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Non-Linear Model Predictive Control Based on Game Theory for Traffic Control on Highways

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Abstract: This paper presents a methodology for traffic control using a distributed model predictive control scheme based on game theory (GT-DMPC). Using the traffic model METANET as prediction model, the control objective is to minimize the total time spent by vehicles in the traffic network. The proposed control methodology is compared with a centralized model predictive control approach and a non-controlled-case. The simulations show that the GT-DMPC controller efficiently distributes the vehicles entering the highway, and presents a similar performance in comparison with centralized MPC.

Keywords: Traffic Control, Model Predictive Control, Second Order Traffic Models, Game Theory

1. INTRODUCTION

Sustainable mobility of the people is a key issue in the modern society. However, nowadays many traffic networks are operating in an inefficient way, producing several negative impacts in the environment and the deterioration of the quality of life in general. Solutions like building new roads or improving the infrastructure now installed is not always possible because of environmental and/or budget regulations. Thus, developing efficient management and control systems for traffic and transportation, to satisfy the always increasing demand for mobility, is nowadays a crucial topic of research.

Among the different approaches existing in the literature, the ones based on model predictive control (MPC) have been quite broadly used and successfully applied to solve traffic problems. These techniques are mainly focused on the optimal use of the information provided by the infrastructure already installed, and on reducing the total travel time while considering explicitly the physical and operational constraints of the system (Kotsialos et al., 2002a; Bellemans et al., 2006; Hegyi et al., 2005; van den Berg et al., 2003). However, despite of the advantages of MPC over other methods, the application of this control scheme in real large-scale systems (such as energy transportation systems, water distribution systems, traffic systems, etc.) resulted impractical due to the computational burden of its centralized applications (Camponogara et al., 2002; Cortes et al., 2008).

In order to make possible the real implementation of MPC in large-scale systems, different distributed model predictive control (DMPC) approaches have been proposed. Specifically, in this paper we will focus on the application of distributed predictive control based on game theory (GT-DMPC) (Giovanini and Balderud, 2006; Maestre et al., 2009). In GT-DMPC, the local control actions of a subsystem are computed in a cooperative way, considering the preferences of other local controllers. Although such approaches have been reported to produce good results, the communication requirements of the schemes are crucial, because the computational burden may increase drastically if each local controller is required to communicate many times its preferences and then to have to solve more than one optimization problem at each time step. This issue is particularly important in traffic control applications, as traffic or models like METANET are non-linear, and thus the controllers have to deal with those non-linearities. In order to tackle this drawback, in (Alvarado et al., 2011; Valencia et al., 2011; Valencia, 2012) a DMPC scheme based on bargaining games was proposed, where the local controllers are able to decide whether to cooperate or not based on the benefit perceived from the cooperative behavior. However, this scheme (as well as in the other GT-DMPC approaches) only has been designed and tested for linear models. Thus, in this paper, we propose to extend the formulation of GT-DMPC to non-linear systems, focusing on traffic control purposes using the macroscopic traffic model METANET as prediction model.

The paper is organized as follows: In Section 2 the METANET macroscopic traffic model is introduced. Then, in Section 3 the formulation of the non-linear MPC for total travel time reduction in a highway is presented. In Section 4 the traffic problem is formulated as a bargaining game. Finally, in Sections 5 and 6 simulation results and concluding remarks are presented.

2. MACROSCOPIC TRAFFIC MODEL METANET

Let us start by introducing some concepts and notations related with the traffic model used in this work, viz. the METANET model described in (Papageorgiou et al., 1990; Kotsialos et al., 1999, 2002b). There are links and nodes. The links have homogeneous properties, so on-ramps, off-ramps, merge/split of links, lane drop, yields node. The links are imaginary divisions of a highway formed by a sequence of adjacent stretches called segments. Each segment is characterized by its length (L_m), number of lanes (λ_m), vehicle density ($\rho_{(m,i)}$), mean speed ($v_{(m,i)}$), and output flow ($q_{(m,i)}$), with m denoting the link number and (m,i) denoting the segment i of the link m. The dynamic evolution of the density of vehicles in the segment (m,i) is given by:

$$\rho_{(m,i)}(k+1) = \rho_{(m,i)}(k) + \frac{T_\mathrm{s}}{\lambda_m L_m}[q_{(m,i-1)(k)} - q_{(m,i)(k)}] \tag{1}$$

where T_s is the sample time (often 10 s). In order to determine the dynamic evolution of the vehicles in the segment (m,i), the following tree terms are considered.

