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Variable Speed Limit Control Based on the Extended Link Transmission Model

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ABSTRACT
In this paper, the link transmission model (LTM) is extended to include the effects of variable speed limits (VSL) and consequently to provide variable speed limit control for traffic networks modeled by LTM. The LTM has recently been developed for route assignment, but in this paper the LTM is modified to be used for control purposes. By this we achieve a model that provides a balanced trade-off between accuracy and computational complexity, and hence it is useful for on-line model-based traffic control.

Nevertheless, the extension of the model for ramp metering and speed limit control needs careful attention. Since the LTM lacks explicit velocity equations focus is on other potential sources that could imitate the influences of VSL. The delays inside the model are manipulated in order to achieve the mentioned goal. Moreover, we take into account different situations that may occur in reality based on changes in VSL and different traffic conditions. Finally, the total extensions are verified using simulation and real data. To this aim, the VSL extension integrated in the LTM is verified using simulations for a benchmark case study (in order to show performance of the extended LTM clearly). Next, the LTM is calibrated by real data collected from the A12 Freeway in The Netherlands. The optimal parameters of the model are identified using a global optimization method. Comparison with real data from a period of time that VSL installed on the freeway are active, shows acceptable performance of the total extended and calibrated LTM.

1 INTRODUCTION

Traffic flows on freeways and urban networks need to be controlled in order to prevent congestion on the roads and to reduce the total travel time. This can be done using efficient traffic management systems. On-ramp metering, speed limits, and route guidance are among the traffic measures that can be used to control flow of vehicles. To this aim, efficient and fast traffic controllers should be designed and implemented. In the literature, Model Predictive Control (MPC) (*1*) has been widely used for on-line model based traffic control (see e.g. (*2*), (*3*), (*4*)). However, inside the MPC controller a fast yet accurate model is needed to predict the evolution of the traffic network system under control over time.

For traffic networks, a wide range of traffic flow models have been developed (*5*), (*6*). Among these models, one that can include the traffic states (flow, density, velocity) and effects of the traffic measures is useful for control proposes. METANET is a second-order aggregate model proposed in (*7*). This model predicts traffic states using an exponential approximation of the fundamental diagram. However, this model needs to be extended to include traffic measures. For ramp metering, in (*2*) and (*8*) required modifications on the outflow of metered ramps have been presented. For speed limit control, based on the results of (*9*) and (*10*), in (*11*) different models for speed control in the METANET framework have been proposed. However, using the extended METANET in the MPC framework as prediction model is computationally complex (not only due to the nonlinear nonconvex optimization itself but also due to the repeated model simulation in each optimization run). Therefore, traffic researchers have focused their efforts on finding a less complex model that has similar properties.

Recently, the link transmission model (LTM) has been proposed in (*12*). The LTM is a first-order model basically developed for route assignment. According to (*12*) and (*13*) the LTM is capable of modeling flows in traffic networks with acceptable accuracy. Thus, it can be used for on-line mode based control purposes. However this model needs to be extended in order to include effects of traffic measures (ramp metering and variable speed limits). In (*14*), we have modified the LTM to include ramp metering and used the extended model in the MPC framework.

In order to have better control and more flexibility variable speed limits (VSL) have been introduced and implemented on freeways to increase safety by decreasing the maximum allowed speed and consequently to dissolve congestions and increase the traffic flow.

In this paper, we further extend the LTM to include the effects of VSL. Since the LTM lacks explicit velocity equations the extension approach would be totally different from e.g. the METANET case (*4*). According to (*15*), (*10*), (*16*), (*17*), there are two aspects of using VSL in the literature and in practice. The first view is about reducing the speed differences to homogenize the traffic densities on the road in order to have more stable flow of vehicles. The second idea is to reduce densities and prevent from congested situations by controlling the flow of vehicles by introducing low speed limits. The latter view of VSL is in the scope of this paper. In order to imitate the effect of a speed limit on a road section, one can manipulate the travel time of vehicles in the traffic model. In our case, this can be done by changing the delays in the LTM model equations. A change in the free-flow speed of a road section results in a change in the travel time of vehicle or in better words the time that vehicles reach the downstream boundary. We elaborate this idea in the following sections and formulate the modifications required for having VSL control using LTM.

