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Technical report 12-014

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If you want to cite this report, please use the following reference instead:
Y. Li and B. De Schutter, "A robust feasibility problem for the design of a reference governor," *Proceedings of the 2012 American Control Conference*, Montréal, Canada, pp. 6745–6750, June 2012.

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* This report can also be downloaded via https://pub.bartdeschutter.org/abs/12_014

A Robust Feasibility Problem for the Design of a Reference Governor

Yuping Li and Bart De Schutter

***Abstract*— We formulate a robust feasibility problem for the design of a reference governor to provide setpoints for the lower-level control in a two-layer hierarchical system. Using linear programming duality, solutions to the robust feasibility problem (i.e. both necessary and sufficient conditions for the existence of an admissible reference) are given. Three cases are considered: 1) Fixed reference; 2) feedforward management; and 3) affine feedback management. The computationally efficient results can be implemented in supervisory control in SCADA networks.**

I. Introduction

Different methods for the design of a reference governor have emerged in process control [3], [4], [6], [13], [15]. These designs are based on the implementation of a two-layer hierarchical control strategy introduced in [2], see Fig. 1. The upper layer of the hierarchical strategy calculates the optimal plant operating point automatically, taking operating constraints into account while maximising economic profits [3] or minimising ecological losses [13]. The lower layer uses as setpoint the output generated by the upper layer to automatically track the operating point despite any disturbances affecting the plant. In fact, this is the strategy applied in most supervisory control approaches in SCADA (supervisory control and data acquisition) networks [12], where robustness is assumed to be dealt with by a lower-level controller while on the upper level the disturbance to the plant is assumed to be known. However, in industrial practices, the references determined in such a way may be aggressive and the lower-level system may run out of safe operation bounds in the presence of large disturbances. For example, in open water channel networks [14], to track the water-level setpoints given out by a supervisory controller and to ensure the system operate within the safety bounds for large water demands, an extra disturbance scheduler is required [9].

In this paper, we consider a robust feasibility problem for the design of a reference governor: Assume that on the lower level of the two-layer hierarchical control (see Fig. 1), a linear feedback controller has already been designed to stabilise the system and to guarantee the output of the linear plant z to track the reference r. On the upper level, a reference governor has to be designed to provide a feasible reference r such that for all disturbances $d \in \mathcal{D}$, the output z belongs to the safe set $\mathcal{Z}$; where $\mathcal{D}$ represents the set of possible disturbances while $\mathcal{Z}$ represents the safety constraints for the operation of the system. Three cases are discussed: 1) the reference is fixed; 2) feedforward management; and 3) affine feedback management. Following the framework investigating the computation of disturbance invariant sets for discrete-time, time-invariant, linear systems given in [8], it is shown that, by using linear programming duality, the resulting necessary and sufficient conditions for robust feasibility problem of the above three cases are affine in the control variables. These conditions can be checked in a computationally efficient manner using standard Linear Programming (LP) solvers. Hence, the results can be implemented in supervisory control in SCADA networks, where the computation load is always a problem being focused on.

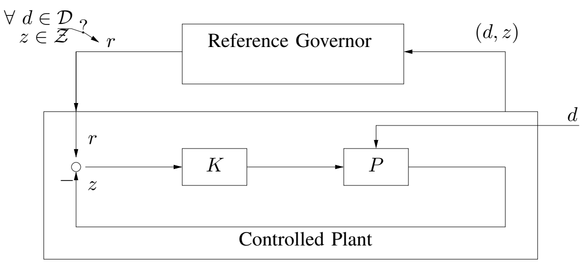

Fig. 1. Hierarchical control configuration

The paper is organised as follows. Section II describes the plant model considered in this paper. The formulation of the robust feasibility problem for the design of the reference governor is given in Section III. In Section IV, the three cases for the robust feasibility problem are discussed. Simulation results are given in Section V. A brief summary is finally given in Section VI.

II. Linear state-space model

For clarity, in the remainder of the paper, the description of the research problem is based on the control of open water networks (which can be seen as a large-scale system composed of many interconnected pools). The discrete-time lower-level controlled plant is represented in the state-space form as

$$x(k+1) = Ax(k) + Br(k) + Gd(k), \tag{1}$$

with the setpoint deviation $r(k)$ and the disturbance perturbation $d(k)$. The controlled output equation is

$$z(k) = Cx(k), \tag{2}$$

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This work has been supported in part by the European 7th framework STREP project "Hierarchical and distributed model predictive control of large-scale systems (HD-MPC)", contract number INFSO-ICT-223854, the Delft Research Center NGI, the BSIK project "Next Generation Infrastructures (NGI)", and the European 7th framework Network of Excellence "Highly-complex and networked control systems (HYCON2)".

with the water-level deviation $z(k)$. We define the control and disturbance vectors up to (and excluding) time k by $\mathbf{r} := (r(0), \ldots, r(k-1))^T$ and $\mathbf{d} := (d(0), \ldots, d(k-1))^T$. Note that for a system composed of N subsystems, $r(l) := (r_1(l), \ldots, r_N(l))^T$, and $d(l) := (d_1(l), \ldots, d_N(l))^T$ for $l = 0, \ldots, k-1$; where the water-level setpoint and the disturbance in pool i is denoted by r_i and d_i, respectively.