(1) Relaxation $R_{(m,i)}(k)$, which expresses the desire of the drivers to achieve a desired speed for the current density:

$$R_{(m,i)}(k) = \frac{T_\mathrm{s}}{\tau}[V(\rho_{(m,i)}(k)) - v_{(m,i)}(k)] \tag{2}$$

(2) Convection $C_{(m,i)}(k)$, which quantifies the effect of the changes in speed in one segment due to the speed difference with the previous segment:

$$C_{(m,i)}(k) = \frac{T_\mathrm{s} v_{(m,i)}(k)[v_{(m,i-1)}(k) - v_{(m,i)}(k)]}{L_m} \tag{3}$$

(3) Anticipation $A_{(m,i)}(k)$, which models the effect of the changes in the speed in one segment due to the difference in density with the next segment:

$$A_{m,i}(k) = -\frac{T_\mathrm{s} n[\rho_{(m,i+1)}(k) - \rho_{(m,i)}(k)]}{\tau L_m(\rho_{(m,i)}(k) + \mu)} \tag{4}$$

By considering the relaxation, convection, and anticipation terms, the dynamic evolution of the mean speed in the segment (m,i) is given by:

$$v_{(m,i)}(k+1) = v_{(m,i)}(k) + R_{(m,i)}(k) + C_{(m,i)}(k) + A_{(m,i)}(k) \tag{5}$$

where the output flow is given by

$$q_{(m,i)}(k) = \lambda_m \rho_{(m,i)}(k) v_{(m,i)}(k) \tag{6}$$

Note that in Eq. (2) the evolution of the relaxation term is determined by the function $V(\cdot)$ for which an empirical relation can be given by:

$$V\left(\rho_{(m,i)}(k)\right) = v_{\mathrm{free},m} \exp\left(-\frac{1}{b_m}\left(\frac{\rho_{(m,i)}(k)}{\rho_{\mathrm{cr},m}}\right)^{b_m}\right) \tag{7}$$

where $v_{\mathrm{free},m}$ is the speed of a vehicle in free-flow. In this relation, $\rho_{\mathrm{cr},m}$ is the critical density of vehicles. The critical density of vehicles determines the behavior of the traffic in a link. If the density of vehicles remains less than its critical value, then the traffic flow is considered as a laminar flow, consequently the mean speed becomes higher and the time spent by the users in the highway decreases as the density of vehicles tends to zero. Otherwise, the traffic flow goes towards an instability region or congestion region, consequently the mean speed goes quickly to zero which is a symptom of congestion in the link.

For a link without on-ramp or other access roads, a model given by Eqs. (1)–(7) is enough. However, if there exist other access roads to the link, the interaction between the traffic on the access road and the traffic on the link should be included. With this purpose, the concept of origin node is introduced. An origin node allows the access of traffic from an external road, where d_o denotes the demand at the origin o. This traffic accessing a link by on-ramp o often is limited or controlled by a traffic light (or ramp metering), where $r_o(k)$ denotes the ramp metering rate, used to regulate the vehicles accessing the highway. Let $q_o(k)$ be the flow of vehicles incoming from the origin o to the link which origin o is connected $(1,m)$. The value of $q_o(k)$ is determined by the minimum value between the number of vehicles waiting to access the highway and the number of vehicles able to access. Let w_o denote the queue of vehicles on the origin node o. Then, the dynamic evolution of the queue is given by

$$w_o(k+1) = w_o(k) + T_\mathrm{s}(d_o(k) - q_o(k)) \tag{8}$$

where q_o is defined as

$$q_o(k) = \min\left(d_o(k) + \frac{w_o(k)}{T_\mathrm{s}}, \quad Q_o r_o(k), \quad Q_o\left(\frac{\rho_{\max,(m,1)} - \rho_{(m,1)}(k)}{\rho_{\max,(m,1)} - \rho_{\mathrm{cr},m}}\right)\right) \tag{9}$$

