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Hybrid model predictive control using time-instant optimization for the Rhine-Meuse Delta

H. van Ekeren, R.R. Negenborn, P.J. van Overloop, B. De Schutter

***Abstract*— In order to provide safety against high sea water levels, in many low-lying countries on the one hand dunes are maintained at a certain safety level and dikes are built, while on the other hand large control structures that can be controlled dynamically are constructed. Currently, these structures are often operated purely locally, without coordination on actions between different structures. Automatically coordinating the actions is particularly difficult, since open water systems are complex, hybrid systems, in the sense that continuous dynamics (e.g., the evolution of the water levels) are mixed with discrete events (e.g., the opening or closing of barriers). In low-lands, this complexity is increased further due to bi-directional water flows resulting from backwater effects and interconnectivity of flows in different parts of river deltas. In this paper, we propose a model predictive control (MPC) approach that is aimed at automatically coordinating the different actions. Hereby, the hybrid nature is explicitly addressed. In order to reduce the computational effort required to solve the hybrid MPC problem we propose to use TIO-MPC, where TIO stands for time-instant optimization. A simulation study illustrates the potential of the proposed controller in comparison with the current setup in the Rhine-Meuse delta in The Netherlands.**

I. Introduction

Floods are one of the most common type of natural disasters that Europe has to face. In the period between 1998 and 2004 there were more than 100 major floods in Europe. Due to the changing climate, in the nearby future, flood prevention will become even more important as sea levels will rise and precipitation will intensify [1].

One of the areas where increased problems are expected is the highly populated Rhine-Meuse delta in The Netherlands, including the large cities of Rotterdam and Dordrecht, and the largest port of Europe (see Figure 1) [1]. To protect the area against floods, storm surge barriers and dikes have been constructed. The barriers each have local control systems consisting of simple if-then-else rules that determine when a barrier should be closed or opened. These local rules do not in the best way utilize the capacity available in the system. We therefore investigate how coordination of the actions of these structures can improve performance.

The storm surge barriers and the water system of the Rhine-Meuse delta can be considered as a hybrid system, in the sense that several barriers are designed to be either

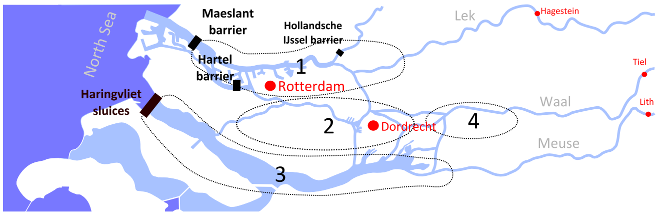

Fig. 1: The Rhine-Meuse delta divided into 4 reservoirs.

opened or closed (discrete) and dikes can overtop (discrete), while at the same time some other barriers are operated with continuous actions and the water system involves continuous dynamics (evolution of water flows and levels). The decision to close the barriers depends on water levels, water flows, and weather conditions in the near future. The main control goal in the Rhine-Meuse delta involves a trade-off between keeping water levels low and minimizing the cost of using the storm surge barriers.

Instead of using local rule-based control, we propose to use model predictive control (MPC), an optimization-based control technique originating from the process industry [2], and now gaining increasing attention also in other fields, including the field of open water systems [3], [4], [5], [6], [7], [8], [9], [10]. In our case, we are considering a hybrid system representation of the water system due to the presence of both continuous and discrete elements. Each of these water applications, however, does not consider this hybrid nature explicitly. Therefore, we propose to use a particular hybrid MPC technique for coordinating the actions of the structures that explicitly takes into account the hybrid nature of the system, here referred to as TIO-MPC, where TIO stands for time-instant optimization. Contrarily to other hybrid MPC techniques (such as the well-known hybrid MPC based on mixed-logical dynamic (MLD) modeling framework [11], [12]), this technique optimizes time instants. This has as advantage that computational time requirements are reduced and nonlinear prediction models can directly be used. Before, such a technique has been used for traffic control [13]; here we investigate its use for water control.

This paper is organized as follows. Section II describes the model of the Rhine-Meuse case study. Section III proposes hybrid MPC using time-instant optimization. Section IV illustrates the potential of the proposed approach in a simulation study. Section V concludes this paper.

