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Multiobjective Model Predictive Control Applied to a Dial-a-Ride System

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Abstract.
A multiobjective model-based predictive control approach is presented for solving a dial-a-ride problem. The dynamic objective function considers two dimensions: user and operator costs. As those two components are usually aimed at opposite goals, the problem is formulated and solved through multiobjective model predictive control.

When a new call asking for service is received, the method first solves a multiobjective optimization problem, providing the Pareto optimal set. Note that from this set just one solution has to be applied to the system. Then, the dispatcher participates in the dynamic routing decisions by expressing his/her preferences in a progressively interactive way, seeking the best trade-off solution at each instant among the Pareto optimal set. The idea is to provide to the dispatcher a more transparent tool for the decisions. Several criteria, emulating different dispatchers, are proposed in order to systematize different ways to use the information provided by the dynamic optimal Pareto front.

An illustrative experiment of the new approach through simulation of the process is presented to show the potential benefits in the operator cost and in the quality of service perceived by the users.

1 Introduction

The dynamic pickup and delivery problem (DPDP) considers a set of online requests of service for passengers traveling from an origin (pickup) to a destination (delivery) served by a fleet of vehicles initially located at several depots (Desrosiers et al., 1986; Savelsbergh and Sol, 1995). The final output of such a problem is a set of routes for the fleet, that dynamically change over time and are required in real-time. Progress in communication and information technologies has allowed researchers to formulate such dynamic problems and to develop efficient algorithms of high computational complexity to solve them. The dynamic pickup and delivery problem (DPDP) designed to operate dial-a-ride systems has been intensely studied in the last decades. A recent and complete review of dynamic pickup and delivery problems can be found in Berbeglia et al. (2009), where general issues as well as solution strategies are described. They conclude that is necessary to develop more studies on policy analysis associated with dynamic many-to-many pickup and delivery problems.

A well-defined DPDP should be based on an objective function that includes prediction of future demands and traffic conditions in current routing decisions. In some previous works (Sáez et al., 2008; Cortés et al., 2008, 2009) we have proposed an analytical formulation for the DPDP as a model-based predictive control problem using state space models. The proposed global optimization problem was large and NP-hard, so, the use of evolutionary algorithms was considered (GA, PSO, and Fuzzy Clustering). However, in those works the trade-off between users' level of service and the associated extra operational costs was completely unknown for the dispatcher. Moreover, some issues regarding users' level of service like postponed users (experiencing very long travel or waiting times) were not considered.

In real life implementations of DPDP the quality of service is very important. In Paquette et al. (2009), the authors conclude that most dial-a-ride studies are focused on the minimization of operational costs, and that it is necessary to develop more studies on user policies. We must notice that these two dimensions represent opposite objectives. On the one hand, the users want to obtain good service, implying more direct trips, resulting in lower vehicle occupancy rates and consequently, higher operational costs to satisfy the same demand, for a fixed fleet. More efficient routing policies from the operator's standpoint will reflect higher occupation rates, longer routes, and consequently, longer waiting and travel time for users. Thus, the question is how to properly balance both components in the objective function to make proper planning and fleet dispatching decisions. To guide the decision maker in this context, we propose the use of a multiobjective model-based predictive control approach for solving the dial-a-ride problem. The dispatcher must express his/her preferences (criterion) in a progressive way (interactively), seeking the best-compromise solution from the dynamic Pareto set. The performance of the system will be related with the criterion used. Multiobjective optimization has been applied to a large number of static vehicle routing problems. For a comprehensive review of multiobjective vehicle routing problems the interested reader is referred to Jozefowiez et al. (2008). As far as we know, all the multiobjective applications in vehicle routing problems are evaluated in static scenarios, one of the aims of this paper being to contribute in the analysis of using multiobjective in dynamic and stochastic environments.

The methods for multiobjective optimization can be classified into groups. The most common are the methods based on (a priori) transformation into scalar objective. Those methods are too rigid in the sense that changes in the preference of the decision-maker cannot easily be considered. Among those methods we can highlight formulations based on prioritizations (Kerrigan et al., 2000; Kerrigan and Maciejowski, 2003; Núñez et al., 2009); based on a goal attainment method (Zambrano and Camacho, 2002); and the most typical used in the literature of model predictive control is the weighted-sum strategy. Recently, Bemporad and Muñoz (2009) provide stability conditions for selecting dynamic

Pareto optimal solutions, using a weighted-sum based method. The other methods are based on the generation and selection of Pareto optimal points. The main drawback in those methods is the fact that to obtain the solution set from the multiobjective problem requires a big computational effort. We claim that new toolboxes for Evolutionary Computation and other efficient algorithms have been developed in recent years, so it is possible to determine a good representative pseudo-optimal Pareto set in a dynamic context. The method used in this paper belongs to this second family of solutions. We will be "users" of an ad-hoc multiobjective optimization algorithm, the main idea being to show the details of a suitable tool for dispatchers that allows making decisions in a more transparent way. The use of MO allows the decision-maker obtaining solutions that are not explored with the typical mono-objective model predictive control (MPC) scheme. This extra information is a crucial support for the decision-maker who is finally looking for reasonable options of service policies not only for users but also for operators.

In the next section the multiobjective model predictive control approach is presented. Next, the dial-a-ride problem, including the model, the objective functions and MO-HPC methods are discussed. Then, simulation results are shown and analyzed. Finally conclusions and future work are highlighted.