Q_o is the on-ramp capacity (veh/h) under free-flow conditions and $\rho_{\max,(m,1)}$ being the maximum density of vehicles able to access the segment 1 of link m. By including $q_{(m,1)}$ in Eqs. (1) and (5) we get:

$$\rho_{(m,1)}(k+1) = \rho_{(m,1)}(k) + \frac{T_\mathrm{s}}{L_m\lambda_m}(q_{\mathrm{in},(m,1)}(k) - q_{\mathrm{out},(m,1)}(k)) \tag{10}$$

$$v_{(m,1)}(k+1) = v_{(m,1)}(k) + R_{(m,i)}(k) + C_{(m,i)}(k) + A_{(m,1)}(k) - \frac{\delta T_\mathrm{s} q_o(k) v_{(m,1)}(k)}{L_m\lambda_m(\rho_{(m,i)}(k) - \mu)} \tag{11}$$

$$q_{\mathrm{in},(m,1)}(k) = q_o(k) + q_{(m-1,i_{last})}(k) \tag{12}$$

$$q_{\mathrm{out},(m,1)}(k) = \lambda_m \rho_{(m,1)}(k) v_{(m,1)}(k) \tag{13}$$

with $-(\delta T_\mathrm{s} q_o(k) v_{(m,1)}(k))/(L_m\lambda_m(\rho_{(m,i)}(k)+\mu))$ describing the merging phenomenon.

3. NON-LINEAR MODEL PREDICTIVE CONTROL FOR TRAFFIC APPLICATIONS

3.1 Non-Linear Model Predictive Control (NMPC)

Consider the discrete-time non-linear system whose dynamic evolution is described by the following state space model

$$x(k+1) = f_\mathrm{d}(x(k), u(k)) \tag{14}$$

with $x(k) \in \mathbb{R}^n$ and $u(k) \in \mathbb{R}^m$, the state and the input vector respectively. The general idea of non-linear model predictive control (NMPC) is to determine the sequence of control actions for the system by solving an optimization problem considering the predicted trajectories given by the non-linear model. Often, a quadratic function of the form

$$J(\widetilde{x}(k), \widetilde{u}(k)) = \sum_{l=1}^{N_\mathrm{p}} x^T(k+l)Qx(k+l) + \sum_{l=0}^{N_\mathrm{u}-1} u^T(k+l)Ru(k+l) + \Delta u^T(k+l)S\Delta u(k+l) \tag{15}$$

is used to measure the performance of the system. Here the superscript T is the transpose operator, $\widetilde{x}(k) = [x^T(k+1), \ldots, x^T(k+N_\mathrm{p})]^T$, $\widetilde{u}(k) = [u^T(k), \ldots, u^T(k+N_\mathrm{u}-1)]^T$, with $u(k+l) = u(k+N_\mathrm{u}-1)$, for $l = N_\mathrm{u}, ..., N_\mathrm{p}$, with $N_\mathrm{p}, N_\mathrm{u}$ the prediction and control horizon respectively; Q, R, S are weighting matrices generally diagonal with positive elements, and $\Delta u(k+l) = u(k+l) - u(k+l-1)$.

Assume $x(k) \in \mathbb{X}$, and $u(k) \in \mathbb{U}$ for all k, where $\mathbb{X}$ and $\mathbb{U}$ determine the feasible values of the states and the inputs respectively, and they are given by the physical and operational constraints of the system. Then, the NMPC problem can be formulated as a non-linear optimization problem:

$$\begin{aligned} &\min_{\widetilde{u}(k)} J(\widetilde{x}(k), \widetilde{u}(k)) \\ s.t.\quad & x(k+1) = f_\mathrm{d}(x(k), u(k)), \quad x(0) = x_0, \\ & x(k) \in \mathbb{X}, \quad u(k) \in \mathbb{U} \end{aligned}$$

3.2 Optimal Reduction of the Travel-Time

From Section 2, the dynamic behavior of the traffic network can be described using Eqs. (6)–(13). The model given by these equations has the form of Eq. (14), with $x(k) = [\rho^T(k), v^T(k), w^T(k)]^T$, and $u(k) = r^T(k)$, where $\rho(k)$, $v(k)$, $w(k)$, $r(k)$ are the vectors containing the densities, mean speeds, queues, and ramp-metering rates of all links, segments and origins of the highway respectively. Since the traffic model is a discrete-time model it will be used to simulate the plant and to predict the trajectories of the states in the NMPC.

