerically using standard methods such as finite elements or finite differences, which may generate instabilities and/or wrong shock speeds [34]. An efficient first-order numerical method to solve such equations is the Godunov scheme [35], which divides the computational domain into segments, the so-called cells, of length Δx_i [km], where it is assumed that the density ρ_i [veh/km] is constant in each of them for a specific time interval. The Godunov scheme is:

$$\rho_i^+ = \rho_i + \frac{\Delta t}{\Delta x_i}\left(\phi^{\text{num}}(\rho_{i-1},\rho_i) - \phi^{\text{num}}(\rho_i,\rho_{i+1})\right), \tag{2}$$

where ρ_i^+ denotes the traffic density in cell i for the next time step, Δt is the time interval length, and $\phi^{\text{num}}(\rho_-,\rho_+)$ is the numerical flow solving the Riemann problem on the interface having density values ρ_- and ρ_+ from the left and the right, respectively [34]. The Courant-Friedrichs-Lewy (CFL) condition $\frac{v\Delta t}{\Delta x_i} < 1$, with the free flow speed v [km/h], is a sufficient condition for (2) to converge to ρ [34]. To treat the boundary conditions ghost cells are added from the left and from the right and the traffic densities in these cells are computed from the traffic flow measurements using (2).

Although $\phi^{\text{num}}(\rho_-,\rho_+)$ might be difficult to compute in general, in [18] the so-called cell transmission model (CTM), based on the simplified numerical flow formulation

$$\phi^{\text{num}}(\rho_-,\rho_+) \overset{\text{def}}{=} \min\left(v_+\rho_-, w_+\left(\rho_+^{\max} - \rho_+\right)\right), \quad (3)$$

was introduced, with the congestion wave speed w [km/h], and the maximal density $\rho^{\max}$ [veh/km]. See Fig. 1 for the graphical representation of the fundamental diagram.

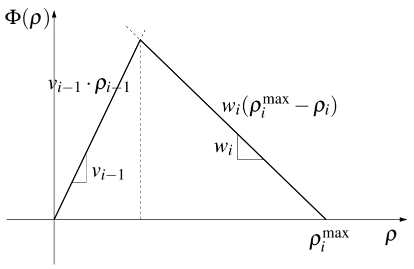

Fig. 1. The CTM fundamental diagram

B. The linear switching model

In this section we present a linear switching model starting from the CTM described in the previous section. This model is similar to those introduced in [31], [36]. To illustrate the derivation, we consider the freeway segment depicted in Fig. 2 and Fig. 3, divided into four cells of the same length. Generalization of the model by adding on-ramps and off-ramps placed at the interfaces between neighboring cells is described in the report [17].

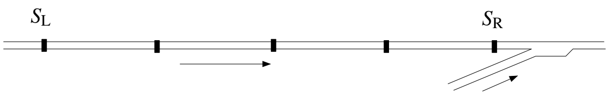

Fig. 2. The considered freeway segment

We call the interface between cells $i-1$ and i (see Fig. 3) free (F) if $v_{i-1}\rho_{i-1} \leq w_i(\rho_i^{\max} - \rho_i)$ and congested (C) if $v_{i-1}\rho_{i-1} > w_i(\rho_i^{\max} - \rho_i)$. If the interface is free, $\phi^{\text{num}}(\rho_{i-1},\rho_i) \overset{\text{def}}{=} v_{i-1}\rho_{i-1}$, if it is congested, $\phi^{\text{num}}(\rho_{i-1},\rho_i) \overset{\text{def}}{=} w_i(\rho_i^{\max} - \rho_i)$. We refer to each of the possible combinations of F and C as the "mode" of the system, while the traffic densities are the "states" of the system.

An alternative approach [31], [36] is to focus on the congestion in the cells instead of the congestion at the cell interfaces. In this approach the cell i is called congested if $\rho_i > \rho_i^{\text{crit}}$, where ρ_i^{crit} is the so-called critical density. If the critical density and the maximum flow and density have the same value for all cells, i.e., $\rho_i^{\text{crit}} = \rho^{\text{crit}}$, $\Phi_i^{\max} = \Phi^{\max}$, $\rho_i^{\max} = \rho^{\max}$, then $\rho^{\text{crit}} = w_i\rho^{\max}/(v_{i-1} + w_i)$. Clearly (see Figure 1) $\rho_i < \rho^{\text{crit}}$ is equivalent to $v_{i-1}\rho_{i-1} < w_i(\rho^{\max} - \rho_i)$ and therefore in such case the definition of congestion via critical cell density [31], [36] is equivalent to the definition of congestion at the cell interfaces introduced in this paper. However, the formulation of the congestion via cell interfaces is more useful for real applications, as quantities usually measured by sensors are speed and traffic flows, which are quantities measured at a point. The placement of the sensors can then coincide with the cell interfaces.

We make the following two assumptions:

Assumption 1: There is only one congestion wavefront within the system in each time step.

Assumption 2: The congestion wave in the considered system propagates from the downstream end.