Assume that the system is initially at steady-state, which is $x(0) = 0$, and hence $z(0) = 0$. Up to time k, the system output can be expressed as

$$\mathbf{z} = \mathbf{B}\mathbf{r} + \mathbf{G}\mathbf{d}, \tag{3}$$

where $\mathbf{z} := (z(1), \ldots, z(k))^T$ with $z(l) := (z_1(l), \ldots, z_N(l))^T$, $\mathbf{B}$ and $\mathbf{G}$ being lower-triangular, Toeplitz matrices with the l-th row written as $\left(CA^{l-1}B, \ldots, CB, 0, \ldots, 0\right)$ and $\left(CA^{l-1}G, \ldots, CG, 0, \ldots, 0\right)$ respectively.

Remark 1: Most water-level setpoints in the practical channel control are calculated from historic data, which can be seen as the nominal setpoints to be filtered in Bemporad's reference governor construction [2]. In this paper, we omitted the nominal setpoints in Fig. 1 and define $r(k)$ as the setpoint deviation at time k. Hence the assumption of the system initial state, i.e. $x(0) = 0$, is reasonable. $\circ$

III. The robust feasibility problem

In this paper we focus on the situation that the reference vector $\mathbf{r}$ and the disturbance vector $\mathbf{d}$ in (3) are unknown but bounded, i.e. $\mathbf{r} \in \mathcal{R}$ and $\mathbf{d} \in \mathcal{D}$, where $\mathcal{R}$ and $\mathcal{D}$ are known, bounded sets that represent admissible management and disturbance trajectories up to time k respectively.

Remark 2: In the control of open water channels $\mathbf{d}$ contains water demands from farmers. Although these demands are normally scheduled, there exists uncertainty in these disturbances (e.g. starting and stopping time of the water offtakes or the flow needed). This motivates the requirement of $\mathbf{d} \in \mathcal{D}$, where $\mathcal{D}$ defines the largest water-demand deviation at the downstream ends of pools. The definition of such a set is based on historic data and the environmental consideration, e.g. weather forecasts. Similarly, the requirement of $\mathbf{r} \in \mathcal{R}$ is motivated by admissible water-levels in the pools (corresponding to 1) water capacity to satisfy water demands, and 2) channel safety, e.g. no water spillage over the banks of the channel). $\circ$

Following the analysis in [8] of the polyhedron which characterises the bounds, the set of admissible reference trajectories here is described by a polytopic model:

$$\begin{aligned}
\mathcal{R} &:= \{\mathbf{r} : \|\mathbf{r}_i\|_\infty \leq \sigma_i\} \\
&= \left\{ \mathbf{r} : \begin{bmatrix} I_{k\times k} \\ -I_{k\times k} \end{bmatrix} \begin{bmatrix} r_i(0) \\ \vdots \\ r_i(k-1) \end{bmatrix} \leq \begin{bmatrix} \sigma_i \\ \vdots \\ \sigma_i \end{bmatrix}_{(2k\times 1)} \right\} \quad (4) \\
&= \{\mathbf{r} : \mathbf{R}\Pi\mathbf{r} \leq \boldsymbol{\sigma}\}, \quad (5)
\end{aligned}$$

where $\mathbf{R} = \operatorname{diag}\{R, \ldots, R\}$ with $R = [I, -I]^T$ (I being the $kN \times kN$ identity matrix), $\boldsymbol{\sigma} = \left[\boldsymbol{\sigma}_1^T, \ldots, \boldsymbol{\sigma}_N^T\right]^T$ with $\boldsymbol{\sigma}_i \in \mathbb{R}_+^{2k\times 1}$ for $i = 1, \ldots, N$, $\boldsymbol{\sigma}_i = [\sigma_i, \ldots, \sigma_i]^T$. Moreover, $\Pi \in \{0,1\}^{(k\times N)\times(k\times N)}$ is a mapping, stacking the variables in an appropriate way:

$$\Pi = \left[\begin{array}{ccc}
1\ 0\ 0\ \ldots\ 0 & & \\
& \ddots & \\
& & 1\ 0\ 0\ \ldots\ 0 \\ \hline
0\ 1\ 0\ \ldots\ 0 & & \\
& \ddots & \\
& & 0\ 1\ 0\ \ldots\ 0 \\ \hline
& \vdots & \\ \hline
0\ 0\ \ldots\ 0\ 1 & & \\
& \ddots & \\
& & 0\ 0\ \ldots\ 0\ 1
\end{array}\right].$$