Although Eq. (15) often is used to measure the performance of the system in MPC schemes, in traffic control applications other cost functions are used. This is motivated by the fact that in traffic control the objective often is to reduce the travel time of the users (the objective of the NMPC approach presented in this paper) or to reduce the emissions, and not to regulate the states of the system to some value. Hence, a different cost function is used. From the definitions, the total number of vehicles in the highway and its on-ramps is given by

$$\sum_{m\in\mathfrak{M}} \sum_{i\in\psi_m} \rho_{(m,i)}(k) L_m \lambda_m + \sum_{o\in\mathfrak{O}} w_o(k) \tag{16}$$

where $\mathfrak{M}$ is the set of links, ψ_m denotes the set of segments of link m, and $\mathfrak{O}$ denotes the set of origin. Therefore, the travel time of the users of the highway and the access roads over a prediction horizon N_p is given by

$$J(\widetilde{x}(k), \widetilde{u}(k)) = T_\mathrm{s} \sum_{l=k}^{k+N_p-1} \left(\sum_{i\in\psi_m} \rho_{(m,i)}(l) L_m \lambda_m + \alpha \sum_{o\in\mathfrak{O}} w_o(l) + \alpha_\mathrm{ramp}(r_o(l) - r_o(l-1))^2 \right) \tag{17}$$

where $\alpha, \alpha_\mathrm{ramp} > 0$ are tuning parameters associated with the time spent by the users in the queues at the origins and with the smoothness of the changes of the control actions. Finally, the NMPC formulation is completed by adding the boundary constraints. In this case it was considered that $\rho_{\min,(m,i)} \le \rho_{(m,i)}(k) \le \rho_{\max,(m,i)}$, $w_{\min,o} \le w_o(k) \le w_{\max,o}$, and $v_{\min,(m,i)} \le v_{(m,i)}(k) \le v_{\max,(m,i)}$, with $\rho_{\min,(m,i)}$, $w_{\min,o}$, $v_{\min,(m,i)} \ge 0$. These constraints determine the feasible set $\mathbb{X}$ for the states (if the prediction model is included in the state constraints the feasible set becomes $\widetilde{\mathbb{X}}(\widetilde{u}(k); x(k))$, where the time evolution is given by the state equation (14) coming from the traffic model). Additionally, the feasible set for the control inputs $\mathbb{U}$ is determined by the inequality $r_{\min,o} \le r_o(k) \le r_{\max,o}$, $r_{\min,o} \ge 0$ and $r_{\max,o} = 1$. Therefore, the NMPC for travel time reduction can be formulated as

$$\min_{\widetilde{u}(k)} J(\widetilde{x}(k), \widetilde{u}(k)) \tag{18}$$

$$s.t \quad \widetilde{x}(k) \in \widetilde{\mathbb{X}}(\widetilde{u}(k); x(k)), \quad \widetilde{u}(k) \in \widetilde{\mathbb{U}} \tag{19}$$

where $\widetilde{\mathbb{X}}(\widetilde{u}(k); x(k))$ denotes the set resulting of the intersection of the set given by the simple bound constraints for the states $\mathbb{X}$ and the set determined by the evolution of the state trajectories along the prediction horizon for input $\widetilde{u}(k) \in \widetilde{\mathbb{U}}$ and the model (14) taking as initial condition $x(k)$. The optimization problem (18)-(19) corresponds to the centralized formulation of the NMPC for travel time reduction in a highway. However, as it was stated in Section 1, traffic systems are large-scale systems, and hence, the implementation of centralized MPC is not advisable. Therefore, in the next section a distributed model predictive control based on game theory scheme is proposed.

