Delft University of Technology

Delft Center for Systems and Control

Technical report 12-027

Optimal trajectory planning for trains under operational constraints using mixed integer linear programming*

Y. Wang, B. De Schutter, T.J.J. van den Boom, and B. Ning

If you want to cite this report, please use the following reference instead:
Y. Wang, B. De Schutter, T.J.J. van den Boom, and B. Ning, "Optimal trajectory planning for trains under operational constraints using mixed integer linear programming," *Proceedings of the 1st European Symposium on Quantitative Methods in Transportation Systems (LATSIS 2012)*, Lausanne, Switzerland, 4 pp., Sept. 2012. Paper 38.

Delft Center for Systems and Control
Delft University of Technology
Mekelweg 2, 2628 CD Delft
The Netherlands
phone: +31-15-278.51.19 (secretary)
fax: +31-15-278.66.79
URL: http://www.dcsc.tudelft.nl

Optimal Trajectory Planning for Trains under Operational Constraints using Mixed Integer Linear Programming

Yihui Wang *, Bart De Schutter, Ton J.J. van den Boom
Delft Center for Systems and Control,
Delft University of Technology, Delft, The Netherlands,

Bin Ning
State Key Laboratory of Traffic Control and Safety,
Beijing Jiaotong University, Beijing, P.R. China,

**Email: yihui.wang@tudelft.nl

1 Introduction

Because of the rising energy prices and environmental concerns, the energy efficiency of transportation systems becomes more and more important. This also includes railway networks since rail traffic plays an important role in public transportation systems and since it is important for the sustainability of transportation systems [1, 2]. Advanced train control systems, e.g. the European Train Control System, enable energy-efficient driving of trains and also fulfill the safety and operational requirements for high-speed lines and dedicated urban rapid transit railway systems with short headways [3]. The automatic train operation (ATO) subsystem of advanced train control system plays a key role in ensuring accurate stopping, operation punctuality, energy saving, and riding comfort [2]. The ATO subsystem is responsible for calculating the optimal speed-position reference trajectory based on the information collected by train control systems, such as line conditions, maximum traction force, and braking performance, etc. Therefore, an efficient algorithm to find the optimal speed-position trajectory is significant to the driving performance of the ATO subsystem. In [4] we have proposed a mixed integer linear programming (MILP) approach to solve the optimal trajectory problem for a single train. The resulting MILP problem can be solved efficiently using existing commercial and free solvers that guarantee finding the global optimum. The varying line resistance, variable speed restrictions, and constant maximum traction force are taken into account in the optimal trajectory planning problem in [4]. However, the traction force is a nonlinear function and there exist operational constraints that could result from the timetable or from the real-time rescheduling process [5, 6]. For example, in order not to get hindered by a preceding train, a train may be required not to pass a location earlier than a certain time. On the other hand, in order not to hinder a following train, a train may have to be at a certain location not later than a scheduled time. These constraints are essential to the success of the real-time operation and the rescheduling process for railway networks. Here, we include these constraints in the MILP approach. In addition, we propose the pseudospectral method to solve the trajectory planning problem for the first time in the railway context. The performance of the new MILP approach and the pseudospectral method are compared with the dynamic programming approach in the literature.

2 Solution approaches

In the proposed approaches, the varying line resistance, variable speed restrictions, varying maximum traction force, and operational constraints are included in the optimal trajectory planning problem for train operation. The objective function is a trade-off between the energy consumption and the riding comfort. Two approaches are proposed to solve this optimal control problem for a single train. First, a state-of-the-art method for optimal control problems, viz. the pseudospectral method, is adopted, that has not been used for train optimal control before. In the pseudospectral method, the optimal control problem of train operation is recast into a multiple-phase optimal control problem, which is then transformed into a nonlinear programming problem. However, the calculation time for the pseudospectral method is too long for the real-time application in an ATO subsystem. To shorten the computation time, a mixed-integer linear programming (MILP) problem is formulated by approximating the nonlinear terms in the trajectory planning problem by piecewise affine functions. The MILP problem can be solved efficiently by existing solvers that guarantee the global optimum.