The rest of the paper is organized as follows. In Section 2, the LTM components are defined and the original mathematical formulations from (*12, 13*) are presented. In Section 3, the original model is extended to include ramp metering and variable speed limits. A model for including ramp metering in the LTM framework is discussed and formulated. For the VSL, different situations that may occur in case a change in the value of speed limits is applied, are studied and discussed. Based on that, mathematical formulation of VSL control inside the LTM framework is derived. The performance of the extended model is shown using a simple traffic network case study. For two demand profiles corresponding to free-flow and congested situations, results of changing the speed limit are depicted and discussed. Next, the LTM is used to model a stretch of the A12 Highway in the Netherlands. To this aim, the parameters of the model should be calibrated using real data. Results of the parameter identification along with an evaluation of the speed limit extensions are presented in Section 4. Conclusions and ideas for further research are discussed in Section 5.

2 THE LINK TRANSMISSION MODEL

In this section, the original LTM is introduced using (*12*) and (*13*). The LTM is capable of determining time-dependent link volumes, link travel times, and route travel times in traffic networks. To this aim, the LTM uses the cumulative number of vehicles to characterize the traffic evolution. The cumulative numbers of vehicles are updated using flow functions of links and nodes defined in the following subsections.

2.1 Link Model

In the LTM framework, a traffic network consists of homogeneous links, which start at an upstream boundary denoted by x_i^0 and end at a downstream boundary denoted by x_i^{L}. The link i has a length L_i and links are connected to each other via nodes. Link and node types are depicted in Figure 1.

The cumulative number of vehicles $N(x,t)$ is defined only for the upstream and downstream boundaries of the links. In order to proceed with update equations for the cumulative number of vehicles we need to define two quantities for each link; the sending and the receiving number of vehicles. The sending number of vehicles for link i is the maximum amount of vehicles that can potentially leave the downstream end of this link during the time interval $[t, t+T_{\mathrm{s}}]$ (T_{s} is the sample time) and it is defined as:

$$S_i(t) = \min\left[N\left(x_i^0, t+T_{\mathrm{s}} - \frac{L_i}{v_{\mathrm{free},i}}\right) - N(x_i^{\mathrm{L}}, t),\ q_{\mathrm{M},i}T_{\mathrm{s}}\right] \tag{1}$$

where $v_{\mathrm{free},i}$ and $q_{\mathrm{M},i}$ are the free-flow speed and the capacity of link i. The sending number is constrained by the boundary conditions at the upstream end of the link. If a free-flow traffic state occurs at the downstream link boundary at time $t+T_{\mathrm{s}}$, then this state must have been emitted from the upstream boundary $\frac{L_i}{v_{\mathrm{free},i}}$ time units earlier.

Similarly, the receiving number of vehicles $R_i(t)$ of link i is the maximum amount of vehicles that can enter the upstream end of this link during time interval $[t, t+T_{\mathrm{s}}]$ and it is formulated as:

$$R_i(t) = \min\left[N\left(x_i^{\mathrm{L}}, t+T_{\mathrm{s}} + \frac{L_i}{w_i}\right) + \rho_{\mathrm{max},i}L_i - N(x_i^0, t), q_{\mathrm{M},i}T_{\mathrm{s}}\right] \tag{2}$$

where w_i and $\rho_{\mathrm{max},i}$ are the negative maximum spillback wave speed and the jam density of link i, respectively.

2.2 Node Models

The transition number of vehicles is defined for each type of nodes and determined using the sending and receiving numbers of vehicles of the connected links. In fact, the transition quantity determines the maximum number of vehicles that can be transferred from incoming links to outgoing links of a node during the time interval $[t, t+T_s]$.