4. BARGAINING APPROACH TO OPTIMAL TRAFFIC CONTROL

In Section 3 an NMPC approach for travel time minimization was presented. Such formulation corresponds to a centralized control scheme. Since real traffic networks are large-scale systems, DMPC arises as an alternative to overcome the computational problems associated with the implementation of the centralized NMPC schemes. Assume that the whole system can be decomposed into M subsystems r such that

$$x_r(k+1) = f_{\mathrm{d}r}(x(k), u_r(k), u_{-r}(k)) \tag{20}$$

where $u_{-r}(k) = [u_1^T(k), \ldots, u_{r-1}^T(k), u_{r+1}^T(k), \ldots, u_M^T(k)]^T$. Let $\mathbb{U}_r$ be the feasible set for the control inputs of subsystem r. Let Ω_r be the set of feasible control actions for $\widetilde{u}_r(k)$, where $\widetilde{u}_r(k) = [u_r^T(k), \ldots, u_r^T(k+N_\mathrm{u}-1)]^T$. Let $\mathbb{X}_r$ be the feasible set for the states of subsystem r determined by the boundary constraints. Let Ξ_r be the set of feasible values of the states for $\widetilde{x}_r(k)$, where $\widetilde{x}_r(k) = [x_r^T(k+1), \ldots, x_r^T(k+N_\mathrm{p})]^T$, and $\widetilde{u}_{-r}(k) = [u_{-r}^T(k), \ldots, u_{-r}^T(k+N_\mathrm{u}-1)]^T$. Let $\widehat{\Xi}_r(\widetilde{u}_r(k), \widetilde{u}_{-r}(k); x(k))$ be the feasible set for the states, considering the boundary constraints, the initial condition $x(k)$, and the prediction model (20). Let $J_r(\widetilde{x}_r(k), \widetilde{u}_r(k), \widetilde{u}_{-r}(k))$ denote the cost function of subsystem r, then the local optimization problem arising from the partition of the whole system can be formulated as follows

$$\min_{\widetilde{u}_r(k)} J_r(\widetilde{x}_r(k), \widetilde{u}_r(k), \widetilde{u}_{-r}(k)) \tag{21}$$

$$s.t. \quad \widetilde{x}_r(k) \in \widehat{\Xi}_r(\widetilde{u}_r(k), \widetilde{u}_{-r}(k); x(k)), \quad \widetilde{u}_r(k) \in \Omega_r$$

Hence, this DMPC approach is composed by the local optimization problem (21) and the procedures used to guarantee the negotiation among subsystems in order to jointly compute the control actions to be applied to the controlled non-linear system.

From (21), the local optimization problems are coupled to each other through the inputs and the states, i.e., the value of the cost function $J_r(\cdot)$ and the decision space of subsystem r depends of the decision of the remaining subsystems. Then, the DMPC problem can be treated as a game where the success of each subsystem is based on the choices of the other subsystems. In a dynamic setting, at each time step k a play of the DMPC game is performed, so a control action is computed based on the rules of the game. At time step k each subsystem has to choice a local control action, under the conditions stated by the sets $\widehat{\Xi}_r(\widetilde{u}_r(k), \widetilde{u}_{-r}(k); x(k))$ and Ω_r, the negotiation algorithm or moves of the DMPC game used to solve the problem (21), and trying to minimize the local cost function.

Let $\Gamma_r(\widetilde{u}_r(k), \widetilde{u}_{-r}(k); x(k))$ denote the set resulting from the cartesian product of $\Xi_r(\widetilde{u}_r(k), \widetilde{u}_{-r}(k); x(k))$ and Ω_r. Then, the DMPC game can be represented as a tuple

$$\begin{aligned} G_{\text{DMPC}} = (N, \{\Gamma_r(\widetilde{u}_r(k), \widetilde{u}_{-r}(k); x(k))\}_{r\in N}, \\ \{J_r(x(k), \widetilde{u}_r(k), \widetilde{u}_{-r}(k))\}_{r\in N}) \end{aligned} \tag{22}$$

where N are the subsystems, and $J_r(x(k), \widetilde{u}_r(k), \widetilde{u}_{-r}(k))$ the cost function of subsystem r respectively. Since the game G_{DMPC} comes from of a distributed control set-up, the idea of the game is to find the optimal control inputs that increases the overall system performance. When all subsystems have a common goal, like to minimize the global cost function, the game is called cooperative. However, in order to achieve near Pareto optimal solutions, the subsystems may have the flexibility to decide whether to collaborate or not, if from their point of view that decision would lead into a more important benefit for the overall system.