2 Multiobjective model-based predictive control (MO-MPC)

2.1 Multiobjective model predictive control

Model predictive control (MPC) involves a family of controllers whose main ingredients are: 1) the use of a predictive model over a prediction horizon, 2) computation of a sequence of future control actions through the optimization of an objective function, considering operation constraints and desired behavior of the system, 3) the use of a rolling horizon strategy, i.e., the optimization process is repeated at each sampling instant, and the first action in the obtained control sequence is applied (Camacho and Bordons, 1999). Multiobjective in MPC (MO-MPC) is a generalization of MPC, where instead of minimizing a single-objective function, we consider more performance indices (Bemporad and Muñoz, 2009). Consider for example the process modeled by the following non-linear discrete-time system:

$$x(k+1) = f(x(k), u(k)) \,, \tag{1}$$

where $x(k) \in \mathbb{R}^n$ is the state vector, $u(k) \in \mathbb{R}^m$ is the input vector, and $k \in \mathbb{N}$ denotes the time step. In MO-MPC, if the process modeled by (1) has conflicts, i.e., a solution that optimizes one objective may not optimize others, the following multiobjective problem is solved:

$$\begin{aligned}
&\min_{U} J(U, x_k) \\
&\text{subject to} \\
&x(k+\ell+1) = f(x(k+\ell), u(k+\ell)), \qquad \ell = 0, 1, \ldots, N-1 \\
&x(k) = x_k, \\
&x(k+\ell) \in \mathbf{X}, \qquad \ell = 1, 2, \ldots, N \\
&u(k+\ell) \in \mathbf{U}, \qquad \ell = 0, 1, \ldots, N-1,
\end{aligned} \tag{2}$$

where $U = \left[u^T(k), \ldots, u^T(k+N-1)\right]^T$ is the sequence of future control actions, $J(U, x_k) = [J_1(U, x_k), \ldots, J_l(U, x_k)]^T$ is a vector-valued function with the l objectives to minimize, N is the prediction horizon, $x(k+\ell)$ is the ℓ-step-ahead predicted state from the initial state x_k. The state as well as the inputs are constrained to X and $\mathbf{U}$. The solution of MO-MPC problem is a set of control action sequences called the Pareto optimal set.

Next we define Pareto optimality. Consider a feasible control sequence $U^P = \left[u_P^T(k), \ldots, u_P^T(k+N-1)\right]^T$. The sequence U^P is said to be Pareto optimal if and only if there does not exist another feasible control action sequence U such that:

1. $J_i(U, x_k) \leqslant J_i\left(U^P, x_k\right)$, for $i = 1, \ldots, l$.
2. $J_j(U, x_k) < J_j\left(U^P, x_k\right)$, for at least one $j \in \{1, \ldots, l\}$.

The Pareto optimal set P_S contains all Pareto optimal solutions. The set of all objective function values corresponding to the Pareto optimal solutions is known as the Pareto optimal front $P_F = \left\{[J_1(U), \ldots, J_l(U)]^T : U \in P_S\right\}$.

Among the algorithms to solve this kind of problem, we can mention conventional methods like the ones based on decomposition, weighting, etc (Haimes et al., 1990). Also, nowadays, there is an important interest in evolutionary multiobjective optimization algorithms, and many researchers are working on more efficient algorithms (for example Durillo et al., 2010, just to mention one very recent work). In this paper, we use an ad-hoc branch-and-bound based algorithm for the simulation results to measure the benefits of the approach (independently of the algorithm used).

From the set of the optimal control solutions, just the first component $u(k)$ of one of those solutions has to be applied to the system, so at every instant, the controller (dispatcher in the context of a dial-a-ride system) has to use a criterion in order to find the control sequence that better suits the current objectives. In this paper, that decision is obtained after the Pareto set is determined. Then, it is not possible to choose a priori some weighting factor and to solve a single-objective optimization problem. The idea is to provide to the dispatcher a more transparent tool for the decisions.

In the context of solving a dial-a-ride problem the MO-MPC is dynamic, meaning that real-time decisions related to a service policy are made as the system progresses; for example, the dispatcher could minimize the operational costs J_2 keeping a minimum acceptable level of service for users (through J_1) when deciding a vehicle-user assignment. Nevertheless, this tool could be implemented as a reference to support the dispatcher decision, which offers the flexibility of deciding which criterion is more adequate. In this kind of problems, MO-MPC suits very well, as it helps the dispatcher to choose a solution to be applied, considering the trade-off between Pareto optimal solutions. Two criteria that could be used in this context are explained next.

2.2 Dispatcher criteria

Once the MO-MPC problem (2) is solved, there are many ways to choose one solution from the Pareto set. In this section, we will explain two criteria that could be used, and their advantages.

2.2.1 Criterion based on weighted-sum

The weighted-sum is the method most used for multiobjective optimization (Haimes et al., 1990). The idea is to transform into a scalar objective the multiobjective optimization. There are three main problems. First, it requires the choice of the appropriate weighting coefficients (a-priori). Second, not all Pareto optimal solutions are accessible by appropriate selection of weights. Finally, when there are multiple solutions most of the optimization algorithm will converge to just one those solutions. We propose as an option for MO-MPC the use the weighted-sum method after the Pareto set is obtained.

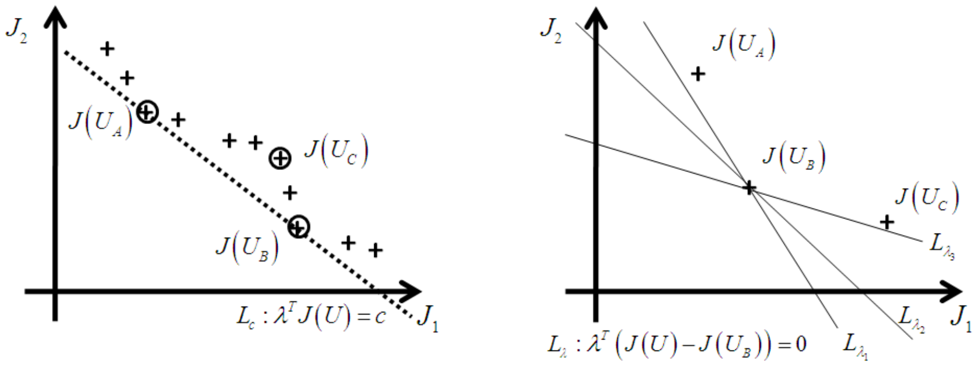

Figure 1: a) Some Pareto optimal points, b) In discrete systems, a Pareto optimal solution minimizes a set of scalar linear weighted functions.