2.2.1 Inhomogeneous Nodes

For an inhomogeneous node, which represents a change in the characteristics of a road such as capacity, speed limits, etc., the transition number $G_{ij}(t)$ is formulated as

$$G_{ij}(t) = \min\left[S_i(t), R_j(t)\right] \tag{3}$$

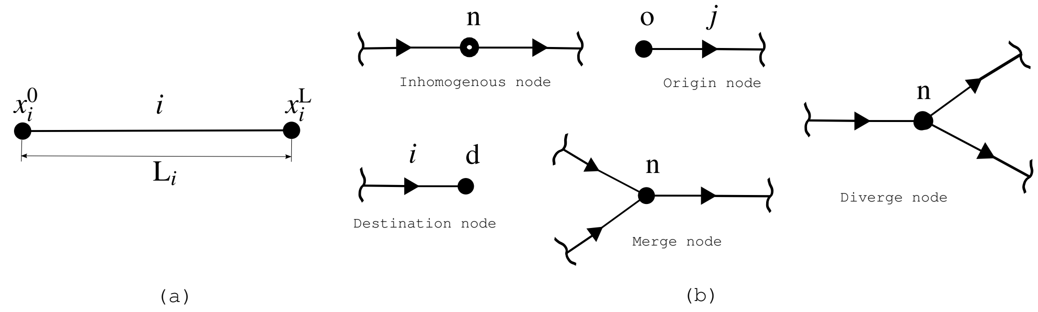


FIGURE 1 (a) Link model, (b) Different node types.

where i is the index of the unique incoming link and j is the index of the unique outgoing link of the given node(other cases with two incoming or outgoing links are discussed later).

2.2.2 Origin Nodes

For origin nodes, the transition number of vehicles is determined as follows:

$$G_{oj}(t) = \min\left[N_o(t+T_s) - N(x_j^0,t),\ R_j(t)\right] \tag{4}$$

where j is the index of the link connected to the origin o (Without loss of generality we assume that there is only one link connected to an origin or a destination) and N_o denotes the traffic demand in origin o in terms of the cumulative number of vehicles. A simple queue model for origin o is defined as:

$$\omega_o(t) = N_o(t) - N(x_j^0,t) \tag{5}$$

where $\omega_o(t)$ and $N(x_j^0,t)$ denote the number of vehicles standing in the queue at origin o and the cumulative number of vehicles that already entered the network at time t, respectively.

2.2.3 Destination Nodes

The transition number of vehicles for a destination node d is equal to the sending number of vehicles of the link connected to the destination node:

$$G_{id}(t) = S_i(t) \tag{6}$$

with i the index of the unique incoming link. It should be noted that we have assumed that the destination accepts all the flows and it is not considered as a bottleneck (see Section 3.2.2 of (*4*)).

2.2.4 Merge Nodes

This type of nodes can represent merging of links and/or on-ramps in traffic networks. Several models for the merge of links have been proposed in the literature (*18*), (*19*), (*20*), (*21*), (*22*). According to (*22*), the model proposed in (*19*) does not satisfy the *invariance principle* (*23*). Thus, among all of them we choose the most accurate priority-based merge model for two incoming links proposed in (*20*). To this aim, the transition number of vehicles from an incoming link i of a merge node n to the unique outgoing link j is formulated as follows:

$$\begin{cases} G_{ij}(t) = S_i(t) & \text{if} \quad R_j(t) \geq \Big(S_i(t) + S_{i'}(t)\Big) \\ G_{ij}(t) = \text{median}\Big[S_i(t), R_j(t) - S_{i'}(t), \alpha_{ij}R_j(t)\Big] & \text{otherwise} \end{cases} \quad \text{for } \ i \in I_n \tag{7}$$

$$\alpha_{ij} = \frac{q_{\mathrm{M},i}}{q_{\mathrm{M},i} + q_{\mathrm{M},i'}}$$

where $I_n = \{i, i'\}$ is the set of indices of the two incoming links of node n. The distribution fractions α_{ij} reflect priorities that are proportional to the capacities of the incoming links $q_{\mathrm{M},i}$. Note that the sum $\sum_{i \in I_n} \alpha_{ij}$ is equal to 1. The reader is referred to (*21*) for a general merge model with two or more incoming links.