Hence, the game G_{DMPC} is defined as a sequence of bargaining games $\{\Upsilon(\widetilde{u}_r(k), \widetilde{u}_{-r}(k); x(k)), \eta(k)\}_{k=1}^{\infty}$, whose solutions are determined at every instant k by the following local optimization problem:

$$\begin{aligned} &\min_{\widetilde{u}_r(k)} \sum_{\sigma=1}^{M} \omega_\sigma \log(\eta_\sigma(k) - J_\sigma(\widetilde{x}_\sigma(k), \widetilde{u}_r(k), \widetilde{u}_{-r}(k))) \\ s.t.\quad &\eta_\sigma(k) > J_\sigma(\widetilde{x}_\sigma(k), \widetilde{u}_r(k), \widetilde{u}_{-r}(k)) \\ &\widetilde{x}_r(k) \in \Xi_r(\widetilde{u}_r(k), \widetilde{u}_{-r}(k); x(k)), \quad \widetilde{u}_r(k) \in \Omega_r \end{aligned} \tag{23}$$

where $\omega_\sigma > 0$, $\sum_{\sigma=1}^{M} \omega_\sigma = 1$. In order to solve (23) it is required that subsystems communicate each other their actual value of the states, actual control actions, and disagreement points. The disagreement points $\eta(k) := (\eta_1(k), \ldots, \eta_M(k))$, given an initial value $\eta(0) = \eta_0$ are defined recursively as:

$$\eta_r(k+1) := \begin{cases} \eta_r(k) - \alpha_1(\eta_r(k) - J_r(\widetilde{x}_r(k), \widetilde{u}_r(k), \widetilde{u}_{-r}(k))) \\ \text{if } \eta_r(k) \geq J_r(\widetilde{x}_r(k), \widetilde{u}_r(k), \widetilde{u}_{-r}(k)) \\ \\ \eta_r(k) + \alpha_2(J_r(\widetilde{x}_r(k), \widetilde{u}_r(k), \widetilde{u}_{-r}(k)) - \eta_r(k)) \\ \text{if } \eta_r(k) < J_r(\widetilde{x}_r(k), \widetilde{u}_r(k), \widetilde{u}_{-r}(k)) \end{cases} \tag{24}$$

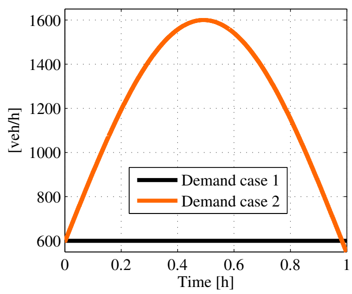

Fig. 1. Demand profile for cases 1 and 2, both origins with the same profile.

where α_1, α_2 are positive tuning parameters, and $\eta_r(0)$ is chosen big enough to satisfy the inequality constraints in (23). Once the subsystems have this information, the optimization problem (23) is locally solved by considering as inputs the ones obtained in the previous instant time for the remaining subsystems (under the same assumption the trajectories of the states of the remaining subsystems are computed to determine the value of the cost function). Moreover,this formulation allows each subsystem to take into account the effect of its decisions in the behavior of the whole system and to promote the cooperation among subsystems.

5. SIMULATION RESULTS

A three-lane highway with two entrances was used for testing the proposed methodology. It consists on one segment of a highway, divided into three links with similar characteristic: same length, same number of lanes, and similar geometry. In this small traffic network benchmark, each link is separated from one another by an on-ramp, modeled as origins which allow the entry of new vehicles to the highway regulated by the traffic signals: $r_1(k)$ and $r_2(k)$. The model parameters we use in this benchmark are the same as the ones used in (Zegeye et al., 2012), $L_1 = 0.5[km]$, $T_s = 0.0028[h]$, $\tau = 0.0041[h]$, $w_{\max,1} = w_{\max,2} = 100[veh]$, $v_{\max,(1,1)} = v_{\max,(1,2)} = v_{\max,(1,3)} = 102[km/h]$, $v_{\text{free},(1,1)} = v_{\text{free},(1,2)} = v_{\text{free},(1,3)} = 102[km/h]$, $\rho_{\text{cr},1} = 33.5[veh/km/lane]$, $\rho_{\max,(1,1)} = \rho_{\max,(1,2)} = \rho_{\max,(1,3)} = 187.6495[veh/km/line]$, $Q_0 = 1751.2[veh/h]$, $b_m = 1.867$, $\delta = 0.8942$, $\omega_1 = \omega_2 = 0.5$, $\alpha_n = 0.4$, $n = 64.2005$, $N_p = 75$, $N_u = 35$, $\lambda_1 = 3$, and $\mu = 32.9010$.