In fact, relaxation of Assumptions 1 and 2 will extend the number of possible modes of the system, but not the solution methodology proposed in this paper. The possible modes of the system $\mathscr{S} \overset{\text{def}}{=} \{1,\dots,6\}$ are depicted in Table I. Let $x_k \overset{\text{def}}{=} (\rho_1(k),\rho_2(k),\rho_3(k),\rho_4(k))^T$, $u_k \overset{\text{def}}{=} (\rho_L(k),\rho_R(k))^T$, $y_k \overset{\text{def}}{=} (\phi_L(k),\phi_R(k))^T$ and let $c \overset{\text{def}}{=} \frac{\Delta t}{\Delta x}$. From the CFL condition it follows that $vc < 1$ has to be satisfied.

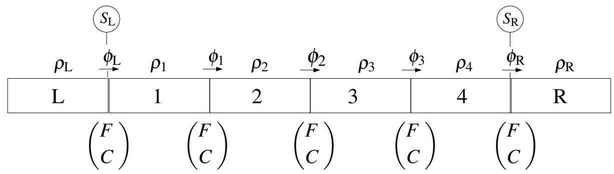

Fig. 3. The considered freeway segment in more detail

The Godunov scheme (2) for each mode $n \in \mathscr{S}$ (see Table I) can be then rewritten as

$$x_{k+1} = A(n)x_k + B(n)u_k + \mu(n), \quad (4)$$

$$y_k = C(n)x_k + D(n)u_k + \psi(n). \quad (5)$$

For example, for $n = 3$ (2) can be rewritten in the form (4)–(5) with $C(3) = 0$ and

$$A(3) = \begin{pmatrix} 1-cv_1 & 0 & 0 & 0 \\ cv_1 & 1-cv_2 & 0 & 0 \\ 0 & cv_2 & 1 & cw_4 \\ 0 & 0 & 0 & 1-cw_4 \end{pmatrix},$$

$$B(3) = \begin{pmatrix} cv_L & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & cw_R \end{pmatrix}, \mu(3) = \begin{pmatrix} 0 \\ 0 \\ -\rho_4^{\max}cw_4 \\ 0 \end{pmatrix},$$

$$D(3) = \begin{pmatrix} v_L & 0 \\ 0 & -w_R \end{pmatrix}, \psi(3) = \begin{pmatrix} 0 \\ w_R\rho_R^{\max} \end{pmatrix}.$$

	interfaces				
mode n	L/1	1/2	2/3	3/4	4/F
1	F	F	F	F	F
2	F	F	F	F	C
3	F	F	F	C	C
4	F	F	C	C	C
5	F	C	C	C	C
6	C	C	C	C	C

TABLE I

THE POSSIBLE MODES n OF THE SYSTEM.

C. The properties of the linear switching model

For each mode the observability properties of the corresponding linear system can be derived (using the Grammian of (4) [37], similarly as it was done in [31] for a piecewise-affine cell transmission model), leading to the following conclusions regarding the model:

- If the freeway segment is free, the system is measurable using the downstream measurements (information propagates downstream at speed v); if the segment is congested, the system is observable using the upstream measurements (information propagates upstream at speed w). Otherwise, the system is unobservable.

Considering the controllability of the considered system by means of on-ramp metering, one can (similarly as with the observability) conclude the following [37]:

- The free section of the segment is controllable from its upstream end, while a congested section is controllable from its downstream end. Without on-ramps the system cannot be controlled by means of on-ramps unless it is fully congested or fully free.

D. The jump Markov linear model

For the linear switching model the mode n is determined by the flow/density condition at the individual cell interfaces. As there is no direct measurement or observation of n, it can be only derived from measured quantities (e.g. traffic speed, traffic density). Therefore, it is reasonable to assume that the mode jumps between possible values following a discrete-time Markov chain with a certain transition probability.Then the proposed linear switching model falls into the class of the jump Markov linear models (JMLM) [38].

III. Estimation objectives

Let $\mathcal{N}(\mu,\Sigma)$ denote the Gaussian distribution with mean μ and covariance Σ. With noise on the measurements and inaccuracies in the fundamental diagram the system (4)–(5) can be rewritten into the JMLM:

$$x_{k+1} = A(r_k)x_k + F(r_k)v_k + B(r_k)u_k + \mu(r_k), \qquad (6)$$

$$y_k = C(r_k)x_k + G(r_k)\varepsilon_k + D(r_k)u_k + \psi(r_k), \qquad (7)$$

where $v_k \sim \mathcal{N}(0,I)$ and $\varepsilon_k \sim \mathcal{N}(0,I)$ are i.i.d. Gaussian sequences and $\{r_s\}_{s=1,\dots,k}$ denotes a discrete time, time-homogeneous, six-state first-order Markov chain with transition probabilities $p_{m,n} \stackrel{\text{def}}{=} \Pr\{r_{k+1} = n | r_k = m\}$, $p_{m,n} \geq 0$, $\sum_{n=1}^{6} p_{m,n} = 1$, for each $m \in \mathcal{S}$. Denote the initial probability distribution as $p_m \stackrel{\text{def}}{=} \Pr\{r_1 = m\}$, for $m \in \mathcal{S}$, such that $p_m \geq 0$ for each $m \in \mathcal{S}$ and $\sum_{m=1}^{6} p_m = 1$. We assume that $x_0 \sim \mathcal{N}(\hat{x}_0, P_0)$, with $\hat{x}_0$ being the estimate of x_0, $P_0 > 0$, and that x_0, v_k, and ε_k are mutually independent for all k. The parameters p_m, p_{mn}, $A(m)$, $B(m)$, $C(m)$, $D(m)$, $F(m)$, $G(m)$, $\hat{x}_0$, P_0 are assumed to be known (for all $m,n \in \mathcal{S}$). Let R denote the set of paths of the finite Markov chain $\{r_s\}_{s=1,\dots,k}$ of non-null prior probability.