II. Rhine-Meuse delta

The Rhine-Meuse delta water system consists of a large number of rivers and sea outlets. The boundary conditions in

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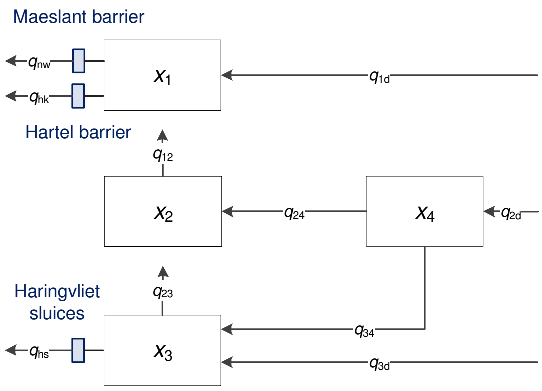

Fig. 2: Structure of the Rhine-Meuse delta model.

the East of the water system consist of the rivers Lek, Waal, and Meuse. The boundary conditions in the West consist of the connections of two rivers (Nieuwe Waterweg and Hartelkanaal) and one outlet (Haringvliet) with the North Sea. There are three main barriers: the Maeslant barrier, the Hartel barrier, the Hollandsche IJssel barrier, and the Haringvliet sluices. The first three barriers are designed to be either completely open or completely closed. In the open state the rivers in which these barriers are build can flow freely and ships can pass without any disturbance. In the closed state river flows and navigation are completely blocked. The last barrier consists of 17 gates that can move independently between a maximum and a minimum height. There, ships can pass via a lock.

A. Area model

Below the most important characteristics of the discrete-time model of the Rhine-Meuse delta considered in this paper are given. The description of the full model, including all equations and parameters can be found in [14]. The model presented is an extension of the model in [15].

The water system under consideration is represented by 4 large reservoirs that are interconnected by rivers, see Figures 1 and 2. The states x_1, x_2, x_3, and x_4 represent the water levels in reservoirs 1, 2, 3, and 4, respectively. The change in each of these water levels is determined using a discretized mass balance as follows:

$$\begin{aligned} x_1(k+1) = x_1(k) + \frac{T_s}{A_{s1}\left(u_{\text{hijb}}(k)\right)} [&q_{12}\left(x_1(k), x_2(k)\right) \\ &+ q_{1d}(k) - q_{nw}\left(x_1(k), h_{\text{hvh}}(k), u_{\text{mb}}(k)\right) \\ &- q_{hk}\left(x_1(k), h_{\text{hvh}}(k), u_{\text{hb}}(k)\right)] \end{aligned} \quad (1)$$

$$\begin{aligned} x_2(k+1) = x_2(k) + \frac{T_s}{A_{s2}} [&-q_{12}\left(x_1(k), x_2(k)\right) \\ &+ q_{23}\left(x_2(k), x_3(k)\right) + q_{24}\left(x_2(k), x_4(k)\right)] \end{aligned} \quad (2)$$

$$\begin{aligned} x_3(k+1) = x_3(k) + \frac{T_s}{A_{s3}} [&-q_{23}\left(x_2(k), x_3(k)\right) \\ &+ q_{34}\left(x_3(k), x_4(k)\right) + q_{3d}(k) \\ &- q_{hs}\left(x_3(k), h_{hs}(k), u_{hs}(k)\right)] \end{aligned} \quad (3)$$

$$\begin{aligned} x_4(k+1) = x_4(k) + \frac{T_s}{A_{s4}} [&-q_{24}\left(x_2(k), x_4(k)\right) \\ &- q_{34}\left(x_3(k), x_4(k)\right) + q_{2d}(k)], \end{aligned} \quad (4)$$

where k is the discrete time step; T_s (s) is the simulation sample time; $A_{s1}\left(u_{\text{hijb}}(k)\right)$, A_{s2}, A_{s3}, and A_{s4} (m^2) are the surface areas of reservoir 1, 2, 3, and 4, respectively; $x_1(k)$, $x_2(k)$, $x_3(k)$, and $x_4(k)$ (m) are the water levels of reservoir 1, 2, 3, and 4, respectively; $q_{1d}(k)$, $q_{2d}(k)$, and $q_{3d}(k)$ (m^3/s) are disturbance inflows from the rivers Lek, Waal, and Meuse, respectively; $q_{nw}\left(x_1(k), h_{\text{hvh}}(k), u_{\text{mb}}(k)\right)$, $q_{hk}\left(x_1(k), h_{\text{hvh}}(k), u_{\text{hb}}(k)\right)$, and $q_{hs}\left(x_3(k), h_{hs}(k), u_{hs}(k)\right)$ (m^3/s) are disturbance inflows from the sea and are controlled by the Maeslant barrier, the Hartel barrier, and the Haringvliet sluices, respectively; $q_{12}\left(x_1(k), x_2(k)\right)$, $q_{23}\left(x_2(k), x_3(k)\right)$, $q_{24}\left(x_2(k), x_4(k)\right)$, $q_{34}\left(x_3(k), x_4(k)\right)$ (m^3/s) are the flows between the reservoirs, described by $q_{ij}\left(x_i(k), x_j(k)\right) = f_{\text{Chézy}}\left(x_i(k), x_j(k)\right)$, where $f_{\text{Chézy}}$ is the formula of Chézy [16]:

$$\begin{aligned} f_{\text{Chézy}}\left(x_i(k), x_j(k)\right) &= A_{c,ij}\left(x_i(k), x_j(k)\right) C_{ij} \text{sign}(x_j - x_i) \\ &\times \sqrt{\frac{R_{ij}\left(x_i(k), x_j(k)\right) \left|x_j - x_i\right|}{l_{ij}}}, \end{aligned} \quad (5)$$

where $q_{ij}\left(x_i(k), x_j(k)\right)$ is the flow between reservoirs i and j; $A_{c,ij}\left(x_i(k), x_j(k)\right)$ (m^2) is the (smallest) water cross section of the transport region of flow $q_{ij}\left(x_i(k), x_j(k)\right)$; C_{ij} ($\text{m}^{1/2}/\text{s}$) is the Chézy roughness coefficient of flow $q_{ij}\left(x_i(k), x_j(k)\right)$; $R_{ij}\left(x_i(k), x_j(k)\right)$ (m) is the hydraulic radius of flow $q_{ij}\left(x_i(k), x_j(k)\right)$; l_{ij} (m) is the length of the river between reservoir i and reservoir j. Note that the sign function is used to indicate the direction of the flow.

The water cross sectional area $A_{c,ij}\left(x_i(k), x_j(k)\right)$ and the hydraulic radius $R_{ij}\left(x_i(k), x_j(k)\right)$ are variables that depend on the water level in the river that connects a reservoir i with a reservoir j. This river water level is approximated by the average of the water levels x_i and x_j. The water cross sectional area and the hydraulic radius depend also on the physical structure of the river cross section. The river cross sections are approximated with straight lines. As a result, $A_{c,ij}\left(x_i(k), x_j(k)\right)$ and $R_{ij}\left(x_i(k), x_j(k)\right)$ are nonlinear functions of $x_i(k)$ and $x_j(k)$. For more details, see [15].

The flows $q_{nw}\left(x_1(k), h_{\text{hvh}}(k), u_{\text{mb}}(k)\right)$ and $q_{hk}\left(x_1(k), h_{\text{hvh}}(k), u_{\text{hb}}(k)\right)$ are also determined with (5), but in addition depend on the state of a barrier:

$$q_{nw}\left(x_1(k), h_{\text{hvh}}(k), u_{\text{mb}}(k)\right) = u_{\text{mb}}(k) f_{\text{Chézy}}\left(x_1(k), h_{\text{hvh}}(k)\right) \quad (6)$$

$$q_{hk}\left(x_1(k), h_{\text{hvh}}(k), u_{\text{hb}}(k)\right) = u_{\text{hb}}(k) f_{\text{Chézy}}\left(x_1(k), h_{\text{hvh}}(k)\right), \quad (7)$$

where $h_{\text{hvh}}(k)$ (m) is the water level of the North Sea at Hoek van Holland, and $u_{\text{mb}}(k)$ is the state of the Maeslant barrier, defined as:

$$u_{\text{mb}}(k) = \begin{cases} 1 & \text{if the barrier is closed at time } k \\ 0 & \text{otherwise.} \end{cases} \quad (8)$$

The state of the Hartel barrier $u_{\text{hb}}(k)$ is defined similarly.

The flow $q_{hs}\left(x_3(k), h_{hs}(k), u_{hs}(k)\right)$ through the Haringvliet sluices depends on the water level of the North Sea near these sluices $h_{hs}(k)$, the water level $x_3(k)$ in reservoir 3 (because the Haringvliet is part of reservoir 3), and the height of the gates of the sluices $u_{\text{hs}}(k)$. The flow is determined by using the equations for free and submerged orifice flow and the equations for free and submerged weir flow, as given in [15].

The influence of the Hollandsche IJssel barrier is modeled via the surface area of reservoir 1 as follows:

$$A_{\text{s1}}\left(u_{\text{hijb}}(k)\right) = A_{\text{s1,normal}} - A_{\text{s1,hij}}\left(1 - u_{\text{hijb}}(k)\right), \quad (9)$$

where $A_{\text{s1,normal}}$ is the surface area of reservoir 1 (m^2) when the Hollandsche IJssel barrier is open ($u_{\text{hb}}(k) = 1$) and where $A_{\text{s1,hij}}$ is the reduction in surface area caused by closure of the Hollandsche IJssel barrier ($u_{\text{hb}}(k) = 0$). The state of the Hollandsche IJssel barrier $u_{\text{hijb}}(k)$ is defined similarly as the state of the Maeslant barrier in (8).