Two different cases are proposed: (i) free flow conditions; and (ii) congestion. In the latter case the demand for the on-ramp at origins 1 and 2 is variable, as shown in Figure 1. In both cases, the results obtained with the proposed distributed control scheme was compared with a uncontrolled situation and with a centralized NMPC approach. In all the simulations a prediction horizon $N_\text{p} = 75$ and a control horizon $N_\text{u} = 35$ were used for both the NMPC controller and in the proposed GT-DMPC controller.

For the GT-DMPC approach, the system is divided in two subsystems in order to perform two concerted optimizations (See Figure 2): the controller MPC1 is responsible for providing the control signal $r_1(k)$ according to a negotiation with the controller MPC2, while the controller MPC2 provides the control signal $r_2(k)$ according to negotiation

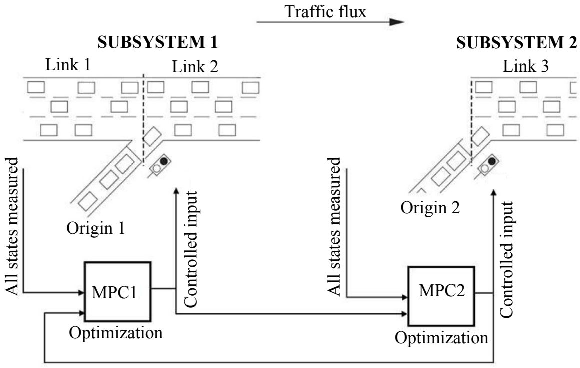

Fig. 2. Distributed MPC for the case of study

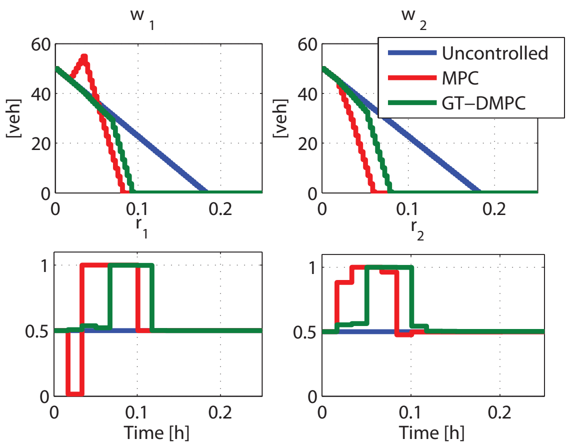

Fig. 3. Queue lengths and the signals of the links of origin 1 and 2 for uncontrolled, centralized MPC, and GT-DMPC schemes, case 1.

with the controller MPC1. In such partition, subsystem 1 is formed by the links $(1,1)$ and $(1,2)$, and by the origin 1, and subsystem 2 is formed by the link $(1,3)$ and the origin 2. As a consequence of the partition of the system the global cost function of Eq. (17) is also partitioned into two local cost functions: $J_1(\widetilde{x}_1(k), \widetilde{u}_1(k), \widetilde{u}_{-1}(k))$ for subsystem 1, and $J_2(\widetilde{x}_2(k), \widetilde{u}_2(k), \widetilde{u}_{-2}(k))$ for subsystem 2. The constraints in both subsystems are the same of the centralized scheme, defined in Section 2.

5.1 *Case 1: Free flow conditions*

The regular traffic conditions proposed for Case 1 are not demanding for the control system, but it is desirable to reduce the total time spent as much as possible. The proposed control techniques reduced more than one half of the time expend by the uncontrolled case to reduce the queue lengths (coincident with the steady-state of density and speed). Figure 3 shows the queue lengths at origins 1 and 2. In Figure 3 are also shown the control signals for the same simulations, the MPC response is faster than the one of the GT-DMPC.