This criterion based on weighted-sum consists of the minimization of the scalar objective function $\lambda^T J(U, x_k)$, where the solution U belongs to the Pareto set (2).

Some advantages of the application of this criterion after obtaining the Pareto set are the following:

- Multiple solutions for a given weighting vector are available to the dispatcher. For example, in Figure 1a, U_A and U_B are Pareto optimal solutions, where $J_1(U_A) < J_1(U_B)$, and $J_2(U_A) > J_2(U_B)$, but also both solutions minimize $\lambda^T J(U, x_k)$.
- When dealing with discrete inputs, a Pareto solution minimizes a set of optimization problems $\lambda^T J(U, x_k)$ with different weights. In Figure 1b, the Pareto optimal solution U_B minimizes for example the optimization problems: ${\lambda_1}^T J(U, x_k)$, ${\lambda_2}^T J(U, x_k)$, ${\lambda_3}^T J(U, x_k)$, etc. With the complete information of the Pareto set, it is possible to change the control sequence to one of the consecutive Pareto solutions U_A or U_C, without guessing the proper weighting factor from a single-objective optimization.

2.2.2 Criterion based on ε-constraint method

The ε-constraint method permits for generating Pareto optimal solutions by making use of a single-objective function optimizer that handles constraints, to generate one point of the Pareto front at a time (Haimes et al., 1990). This method minimizes a primary objective $J_p(U)$ and expresses other objectives as inequality constraints $J_i(U) \leqslant \varepsilon_i$, $i = 1, \ldots, l$ with $i \neq p$. An issue for this method is the suitable selection of ε, for example, if ε is too small, then maybe no feasible solution to be found. Another issue arises when hard constraints are used, requiring a detailed design and knowledge about the different operational points of the process.

We propose as an option for MO-MPC the criterion based on the ε-constraint method that will be used after the Pareto set is obtained. In Figure 2a, given the hard constraint $J_1(U) \leqslant \varepsilon_1$, the Pareto solution that minimizes $J_2(U)$ is shown. In Figure 2b, there is not a Pareto solution satisfying the hard constraint, so, the closer solution to that criterion could be selected. With the Pareto set information, the dispatcher can change the hard constraints and adjust them according to the current conditions of the systems.

In the next section, we provide the details with regard to the implementation of these techniques to a dial-a-ride system.

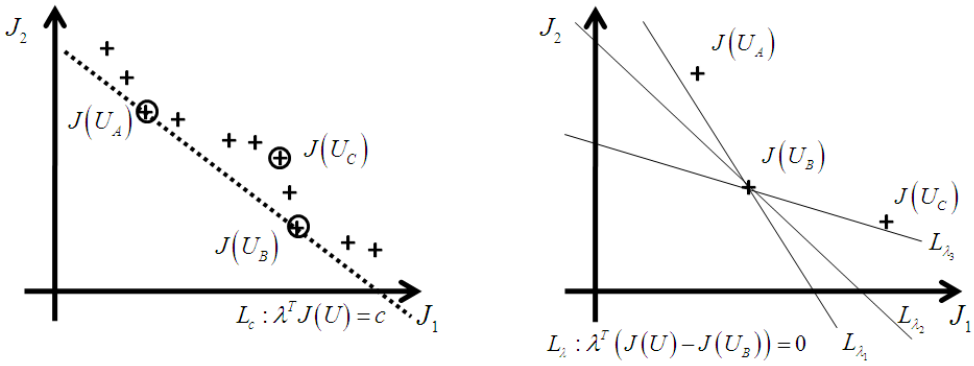

Figure 2: Criterion based on the ε-constraint method, a) feasible solution is found, b) no Pareto solution satisfying the constraint.

3 Dynamic pickup and delivery problem

3.1 Process description

Dial-a-ride systems are transit services that provide a share-ride door-to-door service with flexible routes and schedules. The quality of service of a dial-a-ride is supposed to be in between of public transit buses and taxis. The typical specifications are the users' pickup and delivery destinations and desired pickup or delivery times. We will assume that all the requests are known only after the dispatcher receives the associated call and that all the users want to be served as soon as possible. Thus, even if we will not include explicitly hard time windows, to provide a good service we propose a user-oriented objective function that deals with the problem of undesired assignments to clients, and keeps the service provided as regular (stable) as possible.

The service demand η_k comprises the information of the request and is characterized by two positions, pickup p_k and delivery d_k, the instant of the call occurrence t_k, a label r_k that identifies the passenger who is calling, and the number of passengers waiting there Ω_k. Also we consider the expected minimum arrival time tr_k, which is the best possible time to serve the passenger considering a straight journey from origin to destination (like a taxi service), and a waiting time obtained with the closest available vehicle (in terms of capacity) to pick up that passenger.

We assume a fixed and known fleet size F over an urban area A. The specific characteristics of a request are known only after the associated call is received by the dispatcher. A selected vehicle is then rerouted in real-time to insert the new request into predefined route of the vehicle while the vehicle stays in motion. The assignment of the vehicle and the insertion position of the new request into the previous sequence of tasks associated with such a vehicle are control actions decided in real time by the dispatcher (controller) based on multiple objective functions, which depend on the variables related to the state of the vehicles.

The modeling approach is in discrete time; the steps are activated when a relevant event k occurs, that is, the dispatcher receives a call asking for service. Then, at any event k, each vehicle j is assigned to complete a sequence of tasks which includes several points of pickup and delivery. Only one of those vehicles will serve the last new request. The set of sequences $S(k) = \{S_1(k), \ldots, S_j(k), \ldots, S_F(k)\}$ correspond to the control (manipulated) variable, where the sequence of stops assigned to vehicle j at instant k is given by $S_j(k) = \left[s_j^0(k)\, s_j^1(k)\, \ldots\, s_j^i(k)\, \ldots\, s_j^{w_j(k)}(k) \right]^T$.