Similarly, we model the set of admissible disturbances by

$$\mathcal{D} \;:=\; \{\mathbf{d} : \mathbf{D}\Pi\mathbf{d} \leq \boldsymbol{\pi}\}, \tag{6}$$

where $\mathbf{D} = \operatorname{diag}\{D, \ldots, D\}$ with $D = [I, -I]^T$, $\boldsymbol{\pi} = \left[\boldsymbol{\pi}_1^T, \ldots, \boldsymbol{\pi}_N^T\right]^T$ with $\boldsymbol{\pi}_i \in \mathbb{R}_+^{2k\times 1}$ for $i = 1, \ldots, N$, $\boldsymbol{\pi}_i = [\pi_i, \ldots, \pi_i]^T$.

The output feasible set $\mathcal{Z}$ is then defined as the set of all admissible (controlled) output trajectories up to time k:

$$\mathcal{Z} \;:=\; \{\mathbf{z} : \mathbf{Z}\Pi\mathbf{z} \leq \boldsymbol{\tau}\}, \tag{7}$$

where $\mathbf{Z} = \operatorname{diag}\{Z, \ldots, Z\}$ with $Z = [I, -I]^T$, $\boldsymbol{\tau} = \left[\boldsymbol{\tau}_1^T, \ldots, \boldsymbol{\tau}_N^T\right]^T$ with $\boldsymbol{\tau}_i \in \mathbb{R}_+^{2k\times 1}$ for $i = 1, \ldots, N$, $\boldsymbol{\tau}_i = [\tau_i, \ldots, \tau_i]^T$.

Similar as the problem statement of "feasible control" in [1], we have the definition of "Robustly Feasible Management" as below:

Definition 3.1: An admissible reference $\mathbf{r} \in \mathcal{R}$ is robustly feasible *if and only if* for every admissible disturbance trajectory $\mathbf{d} \in \mathcal{D}$ the output trajectory of system (3) remains admissible, i.e. $\mathbf{z} \in \mathcal{Z}$.

Correspondingly, the following "robust feasibility problem" is formulated.

Problem 3.2: For the system (3) with sets of admissible reference trajectory (5), disturbance trajectory (6), and output trajectory (7), find necessary and sufficient conditions for the existence of a robust feasible reference trajectory.
Therefore, the problem is to find if there exists $\mathbf{r} \in \mathcal{R}$ such that for all $\mathbf{d} \in \mathcal{D}$, $\mathbf{z} = \mathbf{B}\mathbf{r} + \mathbf{G}\mathbf{d} \in \mathcal{Z}$.

IV. Solutions to the robust feasibility problem

The derivation of the results in this section follows the same lines as those given in [1] for the existence conditions of control trajectory. The difference is that an additional linear mapping Π, which was introduced in Section III to characterise the input (and output) variables of large-scale systems, is included in the analysis.

The following lemma is first presented as the basis for the results in this section.

Lemma 4.1: Given a vector $\mathbf{v}$ and a scalar δ, the condition $\mathbf{v}^T\mathbf{d} \leq \delta$ for every $\mathbf{d} \in \mathcal{D}$ is satisfied if and only if there exists a $\boldsymbol{\lambda}$ such that the following conditions are satisfied:

$$\begin{aligned} &\boldsymbol{\lambda} \geq 0, \\ &(\mathbf{D}\Pi)^T \boldsymbol{\lambda} = \mathbf{v}, \\ &\boldsymbol{\lambda}^T \boldsymbol{\pi} \leq \delta. \end{aligned}$$

Proof: $\mathbf{v}^T \mathbf{d} \leq \delta \ \forall \mathbf{d} \in \mathcal{D}$ if and only if

$$\delta \geq \max_{\mathbf{d}} \left\{ \mathbf{v}^T \mathbf{d} : \mathbf{D}\Pi\mathbf{d} \leq \boldsymbol{\pi} \right\}.$$

Directly following linear programming problem (LP) duality (see [11], Section 4.2), one has

$$\max_{\mathbf{d}} \left\{ \mathbf{v}^T \mathbf{d} : \mathbf{D}\Pi\mathbf{d} \leq \boldsymbol{\pi} \right\} = \min_{\boldsymbol{\lambda} \geq 0} \left\{ \boldsymbol{\lambda}^T \boldsymbol{\pi} : (\mathbf{D}\Pi)^T \boldsymbol{\lambda} = \mathbf{v} \right\}. \tag{8}$$

Note that the right-hand side of (8) is equivalent to
$\exists\ \boldsymbol{\lambda} \geq 0$ such that $(\mathbf{D}\Pi)^T \boldsymbol{\lambda} = \mathbf{v}$ and $\boldsymbol{\lambda}^T \boldsymbol{\pi} \leq \delta$. ■

Next, we consider three cases for the robust feasibility problem.