B. Current control systems

The current control systems for the barriers consist of simple rules. E.g., the decision to close the Maeslant barrier and the Hartel barrier is based on predictions of water levels 24 hours ahead of time in the case that these barriers are open. The goal of the local controllers is to achieve the following objectives [17]:

1) To prevent the water level at Rotterdam to rise above 3.87 mMSL (m above mean sea level) and at Dordrecht to rise above 3.25 mMSL.
2) To prevent water levels in the Hollandsche IJssel to rise above 2.25 mMSL, while preventing saline water to flow into this river.
3) To maintain a minimum water level of 0.00 mMSL at Moerdijk (in the Hollandsche Diep).
4) To maintain a minimum discharge, averaged over a tide, through the Nieuwe Waterweg of 1500 m^3/s.
5) To prevent water flowing directly from the North Sea into the Haringvliet.

When the water levels at Rotterdam and Dordrecht stay below their critical value (i.e., the dike height), the whole area is safe, since the most critical (i.e., lowest) dikes are located at these locations [15]. Currently, the Maeslant barrier and the Hartel barrier are used mostly for objective 1; the Hollandsche IJssel barrier for objective 2; and the Haringvliet sluices for objectives 1 and 3–5.

It is noted, however, that the evolution of the water levels when different control actions are applied at the same time, is not taken into account. This may lead to low performance of the local control systems in extreme conditions. The controller proposed in the next section is expected to improve this performance.

III. Time-instant optimization MPC

Instead of having a binary variable[1] for each control cycle step as degrees of freedom (as is common, e.g., when using MLD-MPC), time instants are the degrees of freedom. A time

[1]Note that TIO-MPC is not restricted to binary input variables only, but can also be used for continuous input variables. In contrast to time instant optimization for a binary variable, there are then two (instead of one) continuous variables needed for each time instant: one for the time instant (e.g., t_1) and one for the new input value (e.g., u_1).

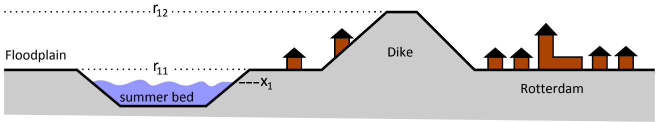

Fig. 3: A schematic view of a river and its surrounding area.

instant t_i is the moment at which the binary input changes its state. The time instants are continuous optimization variables instead of the computationally more demanding binary variables. With this technique the amount of optimization variables will typically be lower. It is hereby noted, however, that if the number of time instants is lower than the number of control cycle steps in the prediction horizon, the input will have less freedom.

To design a TIO-MPC controller for the Rhine-Meuse delta we have to define the objective function, the prediction model, the constraints, and the solution method.

1) Objective function: The objective function has to represent the trade-off between input effort and costs on (too) high water levels. We consider the following function:

$$\begin{aligned} J(k) = {} & J_{\text{x}_1}(\tilde{\mathbf{x}}_1(k)) + J_{\text{x}_2}(\tilde{\mathbf{x}}_2(k)) \\ & + J_{\text{mb}}(\tilde{\mathbf{u}}_{\text{mb}}(k)) + J_{\text{hb}}(\tilde{\mathbf{u}}_{\text{hb}}(k)) + J_{\text{hs}}(\tilde{\mathbf{u}}_{\text{hs}}(k)), \quad (10) \end{aligned}$$

where

$$\begin{aligned} \tilde{\mathbf{x}}_1(k) &= \begin{bmatrix} x_1(k+1) & x_1(k+2) & \cdots & x_1(k+N) \end{bmatrix}^{\text{T}} \\ \tilde{\mathbf{x}}_2(k) &= \begin{bmatrix} x_2(k+1) & x_2(k+2) & \cdots & x_2(k+N) \end{bmatrix}^{\text{T}} \\ \tilde{\mathbf{u}}_{\text{mb}}(k) &= \begin{bmatrix} u_{\text{mb}}(k) & u_{\text{mb}}(k+1) & \cdots & u_{\text{mb}}(k+N-1) \end{bmatrix}^{\text{T}} \\ \tilde{\mathbf{u}}_{\text{hb}}(k) &= \begin{bmatrix} u_{\text{hb}}(k) & u_{\text{hb}}(k+1) & \cdots & u_{\text{hb}}(k+N-1) \end{bmatrix}^{\text{T}} \\ \tilde{\mathbf{u}}_{\text{hs}}(k) &= \begin{bmatrix} u_{\text{hs}}(k) & u_{\text{hs}}(k+1) & \cdots & u_{\text{hs}}(k+N-1) \end{bmatrix}^{\text{T}}, \end{aligned}$$

with N the length of the prediction horizon in discrete time steps. We next describe the terms in this function.