$s_j^i(k)$ contains the information about the ith stop of vehicle j along its route, and $w_j(k)$ is the number of planned stops. A stop is defined by either a pickup or a delivery location. The initial condition $s_j^0(k)$ corresponds to the last point visited by the vehicle. In particular, the sequence of stops assigned to vehicle j at instant k, $S_j(k)$, is given by:

$$S_j(k) = \begin{bmatrix} s_j^0(k) \\ s_j^1(k) \\ \vdots \\ s_j^{w_j(k)}(k) \end{bmatrix} = \begin{bmatrix} r_j^0(k) & P_j^0(k) & z_j^0(k) & \Omega_j^0(k) \\ r_j^1(k) & P_j^1(k) & z_j^1(k) & \Omega_j^1(k) \\ \vdots & \vdots & \vdots & \vdots \\ r_j^{w_j(k)}(k) & P_j^{w_j(k)}(k) & z_j^{w_j(k)}(k) & \Omega_j^{w_j(k)}(k) \end{bmatrix} \tag{3}$$

where $r_j^i(k)$ identifies the passenger who is making the call (label), $P_j^i(k)$ is the geographic position in spatial coordinates of stop i assigned to vehicle j, $z_j^i(k)$ equals 1 if the stop i is a pickup and equals 0 if it is a delivery, and $\Omega_j^i(k)$ is the number of passengers associated with request $r_j^i(k)$. The vehicle follows the sequence in order until completing the list of tasks assigned. Note that the optimization procedure considers all the necessary constraints, such as assigning first the pickup and later the delivery for a specific set of passengers into the same vehicle, without violating its capacity, etc.

Two sources of stochasticity are considered: the first regarding the unknown future demand entering the system in real-time, and the second coming from the network traffic conditions. In the present work, the traffic conditions are modeled by means of a commercial distribution of speeds associated with the vehicles. This distribution considers two dimensions: spatial and temporal. The real distribution of speeds is assumed to be unknown (denoted by $v(t,p,\phi(t))$) which depends on a stochastic source $phi(t)$, representing the traffic conditions of the network as they change in time, and of a position p. We will assume in this work a conceptual network, where the trajectories are defined as a collection of straight lines that join two consecutive stops. Besides, a speed distribution for the urban zone represented by a speed model $\hat{v}(t,p)$, is supposed to be known, which could be obtained from historical speed data. To apply the MO-MPC approach, in the next section a dynamic model is proposed to represent the routing process.

3.2 Process model

For vehicle j, as in Cortés et al. (2008), the state space variables are the position $X_j(k)$, the estimated departure time vector $\hat{T}_j(k)$ and the estimated vehicle load vector $\hat{L}_j(k)$. Let us denote $\hat{T}_j^i(k)$ the expected departure time of vehicle j from stop i, $\hat{L}_j^i(k)$ the expected load of vehicle j when leaving stop i. The dynamic model for the position of vehicle j is as follows:

$$\hat{X}_j(k+1)$$
$$= \begin{cases} \displaystyle\sum_{i=0}^{w_j(k)-1} H_i(t_k+\tau)\left(P_j^i(k) + \int_{s=\hat{T}_j^i(k)}^{t_k+\tau} \hat{v}(s,p(s)) \frac{\left(P_j^{i+1}(k) - P_j^i(k)\right)}{\left\|P_j^{i+1}(k) - P_j^i(k)\right\|_2} ds\right) & \text{if } \hat{T}_j^{w_j(k)}(k) > t_k+\tau \\ P_j^{w_j(k)}(k) & \text{if } \hat{T}_j^{w_j(k)}(k) \leqslant t_k+\tau \end{cases}$$

$$H_i(t) = \begin{cases} 1 & \text{if} \quad \hat{T}_j^i(k) < t \leqslant \hat{T}_j^{i+1}(k) \\ 0 & \text{otherwise} \end{cases} \tag{4}$$

the variable time τ is defined as the time between the occurrence of the future probable call at instant $t_k+\tau$ and the occurrence of the previous call at t_k and can be tuned by means of a sensitivity analysis as described in Cortés et al. (2009). The expected stop visited by the vehicle before instant $t_k + \tau$ is i^*. The stop i^* was visited at instant $\hat{T}_j^{i^*}(k)$. The stop $P_j^0(k)$ denotes the position of the vehicle at instant k. In the model, if the vehicle reaches its last stop $w_j(k)$ and no additional tasks are scheduled for that vehicle, the vehicle will stay at that stop until a new request is assigned to it. For simulation results, the vehicle will proceed in direction to the closest zone with low availability of vehicles and a high probability of having a pickup-request.

The predicted departure time vector depends on the speed and can be described by the following dynamic model:

$$\hat{T}_j^i(k+1) = \begin{cases} T_j^0(k) & i = 0 \\ t_k + \sum_{s=1}^{i} \kappa_j^s(k) & i \neq 0 \end{cases} \qquad i = 0, 1, \ldots, w_j(k)$$

$$\kappa_j^s(k) = \int_{P_j^{s-1}(k)}^{P_j^s(k)} \frac{1}{\hat{v}(t_j(\omega), \omega)} d\omega \tag{5}$$

where $\kappa_j^i(k)$ is an estimation of the time interval between stop $i-1$ and stop i for the sequence associated with vehicle j at instant k.

The dynamic model associated with the vehicle load vector depends exclusively on the current sequence and its previous load. Analytically, we have:

$$\hat{L}_j^i(k+1) = \begin{cases} \min\left\{\bar{L}_j, L_j^0(k)\right\} & i = 0 \\ \min\left\{\bar{L}_j, L_j^0(k) + \sum_{s=1}^{i} \left(2z_j^s(k) - 1\right)\Omega_j^s(k)\right\} & i = 1, \ldots, w_j(k) \end{cases} \tag{6}$$

where $z_j^s(k)$ and $\Omega_j^s(k)$ were defined before in (3) and $\bar{L}_j$ is the capacity of vehicle j. We will assume later a homogeneous fleet of small vehicle with capacity for four passengers. The proposed vehicle sequences and state variables satisfy a set of constraints given by the real conditions of the dial-a-ride problem. Specifically, we must consider constraints of precedence, capacity and consistency in the solution of the MO-MPC problem to generate only feasible sequences.