Neither the continuous-state process $\{x_s\}_{s=1,\dots,k}$ nor the Markov chain process $\{r_s\}_{s=1,\dots,k}$ are observed - instead, we observe the noisy measurement process $\{y_s\}_{s=1,\dots,k}$.

Let $y_{0:k} \stackrel{\text{def}}{=} \{y_0,\dots,y_k\}$, $r_{0:k} \stackrel{\text{def}}{=} \{r_0,\dots,r_k\}$, $x_{0:k} \stackrel{\text{def}}{=} \{x_0,\dots,x_k\}$. Given the JMLM observations and assuming that the model parameters are known, the Bayesian deduction of the most probable modes and states depends on the joint posterior distribution $p(r_{0:k}, x_{0:k}|y_{0:k}) = p(x_{0:k}|y_{0:k}, r_{0:k})p(r_{0:k}|y_{0:k})$.

We will consider the following two estimation problems:

(P1) Find the distribution $p(r_k, x_k|y_{0:k})$.

(P2) Obtain the Minimum Mean Squared Error (MMSE) estimates of ϕ_k given by $\mathrm{I}(\phi_k) \stackrel{\text{def}}{=} \mathbb{E}_{p(r_k,x_k|y_{0:k})}(\phi_k(r_k,x_k))$,

where ϕ_k is an (r_k,x_k)-dependent mapping, typically defined through the MMSE state estimates $\mathbb{E}(x_k|y_{0:k})$ and $\mathrm{cov}(x_k|y_{0:k})$. Given $r_{0:k}$, the Gaussian distribution $p(x_{0:k}|y_{0:k}, r_{0:k})$ and $\mathrm{I}(\phi_k)$ can be computed using a Kalman filter [22], [25]. In theory, $p(r_{0:k}|y_{0:k})$ could be computed exactly, but this discrete distribution has an exponentially growing number of values in time, thus some approximations/filtrations of unprobable states have to be made.

The problems (P1) and (P2) are known to be NP-hard [24]. The particle filtering is chosen to find a suboptimal solution to the problems (P1) and (P2), as this method was already successfully applied in state estimation problems for jump Markov systems [22]–[26].

IV. Particle-filtering based estimation method

A. Basics

The estimation method used in this paper was introduced in [23]. The goal is to estimate sequentially in time the unknown states (x_k, r_k) and the series of posterior distributions $p(x_{0:k}, r_{0:k}|y_{0:k})$ [39].

If we were able to obtain $N \gg 1$ i.i.d. samples $(r_{0:k}^{(i)}, x_{0:k}^{(i)})$, $i = 1,\dots,N$, distributed according to $p(r_{0:k}, x_{0:k}|y_{0:k})$, then, using the strong law of large numbers, MMSE estimates could be computed by averaging, solving the state estimation problem. However, obtaining such i.i.d. samples from the posterior distribution $p(r_{0:k}, x_{0:k}|y_{0:k})$ is not straightforward. Therefore, alternative sampling schemes have to be used.

A solution to estimate $p(r_{0:k}, x_{0:k}|y_{0:k})$ and $I(\phi_k)$ consists in using the so-called importance sampling [40], which will be explained in the following.

B. Importance sampling

Let us introduce an arbitrary proposal distribution $\pi(r_{0:k}, x_{0:k}|y_{0:k})$, from which it is easy to obtain samples, and so that its support includes the support of $p(r_{0:k}, x_{0:k}|y_{0:k})$, i.e., $p(r_{0:k}, x_{0:k}|y_{0:k}) > 0$ implies $\pi(r_{0:k}, x_{0:k}|y_{0:k}) > 0$. Then

$$I(\phi_k) = \frac{\mathbb{E}_{\pi(r_{0:k},x_{0:k}|y_{0:k})}\left(\phi_k(r_k,x_k)\varsigma(r_{0:k}|x_{0:k})\right)}{\mathbb{E}_{\pi(r_{0:k},x_{0:k}|y_{0:k})}\left(\varsigma(r_{0:k},x_{0:k})\right)},$$

with the importance weight $\varsigma(r_{0:k}|x_{0:k}) = p(r_{0:k}, x_{0:k}|y_{0:k})/\pi(r_{0:k}, x_{0:k}|y_{0:k})$. If we have N i.i.d. importance samples $(r_{0:k}^{(i)}, x_{0:k}^{(i)})$, distributed according to $\pi(r_{0:k}, x_{0:k}|y_{0:k})$, then a Monte Carlo estimate of $I(\phi_k)$ is

$$\hat{I}_N^1 = \frac{\sum_{i=1}^{N} \phi_k\left(r_k^{(i)}, x_k\right)\varsigma\left(r_{0:k}^{(i)}, x_{0:k}^{(i)}\right)}{\sum_{i=1}^{N}\varsigma\left(r_{0:k}^{(i)}, x_{0:k}^{(i)}\right)} = \sum_{i=1}^{N} \tilde{\varsigma}_{0:k}^{(i)}\phi_k\left(r_k^{(i)}, x_k^{(i)}\right),$$