A. The case when the reference is fixed

For the case when the reference in the lower-level system is fixed, i.e. no deviation of the reference, set $\mathbf{r} = 0$ in (3). Then Problem 3.2 reduces to checking whether

$$\forall \mathbf{d} \in \mathcal{D},\ \mathbf{z} = \mathbf{G}\mathbf{d} \in \mathcal{Z} \tag{9}$$

Applying Lemma 4.1 row-wise to condition $\mathbf{Z}\Pi\mathbf{z} = \mathbf{Z}\Pi\mathbf{G}\mathbf{d} \leq \boldsymbol{\tau}$, one has

Corollary 4.2: Condition (9) holds if and only if there exists $\mathbf{M} = (M_{ij})$ such that

$$M_{ij} \geq 0, \tag{10}$$
$$(\mathbf{D}\Pi)^T \mathbf{M} = \mathbf{G}^T (\mathbf{Z}\Pi)^T, \tag{11}$$
$$\mathbf{M}^T \boldsymbol{\pi} \leq \boldsymbol{\tau}. \tag{12}$$

Proof: From Lemma 4.1, $\forall\ \mathbf{d} \in \mathcal{D},\ (\mathbf{Z}\Pi\mathbf{G})_h \mathbf{d} \leq \tau_i$ for $h = (i-1)k+1, \ldots, ik$ and $i = 1, \ldots, N$, if and only if $\exists\ (\mathbf{M})_h$ s.t.

$$(\mathbf{M})_h \geq 0, \tag{13}$$
$$(\mathbf{D}\Pi)^T (\mathbf{M})_h = (\mathbf{Z}\Pi\mathbf{G})_h^T, \tag{14}$$
$$(\mathbf{M})_h^T \boldsymbol{\pi} \leq \tau_i, \tag{15}$$

where $(\mathbf{M})_h$ is the h-th row of matrix $\mathbf{M}$. We see that condition (13) is equal to condition (10). Combining condition (14) for $h = (i-1)k+1, \ldots, ik$ and $i = 1, \ldots, N$ and condition (15) for $h = (i-1)k+1, \ldots, ik$ and $i = 1, \ldots, N$, we get conditions (11) and (12) respectively. Hence the corollary is proved. ■

Remark 3: Note that for the case of the fixed reference, the solution given in Corollary 4.2 is equivalent to the "maximal output admissible sets" defined in [7]. In fact, for linear time-invariant systems, such a problem can be formulated as a dual problem of characterising the maximal disturbance set, which is covered in [8]. ○

B. Feedforward management

We then check for the case of feedforward management: Let $\mathbf{r}$ be variable and the robust feasibility problem is to find $\mathbf{r}$ such that

$$\mathbf{r} \in \mathcal{R} \text{ and } \forall \mathbf{d} \in \mathcal{D},\ \mathbf{z} = \mathbf{B}\mathbf{r} + \mathbf{G}\mathbf{d} \in \mathcal{Z} \tag{16}$$

Again, applying Lemma 4.1 row-wise to condition $\mathbf{Z}\Pi\mathbf{z} = \mathbf{Z}\Pi(\mathbf{B}\mathbf{r} + \mathbf{G}\mathbf{d}) \leq \boldsymbol{\tau}$, it follows that

Corollary 4.3: Condition (16) holds if and only if there exists $\mathbf{M} = (M_{ij})$ and $\mathbf{r}$ such that

$$M_{ij} \geq 0, \tag{17}$$
$$\mathbf{R}\Pi\mathbf{r} \leq \boldsymbol{\sigma}, \tag{18}$$
$$(\mathbf{D}\Pi)^T \mathbf{M} = \mathbf{G}^T (\mathbf{Z}\Pi)^T, \tag{19}$$
$$\mathbf{M}^T \boldsymbol{\pi} + \mathbf{Z}\Pi\mathbf{B}\mathbf{r} \leq \boldsymbol{\tau}. \tag{20}$$

Proof: Directly following the definition of $\mathcal{R}$, condition (18) is equal to $\mathbf{r} \in \mathcal{R}$.