3.3 Objective function for the dial-a-ride system

The motivation of this work is to provide to the dispatcher an efficient tool that captures the trade-off between users and operator costs. Besides, we design an objective function able to carry out the fact that some users can become particularly annoyed if their service is postponed (either pickup or delivery). In a proper formulation, a higher cost in the objective function should be considered to penalize differently very-long waiting or travel times. Next in this section, we formalize these ideas in an analytical expression.

In this section, we define the objective functions for choosing the best routes and vehicles to serve the dynamic demand. The optimization variables are the current sequence $S(k)$ that incorporate the new request η_k, and the $h_{\max}$ future sequences $S^h = \{S(k+1), \ldots, S(k+N)\}$, $h = 1, \ldots, h_{\max}$, that incorporate the prediction of future requests (scenarios). Thus, $S_k^{k+N} = \left\{S(k), S^1 \ldots, S^{h_{\max}}\right\}$ comprises all the control actions to be calculated. The scenario h consists of the sequential occurrence of $N-1$ estimated future requests $\hat{\eta}_{k+1}^h, \hat{\eta}_{k+2}^h, \ldots, \hat{\eta}_{k+N-1}^h$, with a probability p_h. This is on top of the actual currently received request. The scenarios are obtained based on historical-data. In this paper we use a fuzzy clustering method, as in Sáez et al. (2008).

A reasonable prediction horizon N is defined, which depends on the intensity of unknown events that enter the system in real-time and on how good the prediction model is. If the prediction horizon is larger than one, the controller adds the future behavior of the system into the current decision.

The proposed objective functions quantify the costs over the system of accepting the insertion of a new request. Such functions normally move towards opposite directions. The first objective function (J_1) that takes into account the users' cost, includes both waiting and travel time experienced by each passenger. The second one (J_2) is associated with the operational cost of running the vehicles of the fleet. Analytically, the proposed objective functions for a prediction horizon N, can be written as follows:

$$
\begin{aligned}
J_1 &= \sum_{\ell=1}^{N} \sum_{j=1}^{F} \sum_{h=1}^{h_{\max}(k+t)} p_h \left(J_{j,h}^{U}(k+\ell) - J_{j,h}^{U}(k+\ell-1) \right) \\
J_2 &= \sum_{\ell=1}^{N} \sum_{j=1}^{F} \sum_{h=1}^{h_{\max}(k+t)} p_h \left(J_{j,h}^{O}(k+\ell) - J_{j,h}^{O}(k+\ell-1) \right)
\end{aligned}
\tag{7}
$$

where

$$
J_{j,h}^{O}(k+\ell) = c_T \left(\hat{T}_j^{w_j(k+\ell)}(k+\ell) - \hat{T}_j^{0}(k+\ell) \right) \Big|_h + c_L \sum_{i=1}^{w_j(k+\ell)} \left(D_j^i(k+\ell) \right) \Big|_h \tag{8}
$$

and

$$
\begin{aligned}
J_{j,h}^{U}(k+\ell) = \theta_v \sum_{i=1}^{w_j(k+\ell)} \left(f_v\left(r_j^i(k+\ell)\right)\left(1 - z_j^i(k+\ell)\right) \left(\underbrace{\hat{T}_j^i(k+\ell) - tr_{r_j^i(k+\ell)}}_{\text{re - routing time}} \right) \right) \Bigg|_h + \\
\theta_e \sum_{i=1}^{w_j(k+\ell)} \left(f_e\left(r_j^i(k+\ell)\right) z_j^i(k+\ell) \left(\underbrace{\hat{T}_j^i(k+\ell) - t_{0r_j^i(k+\ell)}}_{\text{waiting time}} \right) \right) \Bigg|_h
\end{aligned}
\tag{9}
$$

The performance of the vehicle routing scheme will depend on how well the objective function can predict the impact of possible rerouting due to insertions caused by unknown service requests. In (7), $J_{j,h}^{U}$ and $J_{j,h}^{O}$ denote the user and operator costs respectively, associated with the sequence of stops that vehicle j must follow at certain instant. In equations (7)-(9), $k+\ell$ is the instant at which the ℓth request enters the system, measured from instant k. The number of possible call scenarios is $h_{\max}$, p_h is the probability of occurrence of the hth scenario. The occurrence probabilities p_h associated with each scenario are parameters in the objective function and must be calculated based on real time or historical data, or a combination of both. In Sáez et al. (2008) a zoning based for trip patterns estimation based on Fuzzy Clustering was designed. Expressions (8) and (9) represent the operator and users cost functions related to vehicle j at instant $k+\ell$, which depend on the previous control actions and the potential request h which occurs with probability p_h; $w_j(k+\ell)$ is the number of stops estimated for vehicle j at instant $k+\ell$. To explain the flexibility of the formulation and its economic consistency, the term related with the extra time experienced by passengers in this service (delivery time minus the minimum time the user could arrive to its destination) consider a cost θ_v for each minute, and the term related to total waiting time a cost θ_e for each minute. Note that the terms in the objective functions for user are weighted by the functions $f_v(\cdot)$ and$f_e(\cdot)$, which include a service policy for users, so the cost of a user that entered the system a long time ago is considered more important than of another user who has just made the request. We propose the following weighing functions:

$$f_v\left(r_j^i(k+\ell)\right) =$$
$$\begin{cases} 1 & \text{if } \hat{T}_j^i(k+\ell) - t_{0r_j^i(k+\ell)} \leqslant \alpha\left(tr_{r_j^i(k+\ell)} - t_{0r_j^i(k+\ell)}\right) \\ 1 + \hat{T}_j^i(k+\ell) - t_{0r_j^i(k+\ell)} - \alpha\left(tr_{r_j^i(k+\ell)} - t_{0r_j^i(k+\ell)}\right) & \text{if } \hat{T}_j^i(k+\ell) - t_{0r_j^i(k+\ell)} > \alpha\left(tr_{r_j^i(k+\ell)} - t_{0r_j^i(k+\ell)}\right) \end{cases} \tag{10}$$