with the normalized importance weights $\tilde{\varsigma}_{0:k}^{(i)} = \varsigma(r_{0:k}^{(i)}, x_{0:k}^{(i)})/\sum_{j=1}^{N}\varsigma(r_{0:k}^{(j)}, x_{0:k}^{j})$. Optimally, $\pi(r_{0:k}, x_{0:k}|y_{0:k}) = p(r_{0:k}, x_{0:k}|y_{0:k})$, which would correspond to $\tilde{\varsigma}_{0:k}^{(i)} = N^{-1}$ for any i. For N finite, $\hat{I}_N^1(\phi_k)$ is biased, but asymptotically $\hat{I}_N(\phi_k)$ converges almost surely towards $I(\phi_k)$. Under not very restrictive additional assumptions [22], a central limit theorem also holds. In order to be able to carry out a recursive evaluation $\varsigma(r_{0:k}) = \varsigma(r_{0:k-1})\varsigma_k$, the importance function should satisfy $\pi(r_{0:k}|y_{0:k}) = \pi(r_0|y_{0:k})\prod_{h=1}^{k}\pi(r_h|y_{0:h}, r_{0:h-1})$. The incremental weight is then given by

$$\varsigma_k = \frac{p(y_k|y_{0:k-1}, r_{0:k})p(r_k|r_{k-1})}{p(y_k|y_{0:k-1})\pi(r_k|y_{0:k}, r_{k-1})} \propto \frac{p(y_k|y_{0:k-1}, r_{0:k})p(r_k|r_{k-1})}{\pi(r_k|y_{0:k}, r_{k-1})},$$

where "$\propto$" means "is proportional to", and the normalized incremental weight is then $\tilde{\varsigma}_k^{(i)} \stackrel{\text{def}}{=} \left[\sum_{j=1}^{N}\varsigma_k^{(j)}\right]^{-1}\varsigma_k^{(i)}$. The optimal sampling distribution satisfies $p(r_k = m|r_{0:k-1}, y_{0:k}) = p(y_k|y_{0:k-1}, r_{0:k-1}, r_k = m)p(r_k = m|r_{k-1})/p(y_k|y_{0:k-1}, r_{0:k-1})$ and the associated importance weight ς_k is proportional to $p(y_k|y_{0:k-1}, r_{0:k-1}) = \sum_{m=1}^{6} p(y_k|y_{0:k-1}, r_{0:k-1}, r_k =$

$m) \cdot p(r_k = m|r_{k-1})$. Computing $p(y_k|y_{0:k-1}, r_{0:k-1})$ requires the evaluation of six one-step-ahead Kalman filter steps.

As the unconditional variance (i.e., with the observation $y_{0:k}$ being interpreted as random variables) of the importance weights $\varsigma(r_{0:k})$ increases over time [22], it is impossible to avoid the degeneracy phenomenon. That is why we introduce a selection step in the algorithm to discard the particles $r_{0:k}^{(i)}$ with low normalized importance weights $\tilde{\varsigma}(r_{0:k}^{(i)})$ and to multiply the ones with high $\tilde{\varsigma}(r_{0:k}^{(i)})$. Each time a selection step is used the weights are reset to N^{-1}.

We want to obtain N particles $\{x_{0:k}^{(i)}, r_{0:k}^{(i)}\}$ distributed according to $p(x_{0:k}, r_{0:k}|y_{0:k})$. At time k, we extend each $(\tilde{x}_{0:k-1}^{(i)}, \tilde{r}_{0:k-1}^{(i)})$ by $N_i \in \mathbb{N}$ $(\sum_i N_i = N)$ "children" $(x_{0:k}^{(i)}, r_{0:k}^{(i)})$ according to the proposal distribution π to obtain N new particles. Here $x_{0:k}^{(i)} \stackrel{\text{def}}{=} \{\tilde{x}_{0:k-1}^{(i)}, x_k^{(i)}\}$, $r_{0:k}^{(i)} \stackrel{\text{def}}{=} \{\tilde{r}_{0:k-1}^{(i)}, r_k^{(i)}\}$. If $N_i = 0$, then $\tilde{r}_{0:k}^{(i)}$ is discarded; otherwise, it has N_i "children". If we use a selection scheme at each time step, we have a weighted distribution $\tilde{p}_N(r_{0:k}|y_{0:k}) = \sum_{i=1}^{N} \tilde{\varsigma}_k^{(i)} \delta_{\tilde{r}_{0:k}^{(i)}}(\mathrm{d}r_{0:k})$.

We use the sampling importance resampling [27] as the selection step. One samples N times from $\tilde{p}_N(r_{0:k}|y_{0:k})$ to obtain $(r_{0:k}^{(i)}; i = 1, \ldots, N)$. This is equivalent to drawing jointly $(N_i; i = 1, \ldots, N)$ according to a multinomial distribution with parameters N and $\tilde{\varsigma}_k^{(i)}$.