2.2.5 Diverge Nodes

A diverge node connects one incoming link i to outgoing links j in a set J_n where n is the index of the diverge node. For the two outgoing links case, the following model for transition numbers of vehicles has been proposed in (*24*):

$$G_{ij}(t) = \min\left(\beta_{ij} S_i(t), R_j(t), \frac{\beta_{ij}}{\beta_{ij'}} R_j(t)\right) \quad \text{for} \;\; j \in J_n \tag{8}$$

where the outflow of the incoming link is divided over the outgoing links according to turning fractions β_{ij} ($\sum \beta_{ij} = 1$). The turning fractions can be fixed or variable to the route choice (*22*). For the general case with more than two outgoing and/or incoming links and other types of nodes (e.g. intersection nodes) the interested reader is referred to (*22*)).

2.3 Update Equations

Having determined the transition number of vehicles of all nodes, the cumulative number of vehicles for the upstream and downstream boundaries of links can be updated using the following equations:

$$N(x_i^{\mathrm{L}}, t + T_s) = N(x_i^{\mathrm{L}}, t) + \sum_{j \in J_n} G_{ij}(t) \;\; \text{for all } i \in I_n \tag{9}$$

$$N(x_j^0, t + T_s) = N(x_j^0, t) + \sum_{i \in I_n} G_{ij}(t) \;\; \text{for all } j \in J_n \tag{10}$$

for each node $n \in N$ where N is the set of all nodes in the traffic network.

3 EXTENSION OF THE LTM FOR RAMP METERING AND VARIABLE SPEED LIMITS

In this section the LTM model is extended to include traffic control signals. First we investigate the possibility of extending the LTM for metering of on-ramps. Next, variable speed control using the LTM is discussed and required modifications are explained.

3.1 Ramp Metering

An on-ramp can be treated as a combination of an origin node and a merge node connected by a virtual link with a link length that is equal to 0. We place an origin node for the metered ramp with a constraint on its outflow to a virtual link. Thus the transition number of vehicles of the on-ramp o to the virtual link i' can be determined as follows (based on (4)):

$$G_{oi'}(t) = \min\left(N_o(t + T_s) - N(x_{i'}^0, t), q_{\mathrm{M},i'} T_s r_o(t)\right) \tag{11}$$

where $q_{\mathrm{M},i'}$ is the capacity of the virtual link i' (veh/h), T_s is the sample time, and $r_o(t) \in [0, 1]$ is the metering rate. Moreover, $N_o(t + T_s)$ denotes the traffic demand in the on-ramp o and $N(x_{i'}^0, t)$ is the cumulative number of vehicles that already entered the virtual link i'.

Next, the transition numbers of vehicles from the virtual link to the outgoing link of the merge node can be determined by (7) using $G_{oi'}(t)$ and the sending number of vehicles for the other incoming link.

Hence, using the metering rate $r_o(t)$, one can limit the outflow of an on-ramp in order to prevent from traffic congestion in the mainstream road.

3.2 Variable Speed Limits

In this section, we elaborate the modifications required for the original LTM in order to emulate the effects of VSL. One of the effects of VSL is to control the flow of vehicles and their travel time. In other words, VSL can be used to modify the time that vehicles spend to reach the downstream boundaries of links. Hence, by looking at the LTM model we find out that this can be realized by having a time-varying v_{free} which is denoted by $\bar{v}(t)$ from now on. Nevertheless, in order to have an acceptable model for VSL, different circumstances that can occur in reality should be investigated and included in the extended model. In Figure 2a and 2b, the results of changing the value of VSL in the free-flow condition are shown for two cases. Before proceeding, it should be noted that the VSL is assumed to be implemented at the upstream boundary of a link.