From Lemma 4.1, $\forall\ \mathbf{d} \in \mathcal{D},\ (\mathbf{Z}\Pi\mathbf{G})_h \mathbf{d} \leq \tau_i - (\mathbf{Z}\Pi\mathbf{B})_h \mathbf{r}$ for $h = (i-1)k+1, \ldots, ik$ and $i = 1, \ldots, N$, if and only if $\exists\ (\mathbf{M})_h$ s.t.

$$(\mathbf{M})_h \geq 0, \tag{21}$$
$$(\mathbf{D}\Pi)^T (\mathbf{M})_h = (\mathbf{Z}\Pi\mathbf{G})_h^T, \tag{22}$$
$$(\mathbf{M})_h^T \boldsymbol{\pi} \leq \tau_i - (\mathbf{Z}\Pi\mathbf{B})_h \mathbf{r}, \tag{23}$$

where $(\mathbf{M})_h$ is the h-th row of matrix $\mathbf{M}$. We see that condition (21) is equal to condition (17). Combining conditions (22) for $h = (i-1)k+1, \ldots, ik$ and $i = 1, \ldots, N$ and condition (23) for $h = (i-1)k+1, \ldots, ik$ and $i = 1, \ldots, N$, we get conditions (19) and (20) respectively.

Hence the corollary is proved. ■

Note the conditions (10)-(12) for the case of fixed-reference and the conditions (17)-(20) for the case of feedforward management can be checked through Linear Programming.

C. Affine feedback management

For the case of affine feedback management, it is assumed that the disturbance trajectory $\mathbf{d}$ is measured and the management trajectory $\mathbf{r}$ is an affine function of $\mathbf{d}$. In particular,

$$\mathbf{r}(\mathbf{d}) = \mathbf{w} + \mathbf{L}\mathbf{d}. \tag{24}$$

To impose the condition that the reference is an affine function of *past* disturbances, we here require $\mathbf{L}$ to be a block lower-triangular matrix.

Remark 4: In robust MPC this type of reference parameterisations has already been applied (see [5]). Indeed, it is shown in [5] that control parameterization (24) is equivalent to the one where the control is an affine function of past states. Note such a consideration is sensible for the reference governing problem in water management system since the influence of disturbances (i.e. water offtakes in the pools) on the water-level deviations can be modeled as an integrator (see Section V-A). ○

In this case, the robust feasibility problem is to find $\mathbf{w}$ and a block lower-triangular $\mathbf{L}$ such that

$$\forall \mathbf{d} \in \mathcal{D},\ \mathbf{w}+\mathbf{L}\mathbf{d} \in \mathcal{R},\ \mathbf{z} = \mathbf{B}\left(\mathbf{w}+\mathbf{L}\mathbf{d}\right)+\mathbf{G}\mathbf{d} \in \mathcal{Z}. \quad (25)$$

Such a problem can be investigated in the following two steps:

1) The admissibility of the function $\mathbf{r}\left(\mathbf{d}\right) = \mathbf{w}+\mathbf{L}\mathbf{d} \in \mathcal{R}$ (L block lower-triangular) for every $\mathbf{d} \in \mathcal{D}$. Based on the previous development, this is guaranteed by the following necessary and sufficient condition:
$\exists\ \mathbf{N} = (N_{ij})$, $\exists\ \mathbf{w}$, $\exists\ \mathbf{L}$ block lower-triangular, s.t.

$$N_{ij} \geq 0, \quad (26)$$
$$\left(\mathbf{D}\Pi\right)^T \mathbf{N} = \mathbf{L}^T \left(\mathbf{R}\Pi\right)^T, \quad (27)$$
$$\mathbf{N}^T \boldsymbol{\pi} + \mathbf{R}\Pi\mathbf{w} \leq \boldsymbol{\sigma}. \quad (28)$$

The proof of the above *iff* condition follows the same lines as the proof of Corollary 4.3.

2) For fixed $\mathbf{w}$ and $\mathbf{L}$, the admissibility of output $\mathbf{z}$ is such that $\mathbf{B}\left(\mathbf{w}+\mathbf{L}\mathbf{d}\right)+\mathbf{G}\mathbf{d} \in \mathcal{Z}$ for every $\mathbf{d} \in \mathcal{D}$. Such a constraint is guaranteed by the following necessary and sufficient condition: $\exists\ \mathbf{M} = (M_{ij})$ s.t.

$$M_{ij} \geq 0, \quad (29)$$
$$\left(\mathbf{D}\Pi\right)^T \mathbf{M} = \left(\mathbf{B}\mathbf{L}+\mathbf{G}\right)^T \left(\mathbf{Z}\Pi\right)^T, \quad (30)$$
$$\mathbf{M}^T \boldsymbol{\pi} + \mathbf{Z}\Pi\mathbf{B}\mathbf{w} \leq \boldsymbol{\tau}. \quad (31)$$

Again, the proof of the above *iff* condition follows the same lines as the proof of Corollary 4.3.

Hence, we have the following theorem for the robust feasibility problem.

Theorem 4.4: Condition (25) holds if and only if there exists $\mathbf{M} = (M_{ij})$, $\mathbf{N} = (N_{ij})$, $\mathbf{w}$, and block lower-triangular $\mathbf{L}$ such that conditions (26) - (31) are satisfied.