C. The model implementation

For one step, the particle-filtering-based method reads as:

Initialization (time $k-1$)

given: $N \in \mathbb{N}^*$ random samples $(r_{0:k-1}^{(i)}; i = 1, \ldots, N)$

Step 1 (time k): Sequential Importance Sampling Step

for $i = 1, \ldots, N$
    sample $\tilde{r}_k^{(i)} \sim \pi(r_k|y_{0:k}, r_{0:k-1}^{(i)})$;
    set $\tilde{r}_{0:k}^{(i)} \stackrel{\text{def}}{=} \left(r_{0:k-1}^{(i)}, \tilde{r}_k^{(i)}\right)$;
end for;

for $i = 1, \ldots, N$
    evaluate the importance weights up to normalizing constant

$$\varsigma_k^{(i)} \propto \frac{p\left(y_k|y_{0:k-1}, \tilde{r}_{0:k}^{(i)}\right) p\left(\tilde{r}_k^{(i)}|\tilde{r}_{k-1}^{(i)}\right)}{\pi\left(\tilde{r}_k^{(i)}|y_{0:k}, \tilde{r}_{0:k-1}^{(i)}\right)};$$

end for;
for $i = 1, \ldots, N$
    normalize the importance weights

$$\tilde{\varsigma}_k^{(i)} = \left[\sum_{j=1}^{N} \varsigma_k^{(j)}\right]^{-1} \varsigma_k^{(i)};$$

end for;

Step 2 (time k): Selection Step

Copy/Discard particles $(\tilde{r}_{0:k}^{(i)}; i = 1, \ldots, N)$ with respect to high/low normalized importance weights $\tilde{\varsigma}_k^{(i)}$ to obtain N particles $\left(r_{0:k}^{(i)}; i = 1, \ldots, N\right)$.

The complexity of this algorithm at each iteration is $O(N)$ [23].

V. CASE STUDIES

In this section we will empirically test whether in the traffic density estimation for the freeway segment depicted in Fig. 3 the CTM can be replaced by the linear switching model derived in Section II-B. We will also validate the model with the use of microsimulation. Finally, we will perform the particle filtering density estimation for the JMLM proposed in Section II-D. Both model and algorithm were implemented in Matlab.

A. Case study 1: Practical relevance of the proposed model

We consider the freeway segment depicted in Fig. 2, with initial data $v = v_i = 150$ [km/h], $w_i = w = 35$ [km/h] ($i \in \{\mathrm{L}, \mathrm{R}, 1, 2, 3, 4\}$), $\rho^{\max} = 250$ [veh/km], $\Delta x = 0.5$ [km], $\Delta t = 10$ [s]. The purpose of this case study is to observe whether the considered system with noisy boundary conditions stays within the set of possible modes $\mathscr{S}$. Therefore, we will run the CTM with various (but known) boundary traffic densities and we will observe whether during the computations the system does not deviate from behavior corresponding to the modes from $\mathscr{S}$. For computation of initial boundary densities ϕ_{L} and ϕ_{R} we used real-time traffic flow and traffic speed measurements from a three-lane segment of the D383 (Boulevard Laurent Bonnevay) freeway in Lyon on a working day. One of these measurements - the downstream traffic flow - is depicted in Fig. 4. We run the CTM 10000 times with boundary densities set to $\rho_{\mathrm{L}} + \varepsilon_{\mathrm{L}}$, $\rho_{\mathrm{R}} + \varepsilon_{\mathrm{R}}$, $\varepsilon_{\mathrm{L}}, \varepsilon_{\mathrm{R}} \in \mathscr{N}(0, I)$.

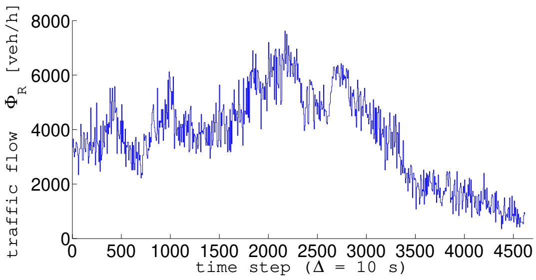

Fig. 4. The traffic flow measured at the downstream boundary of the considered freeway segment

We observed that for all tested scenarios the system modes stayed within $\mathscr{S}$. Additionally, the mode switches for two consequent time steps follow the pattern depicted by the finite automaton in Fig. 5. Here the double circle represents possible finite mode, i.e., each of the modes can be the end mode. The mode 1 is always initial state. From this case study we also computed the probability $p_{m,n}$ of the switch from the mode n to the mode m. This probability will be used as an input for Case study 3 (Section V-C).

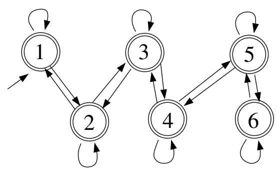

Fig. 5. The representation of the mode switch dynamics by a finite automaton

B. Case study 2: Validation & performance of the proposed model with respect to microsimulation

The next step was to validate the model parameters as well as to compare the performance of the linear switching model with respect to the performance of the traffic microsimulator Aimsun 6. The network in Fig. 2 was considered, with $\Delta x = 0.5$ [km] and $\Delta t = 10$ [s]. Before the computational part of the experiment a one hour long warm-up took place. The following parameters were adopted:

- main road mean traffic flow [veh/h]: 1270 for the first three hours, 1000 for the fourth hour, 800 for the fifth hour, 0 for the sixth hour,

- on-ramp mean traffic flow [veh/h]: 1000 for the first three hours, 800 for the fourth hour, 600 for the fifth hour, 0 for the sixth hour.