As expected, it was possible to achieve much faster the steady state condition. Regarding to the performance comparison between the centralized MPC controller and the GT-DMPC controller, they had similar behavior; however, if we consider a large-scale system, the centralized MPC will be just impractical.

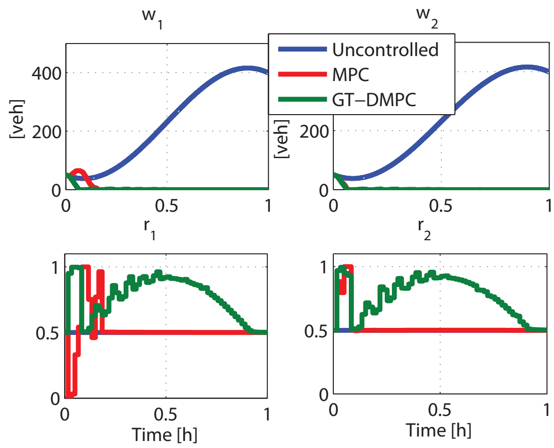

Fig. 4. Queue lengths and the signals of the links of origin 1 and 2 for uncontrolled, centralized MPC, and GT-DMPC schemes, case 2.

5.2 *Case 2: Congestion*

Figure 4 shows the control signals applied; the centralized MPC maintained a similar behavior that the one presented in case 1, and the GT-DMPC controller follows the dynamics of the demand.

The density and speed of vehicles remained within the defined bounds in the controlled cases. But in the uncontrolled case, the Figure 4 clearly shows that the maximum allowed number of vehicles at the on-ramps queues was exceeded (100 vehicles), while the proposed controllers reduced the number of vehicles in the queue to zero.

5.3 *Performance analysis*

In order to determine the performance of the proposed controllers the total time spent (TTS) by the vehicles on the highway and the entrance ramp was evaluated. The TTS is defined as:

$$TTS = \sum_{l=1}^{N_{\text{sim}}} \left(\sum_{m\in\mathfrak{M}} \sum_{i\in\psi_m} \rho_{i,m}(l) L_m \lambda_m + \sum_{o\in\mathfrak{O}} w_o(l) \right) T_s \tag{25}$$

where N_{sim} is the number of simulation steps. Table 1 shows TTS results for the Cases 1 and 2.

Table 1. Performance of the controllers

Configuration	TTS Case 1	TTS Case 2
Uncontrolled	71.23	546.27
Centralized MPC	66.98	89.59
GT-MPC	67.85	101.61

Regarding Case 1, the centralized MPC presents a better performance with respect to the GT-DMPC, but their difference was not significant (1.30 %). Both proposed controllers had an improvement in performance of 20.38 % with respect to the uncontrolled case. Regarding Case 2, again the centralized MPC controller had better performance, in this case the difference with the GT-DMPC was larger (13.42 %), but both had a high difference with the uncontrolled behavior.

Regarding computational times, the total time required by the Centralized MPC was $6.4155e+003(s)$ and $3.2269e+003(s)$ for cases 1 and 2. For the GT-MPC, subsystem 1, $1.3263e+003(s)$ and $1.6533e+003(s)$, and for subsystem 2 $1.5781e+003(s)$ and $1.5693e+003(s)$ for cases 1 and 2 respectively.

6. CONCLUDING REMARKS

In this paper centralized and distributed non-linear model predictive control techniques were implemented for a traffic network. In general, traffic network models are highly non-linear and for real applications, the controllers should be designed including the flexibility to be easily adapted to large-scale setups.

The MPC schemes presented here were able to include explicit constraints in the optimization process, the game theory approach being a good strategy when distributing the MPC for allowing large-scale systems. In our approach, the METANET traffic model was used as a prediction model and the case study was considered using a simple traffic network. The performance of these schemes was analyzed using as performance index the total time spent as comparison, and all the results were consistent with those expected from the literature.

Future work will include implementation on large-scale traffic networks, including several traffic control measures (ramp-metering, speed limits, route guidance, etc.), and the comparison with other hierarchical and distributed MPC approaches for traffic like the ones presented in (Papamichail et al., 2010) and (Frejo and Camacho, 2011).

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