The first part of the objective function consisting of the terms $J_{\text{x}_1}(\tilde{\mathbf{x}}_1(k))$ and $J_{\text{x}_2}(\tilde{\mathbf{x}}_2(k))$ describes the damage and flood risk of high water levels. This part of the objective function is illustrated with $J_{\text{x}_1}(\tilde{\mathbf{x}}_1(k))$ and Figure 3. When the maximum of water level x_1 stays below a reference level r_{11} there will be no damage at all. In this situation the river is in its summer bed. Exceeding reference level r_{11} can lead to some damage (e.g., damaged houses, cattle, and fields on the floodplains) and flood risk (e.g., risk of collapsing dikes), depending on the water level. Therefore, a water level exceeding reference level r_{11} is penalized with a quadratically increasing cost. The dike height r_{12} is the most important reference level. Exceeding reference level r_{12} will suddenly lead to enormous high (economic and social) costs caused by flooding of the crowded area of Rotterdam. Therefore, exceeding level r_{12} is penalized with a constant cost value, but also with a quadratically increasing cost. In case it is impossible to prevent that the water level exceeds the dike height, this quadratic cost ensures that the controller still minimizes the magnitude of the flood. The cost function that we propose, is now defined on the cumulative exceedance of the critical water level by x_1 as follows:

$$J_{\text{x}_1}(\tilde{\mathbf{x}}_1(k)) = \alpha_{11} e_{\text{cum},11}(k) + \alpha_{12} e_{\text{cum},12}(k) + \alpha_{13} \tilde{e}_{13}(k),$$

where

$$e_{\text{cum},11}(k) = \sum_{j=1}^{N} (\max(x_1(k+j) - r_{11}, 0))^2$$
$$e_{\text{cum},12}(k) = \sum_{j=1}^{N} (\max(x_1(k+j) - r_{12}, 0))^2$$
$$\tilde{e}_{13}(k) = \begin{cases} 1 & \text{if } \max(\tilde{\mathbf{x}}_1(k)) > r_{12} \\ 0 & \text{otherwise,} \end{cases}$$

where $e_{\text{cum},11}(k)$ and $e_{\text{cum},12}(k)$ are the cumulative exceedances for reference levels r_{11} and r_{12}, respectively. The parameters α_{11}, α_{12}, and α_{13} are cost weights. The cost function $J_{\mathrm{x}_2}(\tilde{\mathbf{x}}_2(k))$ is defined in a similar way.

The second part of the objective function (10) consisting of $J_{\text{mb}}(\tilde{\mathbf{u}}_{\text{mb}}(k))$, $J_{\text{hb}}(\tilde{\mathbf{u}}_{\text{hb}}(k))$, and $J_{\text{hs}}(\tilde{\mathbf{u}}_{\text{hs}}(k))$ describes the cost of closing and moving the storm surge barriers. Closure of the Maeslant barrier or the Hartel barrier blocks the navigation in the corresponding canals. Secondly, movement of a barrier also costs money due to wear and tear and energy costs. The cost function of the Maeslant barrier is therefore defined as

$$J_{\text{mb}}(\tilde{\mathbf{u}}_{\text{mb}}(k)) = \alpha_{\text{mb}1} \sum_{j=1}^{N} [1 - u_{\text{mb}}(k+j-1)] + \alpha_{\text{mb}2} \sum_{j=1}^{N} |u_{\text{mb}}(k+j-1) - u_{\text{mb}}(k+j-2)|, \quad (11)$$

with $\alpha_{\text{mb}1}$ the cost of closing the Maeslant barrier for one discrete time step, and $\alpha_{\text{mb}2}$ the cost of changing the state of the Maeslant barrier. The cost function of the Hartel barrier $J_{\text{hb}}(\tilde{\mathbf{u}}_{\text{hb}}(k))$ is defined similarly. The cost function for the Haringvliet sluices $J_{\text{hs}}(\tilde{\mathbf{u}}_{\text{hb}}(k))$ is also defined similarly, except for that only costs on the movements are considered (i.e., a term similar to the second term in (11)).

The Hollandsche IJssel barrier is not a part of the objective function, since the control actions for this barrier will not be optimized. The control objective of keeping the water level in the Hollandsche IJssel below 2.25 mNAP results into trivial control actions. Namely, closing the Hollandsche IJssel barrier when the water level $x_1(k)$ at Rotterdam exceeds 2.25 mNAP. This is also the control rule of the current control system of the Hollandsche IJssel barrier. Moreover, the water level in the Hollandsche IJssel is not a state of the model presented in II-A.

2) Model: As already mentioned, the complete nonlinear reservoir model is used as a prediction model for the TIO-MPC approach. The Haringvliet sluices are an input of the TIO-MPC prediction model. The nonlinear relation between the sluice gates, the water levels, and the flow through these sluices fits inside the nonlinear optimization problem. The Hollandsche IJssel barrier and its control system are included in the TIO-MPC prediction model. Thus, the control actions for this barrier are not optimized.