The default microsimulation road and vehicle parameters of Aimsun 6 were used (see [17] for their values). From the traffic densities and flows measured by the sensors the fundamental diagram for each of the interfaces could be reconstructed. The aim was to obtain a unified fundamental diagram for whole segment. The parameters v and w were computed by the L_2-approximation of the collected data. Subsequently the linear switching model was run, with the boundary flows and the fundamental diagram obtained from the microsimulation as inputs. The density results of the linear switching model were compared to those obtained by the traffic simulator Aimsun 6.

Fig. 6–7 show the comparison of the resulting traffic densities of the linear switching model with the measurements of the sensors within the microsimulator Aimsun 6.

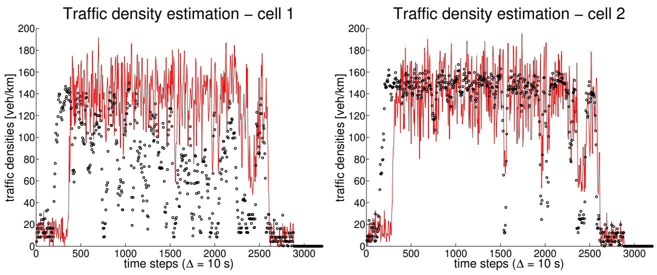

Fig. 6. Comparison of the traffic densities in the first (left) and the second cell (right) computed by the linear switching model (black dots) with the traffic densities measured by the sensors (red solid line)

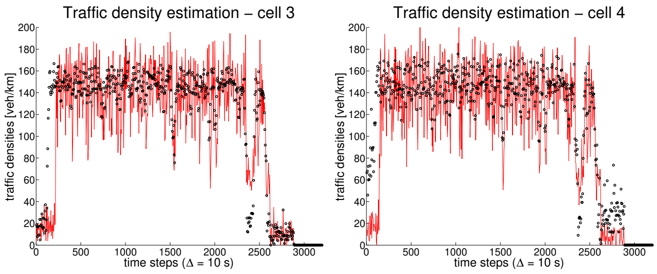

Fig. 7. Comparison of the traffic densities in the third (left) and the forth cell (right) computed by the linear switching model (black dots) with the traffic densities measured by the sensors (red solid line)

One can see that considering the inaccuracies caused by the fundamental diagram approximation as well as the inaccuracies caused by the use of a limited number of possible modes the linear switching model performs quite well, except for the density estimation in the first cell, where the density is highly underestimated. The density obtained by the linear switching model has a lower variance than the density obtained by the microsimulation. We observed that the mode switch during the experiment again followed the behavior depicted by Fig. 5.

C. Case study 3: Particle filtering traffic density estimation

We performed the algorithm introduced in Section IV using the JMLM that we derived in Section II, with the network from Fig. 3. The parameters v, w, $\rho^{\max}$, Δx, Δt, were set as in Section V-C, initial $p_{m,n}$ was obtained in Case study 1. The boundary data were simulated using the traffic microsimulator Aimsun 6, computed from traffic flows with means defined as ϕ_L, ϕ_R in Case study 1. The proposal distribution π assigns equal probability to each of the states for each time step and $p_{m,n}$ was computed as explained in Case study 1.

In Fig. 8 and 9 one can see the results of the density estimation for one of the cells with the number of the sequences $r_{0:k}$ for each k restricted to $N = 500$ and to $N = 1000$, respectively. The solid blue line represents the real density, while the dotted red line represents the estimate computed by particle filtering.

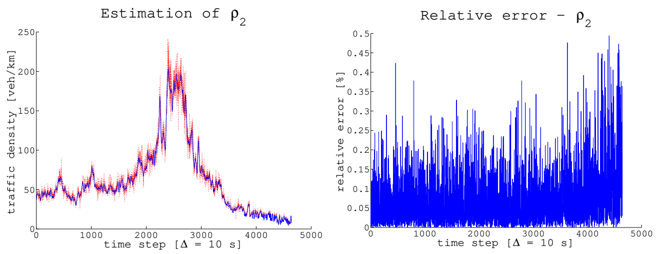

Fig. 8. Estimation of ρ_2 with $N = 500$ (left, red dotted line) with respect to real ρ_2 (left, blue solid line) and relative error of this estimation (right)

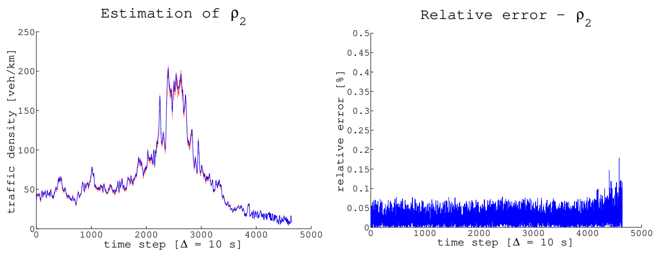

Fig. 9. Estimation of ρ_2 with $N = 1000$ (left, red dotted line) with respect to the real ρ_2 (left, blue solid line) and relative error of this estimation (right)

The estimation results are very promising, if N is sufficiently high, even without optimally chosen π. The computation time of the algorithm is proportional to N. However, the algorithm can be run using parallel processing.