Since the conditions (26) - (31) are affine inequalities in the decision variables $\mathbf{M}$, $\mathbf{N}$, $\mathbf{w}$, $\mathbf{L}$, they can be checked through linear programming.

V. Case Studies

In open water channel control, an important control objective is setpoint regulation of the water-levels in the pools, which enables flow demand at the (often gravity-powered) offtake points to be met without over-supplying [14]. When the number of pools to be controlled is large and the gates widely dispersed, it is natural to employ a decentralised control structure, see Fig. 2. The flow into pool$_i$, denoted by u_i, equals the flow supplied by the upstream pool, v_{i-1}. Note that u_i is actually the control action taken by controller K_i to regulate the water-level y_i to a relevant setpoint r_i, in the face of disturbances associated with variations of the uncontrolled offtake load d_i.

In practice, channel capacity is limited. Moreover, the time delay for water to travel from the upstream end to the downstream end of the pool limits the closed-loop bandwidth, which dampens the performance. Hence, the starting and ending of offtakes (d_i) induce transients (i.e. the water-level drops and rises from its setpoint). Such a transient response propagates to upstream pools as regulators take corrective actions [10]. Hence, the open water channel management objectives can be expressed in terms of constraints on the water-levels in each pool: upper bounds avoid water spillage over the banks of the channel; and lower bounds ensure a minimal channel capacity to supply water. In robust reference management, the setpoints are adjusted, which ensures that the water-level constraints are satisfied, in the face of transients associated with load changes within certain constraints.

A. Plant model

Following [9], the evolution of the water-levels in a channel of N pools with decentralised control can be described by the following continuous state-space model:

$$\dot{x}(t) = \tilde{A}x(t) + \tilde{B}r(t) + \tilde{G}d(t) \quad (32)$$
$$y(t) = \tilde{C}x(t),$$

where $\tilde{A} = \begin{bmatrix} \tilde{A}_1 & \tilde{A}_{p_1} & & \\ & \tilde{A}_2 & \tilde{A}_{p_2} & \\ & & \ddots & \ddots \\ & & & \tilde{A}_N \end{bmatrix}$, $\tilde{B} = \operatorname{diag}\left(\tilde{B}_{r_1}, \ldots, \tilde{B}_{r_N}\right)$, $\tilde{G} = \operatorname{diag}\left(\tilde{B}_{d_1}, \ldots, \tilde{B}_{d_N}\right)$, and $\tilde{C} = \operatorname{diag}\left(\tilde{C}_1, \ldots, \tilde{C}_N\right)$ with $\tilde{A}_i = \begin{bmatrix} 0 & c_{\text{in},i} & -c_{\text{in},i} & 0 \\ 0 & \frac{-2}{t_{\text{d},i}} & \frac{4}{t_{\text{d},i}} & 0 \\ \frac{-\kappa_i}{\rho_i} & 0 & 0 & 1 \\ \frac{-\kappa_i(\rho_i-\phi_i)}{\phi_i\rho_i^2} & 0 & 0 & \frac{-1}{\rho_i} \end{bmatrix}$, $\tilde{A}_{p_i} = \begin{bmatrix} -c_{\text{out},i} \\ 0 \\ 0 \\ 0 \end{bmatrix}$, $\tilde{B}_{d_i} = \begin{bmatrix} -c_{\text{out},i} \\ 0 \\ 0 \\ 0 \end{bmatrix}$, $\tilde{B}_{r_i} = \begin{bmatrix} 0 \\ 0 \\ \frac{\kappa_i}{\rho_i} \\ \frac{\kappa_i(\rho_i-\phi_i)}{\phi_i\rho_i^2} \end{bmatrix}$, $\tilde{C}_i = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}$, where $c_{\text{in},i}$ and $c_{\text{out},i}$ are discharge coefficients, functions of the pool surface area and the gate width; and $t_{\text{d},i}$ is the internal time delay that the water takes to travel from the upstream end to the downstream end of a pool;[1] κ_i, ρ_i and ϕ_i are parameters of the decentralised feedback controller K_i, which is a PI compensator with a low-pass filter as follows:

$$K_i = \frac{\kappa_i}{\phi_i} \frac{(1+s\phi_i)}{s(1+s\rho_i)}$$

Note that the interconnection between neighboring (controlled) pools $v_i = u_{i+1}$ is expressed in the off-diagonal entries of $\tilde{A}$ (i.e. $\tilde{A}_{p_i}$). To build the prediction model, a discrete-time state-space model of the form (1-2) is employed. This can be obtained by directly converting the continuous model through a zero-order hold. The sampling interval T_s should be small enough to capture the whole relevant dynamics of the system. In the case studies in Section V-B, the sampling time is set to 5 minutes.