Expression (10) implies that if the delivery time $\hat{T}_j^i(k+\ell)$ associated with user $r_j^i(k+\ell)$ becomes greater than α times its minimum total time $\left(tr_{r_j^i(k+\ell)} - t_{0r_j^i(k+\ell)}\right)$, the weighting function grows linearly, resulting in a critical service for such a client. Regarding the waiting time factor,

$$f_e\left(r_j^i(k+\ell)\right) = \begin{cases} 1 & \text{if } \hat{T}_j^i(k+\ell) - t_{0r_j^i(k+\ell)} \leqslant TT \\ 1 + \hat{T}_j^i(k+\ell) - t_{0r_j^i(k+\ell)} - TT & \text{if } \hat{T}_j^i(k+\ell) - t_{0r_j^i(k+\ell)} > TT \end{cases} \tag{11}$$

The intuition behind (11) is analogous to (10).

In addition, we will suppose an expression for the vehicle operational cost (8), with a component depending on the total traveled distance, weighted by a factor c_L, and another on the total operational time, in this case at unitary cost c_T. Thus, $D_j^i(k+\ell)$ represents the distance between stops $i-1$ and i in the sequence of vehicle j. The formulation proposed in this paper considers concepts already presented in the literature, like the total service time and dissatisfaction in Psaraftis (1980), the operational cost like in Cortés et al. (2008); being the aim of this paper to present a general framework, where different objective functions proposed in the literature could be included.

3.4 MO-MPC for a dial-a-ride system

A systematic way of incorporating such a trade-off of both objective functions is through a multi-objective approach, which results in a general set of solutions, giving the dispatcher the chance to change the service policies in a more transparent way by looking at the Pareto front. The closed loop of the dynamic vehicle routing system is shown in Figure 3. The MO-MPC represented by the dispatcher makes the routing decisions in real-time based on the information of the system (process) and the values of the fleet attributes, which allow evaluating the model under different scenarios. Service demand η_k and traffic conditions $\phi(t,p)$ are disturbances in this system.

The following multiobjective problem is solved:

$$\min_{S_k^{k+N}} \ \{J_1, J_2\} \tag{12}$$
$$s.t. \text{ operational constraints}$$

with J_1 and J_2 corresponding to the objective functions defined in (7). The solution to this problem corresponds to a set of control sequences, which form the optimal Pareto set. It is considered that $S^i = \left\{S^i(k), \ldots, S^i(k+N-1)\right\}$ is a feasible control action sequence, in the sense that satisfies all the operational constraints. The MO-MPC algorithm is divided in four steps.

Step 1. The scenario h consists of the sequential occurrence of N requests η_k, $\hat{\eta}_{k+1}^h$, $\hat{\eta}_{k+2}^h, \ldots$, $\hat{\eta}_{k+N-1}^h$. For each vehicle $j \in F$, for each scenario h, we will solve 2^N MO problems considering the cases

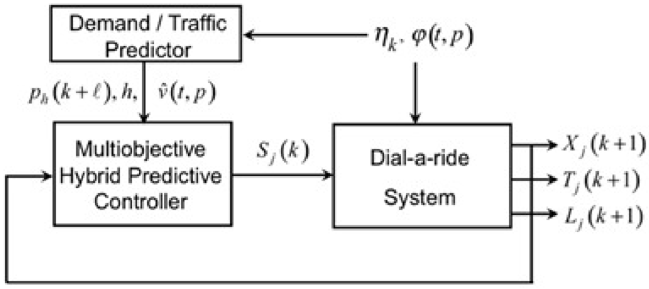

Figure 3: Closed loop diagram of the MO-MPC for the dynamic dial-a-ride problem.

where vehicle j is the one that serves none, one, up to N of those requests. For example, if $N=2$, for each vehicle we solve four MO problems considering the cases to serve none, to serve η_k, to serve $\hat{\eta}^h_{k+1}$, and to serve η_k and $\hat{\eta}^h_{k+1}$. The MO problem in this step is the following:

$$\min_{\left\{S_j(k), S_j^h(k+1), \ldots, S_j^h(k+N)\right\}} \left\{ \sum_{\ell=1}^{N} \left(J^U_{j,h}(k+\ell) - J^U_{j,h}(k+\ell-1)\right), \sum_{\ell=1}^{N} \left(J^O_{j,h}(k+\ell) - J^O_{j,h}(k+\ell-1)\right) \right\}$$
$$s.t. \text{ operational constraints}$$

In this problem, the operational constraints are capacity, consistency and no-swapping constraint (insertion keeping the previous sequence), so the Pareto set contains just feasible sequences. Note that some of those MO problems are easy to solve, but the more requests the vehicle serves, the more possible solutions we will have. In fact, considering the no-swapping constraint, the number of possible solutions when the N request are served by vehicle j only is $0.5 \prod_{i=0}^{N-1} (w_j(k)+i)(w_j(k)+i-1)$, where $w_j(k)$ is the number of stops of vehicle j at instant k. The MO problems in this step are the most time consuming, but they can be solved simultaneously and in parallel because they are not related with each other.

Step 2. Then for a given scenario h, considering the constraint that just one vehicle can serve each request, we obtain the Pareto set of the following MO problem:

$$\min_{\left\{S(k), S^1, \ldots, S^{h_{\max}}\right\}} \left\{ \sum_{j\in F} \sum_{\ell=1}^{N} \left(J^U_{j,h}(k+\ell) - J^U_{j,h}(k+\ell-1)\right), \sum_{j\in F} \sum_{\ell=1}^{N} \left(J^O_{j,h}(k+\ell) - J^O_{j,h}(k+\ell-1)\right) \right\}$$

The solution of this MO problem is obtained with the Pareto sets from Step 1 by combining the $|F|^N$ possible cases in a way that the current request and each future request are served by just one vehicle. For example, if we have three vehicles $F=\{a,b,c\}$, for $N=2$, the cases are $a-a$, $a-b$, $a-c$, $b-a$, $b-b$, $b-c$, $c-a$, $c-b$ and $c-c$, where v_1-v_2 means that η_k is served by vehicle v_1 and $\hat{\eta}^h_{k+1}$ is served by vehicle v_2. The MO problem of this step can be solved in parallel.