The TIO-MPC prediction model has time instants as inputs and also regular (conventional) inputs. The state (open or closed) changes of the Maeslant barrier and the state changes of the Hartel barrier are both modeled with four time instants. These time instants represent the moments at which the state

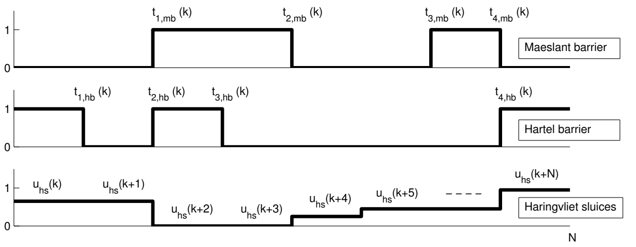

Fig. 4: Illustration of typical TIO-MPC inputs.

of the barriers change: from open to closed or from closed to open. The positions of the gates of the Haringvliet sluices are modeled with one regular input. This results in the following TIO-MPC prediction model of the Rhine-Meuse delta:

$$\tilde{\mathbf{x}}(k) = f\big(\tilde{\mathbf{t}}(k), \tilde{\mathbf{u}}_{\text{hs}}(k), \mathbf{x}(k)\big), \quad (12)$$

with:

$$\tilde{\mathbf{x}}(k) = \begin{bmatrix} \mathbf{x}^{\mathrm{T}}(k+1) & \mathbf{x}^{\mathrm{T}}(k+2) & \cdots & \mathbf{x}^{\mathrm{T}}(k+N) \end{bmatrix}^{\mathrm{T}}$$
$$\tilde{\mathbf{u}}_{\text{hs}}(k) = \begin{bmatrix} u_{\text{hs}}(k) & u_{\text{hs}}(k+1) & \cdots & u_{\text{hs}}(k+N-1) \end{bmatrix}^{\mathrm{T}}$$
$$\tilde{\mathbf{t}}(k) = [t_{1,\text{mb}}(k)\ t_{2,\text{mb}}(k)\ t_{3,\text{mb}}(k)\ t_{4,\text{mb}}(k)\ t_{1,\text{hb}}(k)\ t_{2,\text{hb}}(k)\ t_{3,\text{hb}}(k)\ t_{4,\text{hb}}(k)]^{\mathrm{T}},$$

where $u_{\text{hs}}(k)$ is the gate position (m) of the Haringvliet sluices at time step k. The time instants $t_{1,\text{mb}}(k)$, $t_{2,\text{mb}}(k)$, $t_{3,\text{mb}}(k)$, and $t_{4,\text{mb}}(k)$ (s) are the moments at which the Maeslant barrier changes its state for the kth MPC step. Similarly, the time instants $t_{1,\text{hb}}(k)$, $t_{2,\text{hb}}(k)$, $t_{3,\text{hb}}(k)$, and $t_{4,\text{hb}}(k)$ (s) are the moments at which the Hartel barrier changes its state for the kth MPC step. The inputs of the TIO-MPC model are illustrated in Figure 4. The time instants are possibly beyond the length of the prediction horizon, which makes it possible to have no discrete state changes at all in the prediction horizon. The discrete-time nonlinear reservoir model of the Rhine-Meuse delta requires regular input sequences for the state of the Maeslant barrier and the Hartel barrier. Therefore, a transformation is needed from the time instants into regular input sequences. This transformation is done as follows:

$$\tilde{\mathbf{u}}_{\text{mb}}(k) = \begin{bmatrix} u_{\text{mb}}(k) & u_{\text{mb}}(k+1) & \cdots & u_{\text{mb}}(k+N-1) \end{bmatrix}^{\mathrm{T}},$$

with

$$u_{\text{mb}}(k+j) = \begin{cases} u_{\text{mb}}(k-1) & \text{if } j \le k_{1,\text{mb}} \\ & \text{or } k_{2,\text{mb}} \le j \le k_{3,\text{mb}} \\ & \text{or } j \ge k_{4,\text{mb}} \\ 1 - u_{\text{mb}}(k-1) & \text{otherwise,} \end{cases}$$

for $j = 0, .., N-1$, where $\tilde{\mathbf{u}}_{\text{mb}}(k)$ is the regular input sequence created from the time instants. The integer variables $k_{1,\text{mb}}$, $k_{2,\text{mb}}$, $k_{3,\text{mb}}$, and $k_{4,\text{mb}}$ are the discrete-time rounded equivalents of the continuous variables $t_{1,\text{mb}}(k)$, $t_{2,\text{mb}}(k)$, $t_{3,\text{mb}}(k)$, and $t_{4,\text{mb}}(k)$.