B. Simulation results

The robust reference governing approach is applied to two pools (i.e. Campbells and Schifferlies) of the East Goulburn Main (EGM) Channel, Victoria, Australia. The parameters of controlled pools are given in Table I. The

[1] A first-order Padé approximation is used to represent the transportation time delay $t_{\text{d},i}$. This is reasonable in the modelling since the feedback controller K_i involves a low-pass filter such that high-frequency resonance (caused by the time delay) is dampened.

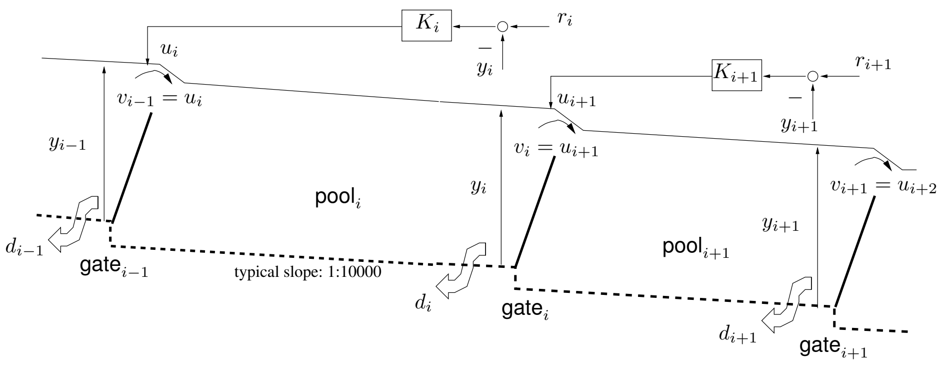

Fig. 2. Decentralised control of an open water channel

Pool	$c_{\text{in},i}$	$c_{\text{out},i}$	τ_i
1	0.055	0.036	5 min
2	0.017	0.026	6 min
Controller	κ_i	ϕ_i	ρ_i
1	0.74	71.83	8.52
2	1.19	141.27	16.75

TABLE I
PARAMETERS OF (CONTROLLED) POOLS

Case	Maximum disturbance deviation		Maximum reference deviation		Maximum water-level deviation	
	π_1	π_2	σ_1	σ_2	τ_1	τ_2
	(Ml/day)		(m)		(m)	
Fixed reference	7	6	0	0	0.10	0.06
Feedforward management	11	7	0.08	0.05	0.10	0.06
Feedback management	50	35	0.08	0.05	0.10	0.06

TABLE II
PARAMETERS DESCRIBING ADMISSIBLE DISTURBANCE, REFERENCE AND WATER-LEVEL TRAJECTORIES

steady-state water-levels of the two pools are 1.5 and 1.56 m, respectively. The prediction horizon is 480 steps (of 5 minutes), which corresponds to a forecast of 40 hours[2]. Following the procedure outlined in Section II, the matrices $\mathbf{B}$ and $\mathbf{G}$ in (3) are constructed.

The admissible output trajectories for the two pools, which are represented by the polytope $\mathcal{Z}$, are set as: $\boldsymbol{\tau}_1$ and $\boldsymbol{\tau}_2$ are constant vectors with entries 0.1 m and 0.06 m, respectively. These requirements impose the constraint that the water-level deviations must remain within ± 0.1 m (in pool$_1$) and ± 0.06 m (in pool$_2$) over the prediction horizon. The robust feasibility problem is solved for the following cases: 1) without reference deviation, 2) feedforward management, and 3) feedback management. So, we check for each of the three cases by equations (10-12), (17-20), and (26-31), respectively, for the existence of a robustly feasible solution.

For the cases of feedforward management and of disturbance feedback management, the admissible reference trajectories for the two pools, which are represented by the polytope $\mathcal{R}$, are set as: $\boldsymbol{\sigma}_1$ and $\boldsymbol{\sigma}_2$ are constant vectors with entries 0.08 m and 0.05 m, respectively. These requirements impose the constraint that the water-level setpoint deviations must remain within ± 0.08 m (in pool$_1$) and ± 0.05 m (in pool$_2$) over the prediction horizon. The set of admissible disturbance trajectories, modeled by the polytope $\mathcal{D}$, is defined by setting $\boldsymbol{\pi}_1$ and $\boldsymbol{\pi}_2$ as constant vectors with entries π_1 and π_2, respectively. We start from small π_1 and π_2 and increase the set $\mathcal{D}$ systematically until the conditions for existence of the robustly feasible solution are no longer feasible.[3] Table II lists the maximum values of π_1 and π_2 for which these conditions remain feasible for the three cases respectively. We see that for the case without reference variation and for the case of feedforward management, the admissible set of disturbance trajectories is much smaller than for the case of feedback management, which is within expectation.