Step 3. Then, using the Pareto set of all the scenarios h, we solve the following MO problem:

$$\min_{S_k^{k+N}} \left\{ \sum_{h} \sum_{j\in F} \sum_{\ell=1}^{N} p_h \left(J^U_{j,h}(k+\ell) - J^U_{j,h}(k+\ell-1)\right), \sum_{h} \sum_{j\in F} \sum_{\ell=1}^{N} p_h \left(J^O_{j,h}(k+\ell) - J^O_{j,h}(k+\ell-1)\right) \right\}$$

The solution of this MO problem is obtained using the Pareto sets from Step 2 (can be done in parallel), by multiplying each Pareto front by the probability of occurrence of the associated scenario p_h and then combining the different cases considering all the scenarios.

Step 4. A relevant step of this approach in the controller's dispatch decision is the definition of criteria to select the best control action at each instant under the MO-MPC approach. For example, once the Pareto front is found, different criteria regarding a minimum allowable level of service can be dynamically used to take policy dependent routing decisions. In this work, we will evaluate MO-MPC based on a weighted-sum and an ε-constraint criterion.

In those cases where the policy for users is accomplished for several solutions, the one that is the closest to the pursued policy will be selected. So, we include soft constraints directly, without incorporating them into the optimization problem, although they are considered in the choice process that emulates the dispatcher.

4 Simulation results

In this section we present the simulation tests conducted to show the MO-MPC strategy application. A period of two hours representative of a labor day (14:00-14:59, 15:00-15:59) is simulated, over an urban area of approximately 81 km^2. A fixed fleet of fifteen vehicles is considered, with a capacity of four passengers each. We assume that the vehicles travel in a straight line between stops and that the transport network behaves according to a speed distribution with a mean of 20 [km/h].

Two hundred and fifty calls were generated over the simulation period of two hours following the spatial and temporal distribution observed from the historical data. Regarding the temporal dimension, a negative exponential distribution for time intervals between calls with rate 0.5 [call/minute] for both hours of simulation was assumed. Regarding the spatial distribution, the pickup and delivery coordinates were generated randomly within each zone. The 15 first calls at the beginning and the 15 last calls at the end of the experiments were deleted from the statistics to avoid limit distortion (warm up period). A total of 10 replications of each experiment were carried out to obtain the global statistics. Each replication (emulating two hours and 250 on-line decisions) took 20 minutes in average, on a Intel Core 2 CPU, 2.40 GHz processor.

We suppose that the future calls are unknown for the controller. From historical data, the typical trip patterns can be extracted by using a systematic methodology. Thus, Cortés et al. (2008) and Sáez et al. (2008) obtained a speed distribution model as well as the trip patterns, the latter through a fuzzy zoning method to define the most likely origin-destination configurations.

The objective function is formulated considering the parameters prediction horizon $N = 2$, $\theta_v = 16.7$ [Ch\$/min], $\theta_e = 50$ [Ch\$/min], $c_T = 25$ [Ch\$/min], $c_L = 350$ [Ch\$/km], $\alpha = 1.5$, $TT = 5$ [min]. We report results of MO-MPC based on a weighted-sum and an ε-constraint criterion. Five different criterion are used for MO-MPC based weighted sum $\lambda^T = [\,\lambda_1 \quad 1-\lambda_1\,]$, $\lambda_1 = 1$, 0.75, 0.5, 0.25 and 0 (for the first five rows of Table 1a and 1b). The results using ε-constraint (in the last five rows of Table 1a and 1b) are associated to the following criteria:

- Criterion 1: Minimum users' cost component
- Criterion 2: Minimum operational cost component
- Criterion 3: The nearest value to a given user cost (traveled time plus waiting time costs).

In case of Criterion 3 (the nearest value to a given user cost), we considered three references: 400, 500 and 600 [Ch$] for sub-cases a), b) and c) respectively.

In Table 1a are the results in terms of the user indices effective travel and waiting times per user, as well as user cost. In Table 1b, the operator indices effective total travel time, distance traveled per vehicle, and total operator costs are presented. Note from Table 1a and 1b, Criterion 1 and Criterion 2 are equivalent to cases with $\lambda^T = [\,1 \;\; 0\,]$ and $\lambda^T = [\,0 \;\; 1\,]$.

Tables 1a and 1b show a clear trade-off between opposite components. Besides, small values for standard deviation imply that the travel and waiting times are more balanced among passengers as a variable weight for them was included in the objective function specification. Notice the extreme case $\lambda^T = [\,0 \;\; 1\,]$ in favor of the operator results in a very poor service for users, not only around the mean but also in terms of bounding the standard deviation. With regard to Criterion 3, the resulting mean user cost over the whole simulation fitted quite well the thresholds defined at each sub-case.

5 Conclusions

This work presents a new approach to solve the dial-a-ride problem under a model predictive control scheme using dynamic multiobjective optimization. We propose different criteria to obtain control actions over real-time routing using the dynamic Pareto front. The criteria allow giving priority to a service policy for users, ensuring a minimization of operational costs under each proposed policy. The service policies are verified approximately on the average of the replications. Under the implemented on-line system it is easier and more transparent for the operator to follow service policies under a multiobjective approach instead of tuning weighting parameters dynamically. The multiobjective approach allow us to obtain solutions that are directly interpreted as part of the Pareto front instead of results obtained with single-objective functions, which lack of direct physical interpretation (the weight factors are tuned but they do not allow applying operational or service policies such as those proposed here).