3) Optimization: The TIO-MPC receding horizon optimization problem consists of the model and the objective function that are discussed in the previous paragraphs. The model (12) relates the inputs of the water system to the evolution of the states (water levels) over the prediction horizon. The objective function (10) is a function of the inputs and the state evolution. These two relations together form a function that relates the inputs (degrees of freedom) to the value of the objective function:

$$J\big(\tilde{\mathbf{t}}(k), \tilde{\mathbf{u}}_{\rm hs}(k)\big) = f_{\rm opt}\big(\tilde{\mathbf{t}}(k), \tilde{\mathbf{u}}_{\rm hs}(k)\big). \tag{13}$$

In fact, the actual state $\mathbf{x}(k)$ of the system and the predicted disturbances $q_{1\rm d}(k+j)$, $q_{2\rm d}(k+j)$, $q_{3\rm d}(k+j)$, $h_{\rm hvh}(k+j)$, and $h_{\rm hs}(k+j)$, for $j = 0,\ldots,N-1$, are also inputs of this function. However, these inputs are not degrees of freedom in the optimization problem. They are constant in the optimization problem. Therefore, they are left out of (13). Function $f_{\rm opt}\big(\tilde{\mathbf{t}}(k), \tilde{\mathbf{u}}_{\rm hs}(k)\big)$ is the function to be minimized by the optimization algorithm, subject to the following constraints:

$$0 \le t_{1,\rm mb} \tag{14}$$
$$t_{1,\rm mb}(k) - t_{\rm min} \le t_{2,\rm mb}(k) \tag{15}$$
$$t_{2,\rm mb}(k) - t_{\rm min} \le t_{3,\rm mb}(k) \tag{16}$$
$$t_{3,\rm mb}(k) - t_{\rm min} \le t_{4,\rm mb}(k) \tag{17}$$
$$t_{4,\rm mb}(k) \le t_{\rm max} \tag{18}$$
$$0 \le t_{1,\rm hb} \tag{19}$$
$$t_{1,\rm hb}(k) - t_{\rm min} \le t_{2,\rm hb}(k) \tag{20}$$
$$t_{2,\rm hb}(k) - t_{\rm min} \le t_{3,\rm hb}(k) \tag{21}$$
$$t_{3,\rm hb}(k) - t_{\rm min} \le t_{4,\rm hb}(k) \tag{22}$$
$$t_{4,\rm hb}(k) \le t_{\rm max} \tag{23}$$
$$u_{\rm hs,min} \le u_{\rm hs}(k+j) \le \tilde{u}_{\rm hs,max}(k+j) \tag{24}$$

for $j = 0,..,N-1$, with:

$$\tilde{u}_{\rm hs,max}(k+j) = \begin{cases} u_{\rm hs,max} & \text{if } x_3(k+j) \ge h_{\rm hs} \\ u_{\rm hs,min} & \text{otherwise,} \end{cases} \tag{25}$$

where $t_{\rm min}$ (s) is the minimum time between two state changes, $t_{\rm max}$ (s) is the maximum value of $t_{4,\rm mb}$ (k) and $t_{4,\rm hb}(k)$ and is larger than the prediction horizon, $u_{\rm hs,min}$ and $u_{\rm hs,max}$ (m) are respectively the minimum and maximum gate positions of the Haringvliet sluices. The relation in (25) is the constraint of a one-directional flow through the Haringvliet sluices. The constraints (14)–(22) are constraints for the Maeslant barrier and the Hartel barrier and describe the order of the time instants.

The cost function is minimized using the nonlinear derivative-free optimization algorithm pattern search. The pattern search algorithm is started i times from i different initial solutions (i.e., multi-start optimization) until the end of the control cycle length. See for more information on pattern search [18].

The TIO-MPC optimization is now as follows:

1) A large set[2] of initial solutions is created.
2) The cost function values $f_{\rm opt}\big(\tilde{\mathbf{t}}(k), \tilde{\mathbf{u}}_{\rm hs}(k)\big)$ of the initial solutions of step 1 are calculated.
3) The initial solutions are ranked based on the cost function values calculated in the previous step. An initial solution with a lower cost function value is usually more promising than a initial solution with a higher cost function value.
4) A pattern search optimization is started with the most promising initial solution based on the ranking calculated in step 3. After convergence of the pattern search optimization, a new optimization is started with the next most promising initial solution. This procedure is repeated until time runs out.
5) The best solution calculated in Step 4 is selected as the output of the multi-start pattern search optimization.

[2]The initial solution set has to be larger than the number of pattern search optimizations that can be performed in the control cycle length. This ensures that the multi-start pattern search optimization algorithm uses the complete control cycle length to search for good control actions.

IV. Simulation experiments

A. Setup

We consider a simulation study in which the nonlinear reservoir model is used as the simulation model. The current local control systems and the TIO-MPC approach are implemented with a prediction horizon of 24 hours (equal to the current practice [17]) and a control cycle length of 10 and 30 minutes, respectively. It is assumed that the controllers have perfect predictions of the boundary conditions (the three river inflows and the two sea water levels) over the prediction horizon. The total simulation time span is 48 hours.