VI. Discussion

A. Discussion about the chosen models

The characteristics of the CTM can be listed as follows:

- The model is nonlinear and, therefore, more complex control/estimation measures are needed.
+ The model is identified automatically.

The characteristics of the JMLM can be listed as follows:

- The model restricts the possible congestion wavefront behavior. This might have serious drawbacks when unexpected behavior (e.g. accident) occurs in the network.
+ If the congestion status of the interfaces is known linear control/estimation methods can be applied.

B. Discussion about the chosen method

- While for the proposed case study the computation time is reasonable (in the range of minutes), for more general networks the computation time may become high.
- It is a suboptimal method.
- It is necessary to know the transition probabilities as well as initial probabilities of individual modes.
+ The method performs well even with biased boundary data.
+ Extension of the set of modes does not change the solution method.
+ The proposed algorithm can be parallelized [22]; this is promising for online estimation.

VII. Conclusions & Future Research

In this paper we have proposed a particle-filtering-based method for traffic density estimation/prediction for Daganzo's cell transmission model-based jump Markov linear system. The sampling importance resampling method was used as the selection step.

We have focused on the system properties as well as on the performance of the proposed estimation method, which turned out to perform very well. As the proposed algorithm can be parallelized, the proposed method seems to be a promising tool for the real-time traffic density estimation. Validation of this claim will be a topic for future work.

For the sake of simplicity all derivations and case studies were done for a freeway segment without on-ramps and off-ramps. Additionally, it was assumed that maximally one congestion wavefront occurs in the segment and that the congestion always proceeds upstream. Although for the case study set-up this congestion behavior was verified for a specific set of real input data, such assumptions do not hold in general. Extension of the model to the one in which all possible congestion modes that may occur are included is straightforward. However, additional research is needed to study possible improvements of the proposed particle filtering method with respect to performance in time, variance, etc., when applied to the more general problems.

Acknowledgements

The authors thank dr. Nils van Velzen from the Delft Institute of Applied Mathematics, Technical University Delft, The Netherlands, for numerous discussions about data assimilation methods that led to the choice of the solution method presented in this paper.

References

[1] C. Chen, Z. Jia, and P. Varaiya, "Causes and cures of highway congestion," *IEEE Control Systems Magazine*, vol. 21, no. 6, pp. 26–32, 2001.

[2] A. Hegyi, B. De Schutter, and J. Hellendoorn, "Optimal coordination of variable speed limits to suppress shock waves," *IEEE Transactions on Intelligent Transportation Systems*, vol. 6, no. 1, pp. 102–112, 2005.

[3] L. Alvarez, R. Horowitz, and C. V. Toy, "Multi-destination traffic flow control in automated highway systems," *Transportation Research Part C: Emerging Technologies*, vol. 11, no. 1, pp. 1–28, 2003.

[4] S. P. Hoogendoorn, B. De Schutter, and H. Schuurman, "Decision support in dynamic traffic management. Real-time scenario evaluation," *European Journal of Transport and Infrastructure Research*, vol. 3, no. 1, pp. 21–38, 2003.

[5] R. Boel and L. Mihaylova, "A compositional stochastic model for real-time freeway traffic simulation," *Transportation Research B*, vol. 40, no. 4, pp. 319–334, 2006.

[6] G. Gomes and R. Horowitz, "Optimal freeway ramp metering using the asymmetric cell transmission model," *Transportation Research Part C*, vol. 14, no. 4, pp. 244–262, 2006.

[7] T. L. Friesz, C. Kwon, and R. Mookherjee, "A computable theory of dynamic congestion pricing," in *Transportation and Traffic Theory*, R. Allsop, M. Bell, and B. Heydecker, Eds. Amsterdam, The Netherlands: Elsevier, 2007, pp. 1–26.

[8] L. Mihaylova, R. Boel, and A. Hegyi, "Freeway traffic estimation within particle filtering framework," *Automatica*, vol. 43, no. 2, pp. 290–300, 2007.

[9] I. Jonsson, H. Harris, and C. Nass, "How accurate must an in-car information system be?: consequences of accurate and inaccurate information in cars," in *Proceedings of the Twenty-Sixth Annual SIGCHI Conference on Human Factors in Computing Systems*, Florence, Italy, 2008, pp. 1665–1674.

[10] A. Muralidharan and R. Horowitz, "Imputation of ramp flow data for freeway traffic simulation," *Transportation Research Record*, no. 2099, pp. 58–64, 2009.

[11] K. Staňková, G. J. Olsder, and M. C. J. Bliemer, "Comparison of different toll policies in the dynamic second-best optimal toll design problem: Case study on a three-link network," *European Journal of Transport and Infrastructure Research*, vol. 9, no. 4, pp. 331–346, 2009.

[12] A. D. May, *Traffic Flow Fundamentals*. Upper Saddle River, NJ: Prentice-Hall, Inc., 1990.

[13] M. Papageorgiou, C. Diakaki, V. Dinopoulou, A. Kotsialos, and Y. Wang, "Review of road traffic control strategies," *Proceedings of the IEEE*, vol. 91, no. 12, pp. 2043–2067, 2003.

[14] J. K. Tugnait, "Adaptive estimation and identification for discrete systems with Markov jump parameters," *IEEE Transactions on Automatic Control*, vol. 27, no. 5, pp. 1054–1065, 1982.