We then test the performance of the feedback management, the disturbance trajectory is set as shown in Fig. 3; note that the largest disturbance deviations (in pool$_1$ and pool$_2$) correspond to the maximum admissible disturbances listed in Table II. For comparison, the response of the lower-level system with the original references (the thick dash-dotted lines) is also given (see the thin dash-dotted lines in Fig. 4). The upper bound and lower bound constraints on the water-levels are violated at some time instants (around 275 min and around 1500 min) in the prediction horizon. In contrast, under the calculated references (the thick solid lines), the dynamics of the system is within the water-level constraints (see thin solid lines in Fig. 4).

VI. CONCLUSION AND FUTURE WORK

This paper has discussed the formulation of a robust feasibility problem for the design of reference governors in a two-layer hierarchical control. The constraints on the

[2] A larger forecast horizon should be selected when the influence of environment, e.g. rain, on the water-levels in the pools is included in the plant model (32), which is not the case for the simulation in this section; note that d_i, as introduced, represents water offtakes from pool$_i$.

[3] The bisection method has been used for the selection of π_1 and π_2. Note that in the simulation, priority was given to π_2, considering the propagation of the system transients in the upstream direction [10].

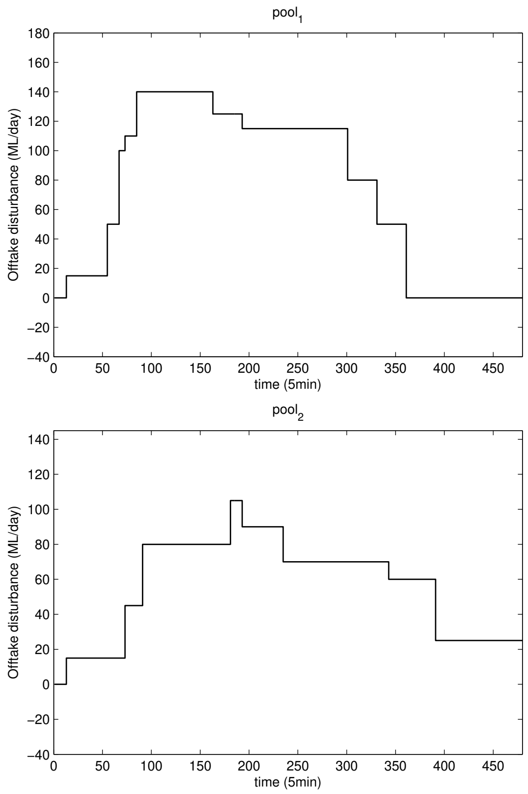

Fig. 3. Offtake disturbances in pool$_1$ and pool$_2$

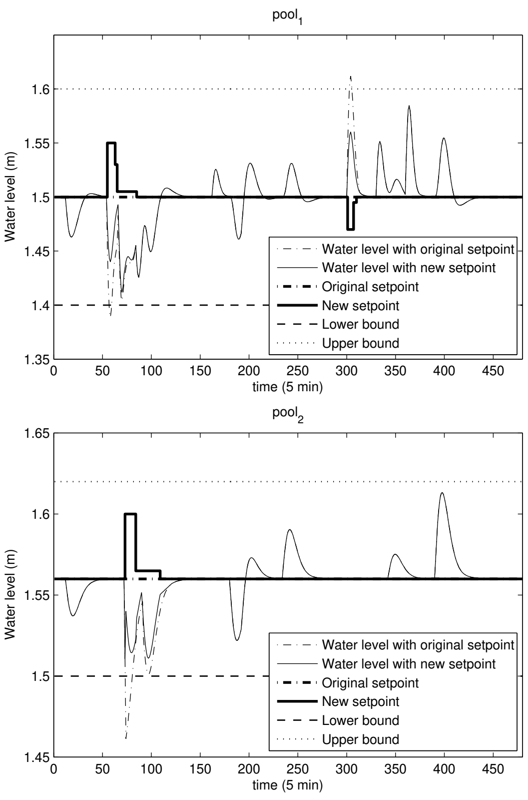

Fig. 4. Reference governing for feedback case with constraint on offtake demand; pool$_1$ and pool$_2$

admissible set of disturbance, reference, and output trajectories are incorporated in the formulation of the robust governor. Necessary and sufficient conditions that are affine in the decision variables are given. Using LP solvers, these conditions can be checked efficiently. The proposed reference governor design approach can be applied in supervisory control in SCADA networks.

Future work will extend to considering other forms of admissible constraints in the design of the reference governor. For example, by appropriately modifying the polytopes, the admissible set of trajectories can be allowed to be time varying. Furthermore, the result of this paper, which is based on the assumption of the lower-level system being linear, could be somehow conservative when applied in real system operation. To deal with this issue, an appropriate strategy for the cooperation between the reference governor and the disturbance scheduler might be a solution.

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