Many scenarios have been investigated to determine the potential of the proposed control system [14]. The scenarios have been created based on historical measurement data of Rijkswaterstaat [19]. This data set consists of historical data of November 7–9, 2007. This also includes the period in which the Maeslant barrier was closed due to storm conditions at sea. Due to space restrictions, here, we show illustrative results from one scenario only.

The considered scenario involves conditions due to a storm surge at sea and a sea level rise of 0.65 m. The flow of the river Rhine at Lobith (which gives an indication of the amount of water flowing into the Rhine Meuse delta) is 1 600 $\mathrm{m^3/s}$. This results in a maximum sea water level of 3.81 mMSL with relatively low discharges of the three rivers.

B. Result

Figure 5 shows the results of the simulation using the current control systems of the Rhine-Meuse delta. As can be observed, the Maeslant barrier and Hartel barrier are both closed for 20 hours. This long closure in combination with the relatively low inflows of the rivers Lek, Waal, and Meuse keeps the water levels at Rotterdam (y_1) and Dordrecht (y_2) very low. The area is therefore well protected against floods. However, the long closure is very expensive, since ocean vessels are blocked for more than 24 hours (4 hours before closure of the Maeslant barrier no navigation is allowed anymore. The discharge through the Haringvliet sluices is quite low (a volume of $15 \times 10^6\,\mathrm{m^3}$), since it is related to the relatively low flow of the river Rhine at Lobith.

Figure 6 shows the results of the simulation when using the TIO-MPC approach. We observe that instead of closing the Maeslant and the Hartel barrier both for a long period, the TIO-MPC approach only closes the Maeslant barrier for two short periods (3 hours in total). The maximum water

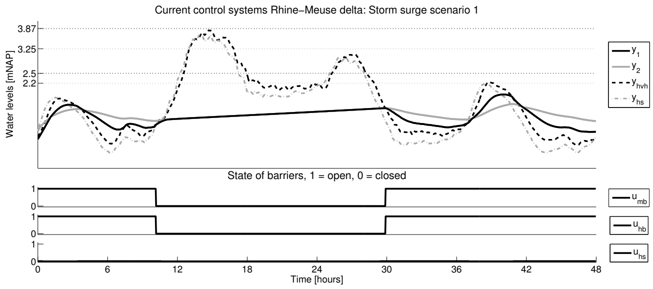

Fig. 5: Simulation results of the current control systems.

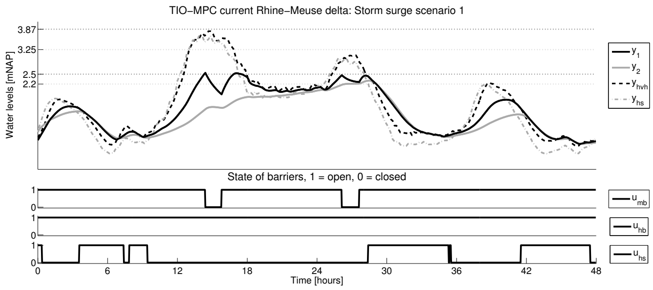

Fig. 6: Simulation results of the TIO-MPC approach. The dotted horizontal lines are respectively from down to top reference levels r_{21}, r_{11}, r_{22}, and r_{12}.

levels at Rotterdam and Dordrecht are just above respectively the first reference levels r_{11} and r_{12}. This clearly illustrates the trade-off between exceeding the first reference levels (where damage starts) and input effort (cost on closing the barriers) that the TIO-MPC controller considers. The Haringvliet sluices are maximally open when possible (i.e., when constraint $y_3 > y_{\text{hvh}}$ is not violated), resulting in a large discharge volume of $705 \times 10^6\,\text{m}^3$.

V. Conclusions & future research

In this paper we have proposed a model predictive control (MPC) approach for water systems represented as hybrid systems (i.e., involving both continuous and discrete dynamics). The approach proposed is based on so-called time-instant optimization (TIO). The idea of TIO-MPC is that the moments at which actions should take place are determined, rather than that for each time step it is determined whether an action should be taken or not (as is typically the case in more conventional predictive control approaches). In particular when considering hybrid MPC problems, involving discrete and continuous variables, this approach can be promising in terms of reduced computational requirements.In a simulation study based on the Rhine-Meuse delta in The Netherlands the potential of the approach has been illustrated, in particular when a trade-off has to be made between input effort and damage costs.

Future research focuses on the computational performance analysis of the proposed approach for future Rhine-Meuse delta setups and coordination among MPC controllers that control different, but interconnected, parts of large water systems.

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