[15] Y. Bar-Shalom, *Multitarget-Multisensor Tracking: Principles & Techniques*. YBS Publishing, 1995.

[16] E. Mazor, A. Averbuck, Y. Bar-Shalom, and J. Dayan, "Interacting multiple model methods in target tracking: A survey," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 34, no. 1, pp. 103–123, 1998.

[17] K. Staňková, "On traffic state estimation for an LWR-based highway traffic model," INRIA, Grenoble, France, Tech. Rep., February 2010.

[18] C. F. Daganzo, "The cell transmission model: A dynamic representation of highway traffic consistent with the hydrodynamic theory," *Transportation Research B*, vol. 28, no. 4, pp. 269–287, 1994.

[19] M. J. Lighthill and G. B. Whitham, "On kinematic waves II: A theory of traffic flow on long crowded roads," *Proceedings of the Royal Society of London. Series A, Mathematical and Physical Sciences*, vol. 229, no. 1178, pp. 317–345, 1955.

[20] P. I. Richards, "Shock Waves on the Highway," *Operations Research*, vol. 4, no. 1, pp. 42–51, 1956.

[21] G. Gomes, R. Horowitz, A. A. Kurzhanskiy, P. Varaiya, and J. Kwon, "Behavior of the cell transmission model and effectiveness of ramp metering," *Transportation Research C*, vol. 16, pp. 485–513, 2008.

[22] A. Doucet, N. J. Gordon, and V. Krishnamurthy, "Particle filters for state estimation of jump Markov linear systems," *IEEE Transactions on Signal Processing*, vol. 49, no. 3, pp. 613–624, 2001.

[23] A. Doucet and C. Andrieu, "Iterative algorithms for state estimation of jump Markov linear systems," *IEEE Transactions on Signal Processing*, vol. 49, no. 6, pp. 1216–1227, 2001.

[24] M. S. Arulampalam, S. Maskell, N. Gordon, and T. Clapp, "A tutorial on particle filters for online nonlinear/non-Gaussian Bayesian tracking," *IEEE Transactions on Signal Processing*, vol. 50, no. 2, pp. 174–188, 2002.

[25] B. Ristic, S. Arulampalam, and N. Gordon, *Beyond the Kalman Filter: Particle Filters for Tracking Applications*. London, UK: Artech House Publishers, 2004.

[26] H. Driessen and Y. Boers, "Efficient particle filter for jump Markov nonlinear systems," *IEE Proceedings Radar, Sonar and Navigation*, vol. 152, no. 5, pp. 323–326, 2005.

[27] N. J. Gordon, D. J. Salmond, and A. F. M. Smith, "Novel approach to nonlinear/non-Gaussian Bayesian state estimation," *IEE Proceedings - Part F: Radar & Signal Processing*, vol. 140, no. 2, pp. 107–113, 1993.

[28] N. J. Gordon, "A hybrid bootstrap filter for target tracking in clutter," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 33, no. 1, pp. 353–358, 1997.

[29] M. Khalaf-Allah, "Nonparametric Bayesian filtering for location estimation, position tracking, and global localization of mobile terminals in outdoor wireless environments," *EURASIP Journal on Advances in Signal Processing*, vol. 2008, 2008, article ID: 317252, 14 p.

[30] X. Sun, L. Muñoz, and R. Horowitz, "Highway traffic state estimation using improved mixture Kalman filters for effective ramp metering control," in *Proceedings of the 42nd IEEE Conference on Decision and Control*, Maui, Hawaii, 2003.

[31] L. Muñoz, X. Sun, R. Horowitz, and L. Alvarez, "A piecewise-linearized cell transmission model and parameter calibration methodology," in *Proceedings of the 85th Annual Meeting of the Transportation Research Board (TRB)*, Washington D.C., 2006.

[32] J. Stadler and S. Roy, "Adaptive importance sampling," *IEEE Journal on Selected Areas in Communications*, vol. 11, no. 3, pp. 309–316, 1993.

[33] R. Ansorge, "What does the entropy condition mean in traffic flow theory?" *Transportation Research B*, vol. 24, no. 2, pp. 133–143, 1990.

[34] L. Veque, *Numerical Methods for Conservation Laws*. Birkhäuser, 1992.

[35] S. Godunov, "A difference scheme for numerical solution of discontinuous solution of hydrodynamic equations," *Matematicheskii Sbornik*, vol. 47, pp. 271–306, 1969.

[36] A. Aligawesa and I. Hwang, "Hybrid system's model and algorithm for highway traffic monitoring," in *Proceedings of the 2010 American Control Conference*, Baltimore, MD, 2010, pp. 2254–2259.

[37] F. M. Callier and C. A. Desoer, *Linear System Theory*. Springer, 1991.

[38] E. Pardoux, *Markov Processes and Applications: Algorithms, Networks, Genome and Finance*. Wiley, 2008.

[39] C. Andrieu, M. Davy, and A. Doucet, "Efficient particle filtering for jump Markov systems. Application to time-varying autoregressions," *IEEE Transactions on Signal Processing*, vol. 51, no. 7, pp. 1762–1770, 2003.

[40] J. M. Bernardo and A. F. M. Smith, *Bayesian Theory*. New York: Wiley, 1994.