The operational constraints result from the timetable, real-time operation restrictions, or the real-time rescheduling process. Albrecht et al. [6, 7] classified these operational constraints into two groups: target points and target windows. Target points correspond to fixed passing times,

Table 1: Performance comparison of the pseudospectral method, the MILP approach, and the dynamic programming approach

	Pseudospectral method	MILP	Dynamic programming
$J_{\min}$	$2.625 \cdot 10^8$	$2.819 \cdot 10^8$	$2.683 \cdot 10^8$
CPU time [s]	1147	0.54	134
End position s_{end} violation [m]	0	0	0
End kinetic energy E_{end} violation [m/s^2]	0.1	0.005	0.0328
End time T_{end} violation [s]	0	4.170	5.404
Speed limit violation	No	No	No

which could be arrival and departure times at stations. In dense networks, target points could also be passing times at certain places where overtaking and crossing of trains is planned. If the passing time is not that strict but is characterized by an earliest arrival time and a tolerated delay, then it forms a target window constraint. The scheduled arrival times at minor stations without connections with other trains can be regarded as target windows. If the train reaches a certain place exactly on time according to the defined target point or in the target window, then conflicts can be avoided. It is worth to note that the operational constraints can be easily included in the trajectory planning problem as linear constraints in both the pseudospectral method and the MILP approach.

3 Simulation results and conclusions

Based on the simulation results of the pseudospectral method, the MILP approach, and a discrete dynamic programming approach as shown in Table 1, it is concluded that the pseudospectral method has the best control performance, but that if the computation time is included, the MILP approach provides an excellent trade-off between control performance and computation time. More specifically, the control performance of the pseudospectral approach is about 10% better than that of the MILP approach and the computation time of the MILP approach is two to three orders of magnitude smaller than that of the pseudospectral method and the discrete dynamic programming approach.

4 Future Work — optimal trajectory planning for multiple trains

Based on the optimal trajectory planning problem for a single train, solving the optimal trajectory planning problem for multiple trains running simultaneously on the same line or network will be a topic for future work. As a beginning point, we will consider two trains running on the same line. Therefore, more constraints need to be considered during the trajectory planning process for the following train. These constraints are caused by the leading train and depend on the signaling system, e.g. the fixed blocking signaling system and the moving blocking signaling system. The mixed-integer linear programming approach as well as other optimization methods will be investigated to solve this planning problem for multiple trains.

References

[1] R. Liu and I. M. Golovicher, “Energy-efficient operation of rail vehicles,” *Transportation Research Part A: Policy and Practice*, vol. 37, no. 10, pp. 917–931, Oct. 2003.

[2] H. Peng, *Urban Rail Transit System*. Beijing, China: China Communication Press, 2008.

[3] I. Hansen and J. Pachl, *Railway, Timetable & Traffic: Analysis, Modelling, Simulation*. Hamburg, Germany: Eurailpress, 2008.

[4] Y. Wang, B. De Schutter, B. Ning, N. Groot, and T. van den Boom, “Optimal trajectory planning for trains using mixed integer linear programming,” in *Proceedings of the 14th International IEEE Conference on Intelligent Transportation Systems (ITSC 2011)*, Washington, DC, USA, Oct. 2011, pp. 1598–1604.

[5] M. Lüthi, *Improving the Efficiency of Heavily Used Railway Networks through Integrated Real-time Rescheduling*. ETH Zürich, Switzerland: Ph.D. thesis, 2009.

[6] T. Albrecht, C. Gassel, A. Binder, and J. van Luipen, “Dealing with operational constraints in energy efficient driving,” in *Proceedings of the IET Conference on Railway Traction Systems (RTS 2010)*, Birmingham, UK, Apr. 2010, pp. 1–7.

[7] T. Albrecht, C. Gassel, and A. Binder, “An overview on real-time speed control in rail-bound public transportation systems,” in *Proceedings of the 2nd International Conference on Models and Technologies for Intelligent Transportation Systems*, Leuven, Belgium, Jun. 2011, pp. 1–4.