This paper shows a potential a real application of a transportation system for point-to-point transport in a real-time setting. Then, we highlight the effort to combine a dynamic transport problem (dynamic dial-a-ride) optimized in real-time by means of control theory and multiobjective optimization. However, the method we use in this paper has one main drawback: obtaining the solution set of the MO problem requires a significant computational effort, and in a real application context, this can be a serious issue. Then, the next step is to find an efficient algorithm from the evolutionary computation literature for a real-time implementation of the scheme (such a Genetic algorithms or Partial Swarm Optimization, for example) and find an application to simulate the system operating in real world context. From the new toolboxes for Evolutionary Computation and other efficient algorithms developed in recent years, we think it is possible to determine a good representative pseudo-optimal Pareto set in a dynamic context. Future work will focus on efficient optimization algorithms.

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Table 1a: MO-MPC, user indices

MO Criterion	Travel time [min/pax]		Waiting time [min/pax]		Mean user cost [Ch$]
	Mean	Std	Mean	Std	
$\lambda^T = [1\ \ 0]$	9.36	3.66	4.52	2.74	382.27
$\lambda^T = [0.75\ \ 0.25]$	9.79	4.25	4.47	2.49	386.89
$\lambda^T = [0.50\ \ 0.50]$	10.19	4.49	4.60	2.99	399.88
$\lambda^T = [-0.25\ \ 0.75]$	10.48	4.75	5.38	3.06	444.12
$\lambda^T = [0\ \ 1]$	10.01	7.38	15.44	10.80	939.15
Criterion 1	9.36	3.66	4.52	2.74	382.27
Criterion 2	10.01	7.38	15.44	10.80	939.15
Criterion 3a	10.32	4.75	4.62	2.67	403.17
Criterion 3b	10.76	5.36	5.63	3.58	461.30
Criterion 3c	10.63	6.09	7.25	4.59	540.20

Table 1b: MO-MPC, operator indices

MO Criterion	Time Traveled [min/veh]		Distance Traveled [km/veh]		Mean operator cost [Ch$]
	Mean	Std	Mean	Std	
$\lambda^T = [1\ \ 0]$	88.16	7.55	24.84	1.86	10898.28
$\lambda^T = [0.75\ \ 0.25]$	75.17	11.06	20.61	2.94	9094.22
$\lambda^T = [0.50\ \ 0.50]$	67.57	12.78	18.62	3.51	8207.24
$\lambda^T = [0.2\,5\,0.75]$	61.67	12.57	16.95	3.17	7476.06
$\lambda^T = [0\ \ 1]$	43.90	17.94	12.58	5.09	5500.82
Criterion 1	88.16	7.55	24.84	1.86	10898.28
Criterion 2	43.90	17.94	12.58	5.09	5500.82
Criterion 3a	74.99	8.76	20.91	2.19	9193.45
Criterion 3b	69.56	11.52	19.92	3.05	8713.60
Criterion 3c	71.40	10.53	20.39	2.80	8924.53

References

Bemporad, A. and Muñoz de la Peña, D. (2009). Multiobjective model predictive control. Automatica 45(12), 2823-2830.

Berbeglia, G., Cordeau, J. and Laporte, G. (2009). Dynamic pickup and delivery problems. European Journal of Operational Research 202(1), 8-15.

Camacho, E.F. and Bordons, C. (1999). Model Predictive Control. London, U.K.: Springer-Verlag.

Cortés, C.E., Sáez, D. and Núñez, A. (2008). Hybrid adaptive predictive control for a dynamic pickup and delivery problem including traffic congestion. International Journal of Adaptive Control and Signal 22(2), 103-123.

Cortés, C.E., Sáez, D., Núñez, A. and Muñoz, D. (2009). Hybrid adaptive predictive control for a dynamic pickup and delivery problem. Transportation Science 43(1), 27-42.

Desrosiers, J., Soumis, F. and Dumas, Y. (1986). A dynamic programming solution of a large-scale single-vehicle dial-a-ride with time windows. American Journal of Mathematical and Management Sciences 6, 301-325.

Durillo, J., Nebro, A., Coello, C., García-Nieto, J., Luna, F. and Alba, E. (2010). A study of multiobjective metaheuristics when solving parameter scalable problems. IEEE Transactions on Evolutionary Computation, in press.

Haimes, Y., Tarvainen, K., Shima, T. and Thadatnil, J. (1990). Hierarchical multiobjective analysis of large-scale systems, by Hemisphere Publishing Corp.

Jozefowiez, N., Semet, F. and Talbi, E. (2008). Multiobjective vehicle routing problems. European Journal of Operational Research 189(2), 293-309.

Kerrigan E.C., Bemporad, A., Mignone, D., Morari, M., Maciejowski, J.M., (2000). Multi-objective Priorisation and Reconfiguration for the Control of Constrained Hybrid Systems. Proceedings of the American Control Conference, Chicago Illinois, June 2000, 1694-1698.

Kerrigan, E.C., Maciejowski, J.M., (2003). Designing model predictive controllers with prioritised constraints and objectives. Proceedings 2002 IEEE International Symposium on Computer Aided Control System Design, 33-38.

Núñez, A., Sáez, D., Oblak, S., Skrjanc, I., (2009). Hybrid Predictive Control based on Fuzzy Model. ISA Transactions 48, 24-31.

Paquette, J., Cordeau, J. and Laporte, G. (2009). Quality of service in dial-a-ride operations. Computers & Industrial Engineering 56(4), 1721-1734.

Psaraftis, H. (1980). A dynamic programming solution to the single many-to-many immediate request dial-a-ride problem. Transportation Science 14(2), 130-154.

Sáez, D., Cortés, C.E. and Núñez, A. (2008). Hybrid adaptive predictive control for the multi-vehicle dynamic pickup and delivery problem based on genetic algorithms and fuzzy clustering. Computers & Operations Research 35(11), 3412-3438.

Savelsbergh, M. and Sol, M. (1995). The general pickup and delivery problem. Transportation Science 29(1), 17-29.

Zambrano, D., Camacho E.F., (2002). Application of MPC with multiple objective for a solar refrigeration plant. Proceedings of the 2002 International Conference on Control Applications 2, 1230 - 1235.