We start with the case that the speed limit increases to a higher value (Figure 2a). In this case, the vehicles are supposed to reach the downstream boundary with less travel time. However, from the time instant the value of the speed limit is changed on there might be some vehicles reaching the downstream boundary that did not face the new speed limit. Therefore, in order to obtain a better update for the cumulative number of vehicles, this situation should be taken into account.

On the other hand, when the limit on the speed goes to a lower level, the evolution of the cumulative number of vehicles may look like in Figure 2b (if a free-flow condition is applied, otherwise in congested situations the influence of VSL may not be as apparent as what is depicted here). Vehicles that enter the link after the time instant at which the VSL value is changed are affected by the new limit and will try to follow the restriction. However, for the vehicles that are already in the link by the time the VSL sign is altered, the new limit is not applicable. They reach the upstream boundary with their previously assigned limit or the free speed of the road. Moreover, since the new speed limit is lower, there will be a time interval in which the cumulative number of vehicles remains constant (this means that no vehicle departs from the downstream end). In the following we formulate these conditions.

3.2.1 Increase in the Value of VSL

As shown in Figure 2a, it takes some time for the vehicles that did not face the new limit to leave the link. Before this time, the sending number of vehicles should be determined using the old value of $\bar{v}$. Moreover, the capacity of the road $q_{\mathrm{M},i}$ is then calculated using a triangular fundamental diagram constructed on the old $\bar{v}$ (we assume that speed of the backward propagation of the congestion wave remains unchanged). It can be inferred from Figure 2c, the capacity $q_{\mathrm{M},i}$ is determined as:

$$q_{\mathrm{M},i} = \rho_{\max,i} \frac{\bar{v}_i w_i}{\bar{v}_i + w_i} \tag{12}$$

If the value of the VSL increases at time instant t^* (t and t^* are assumed to be multiple integer of the

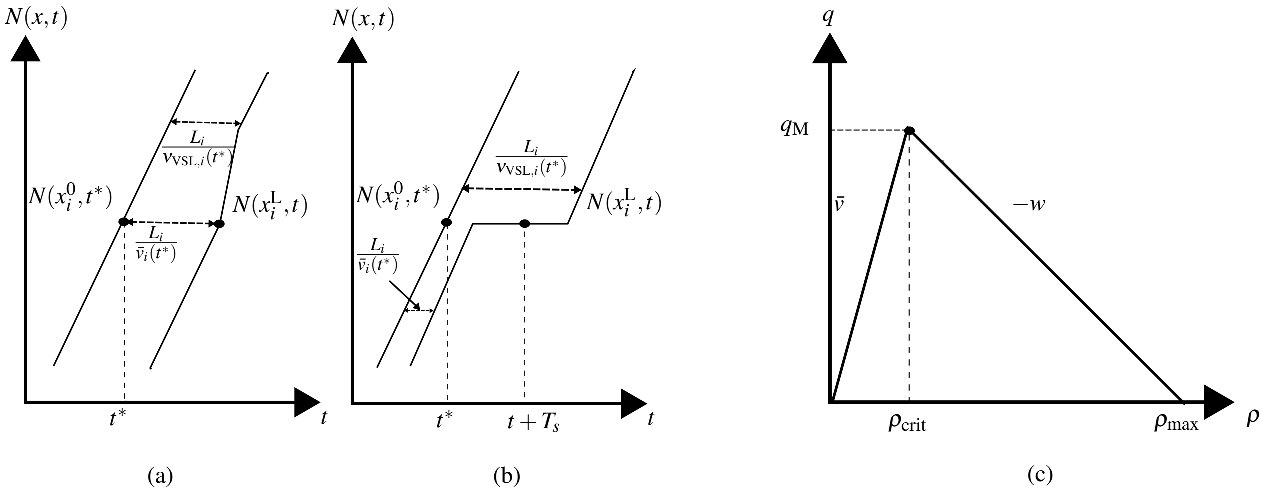


FIGURE 2 (a) Increase in the value of VSL, (b) Decrease in the value of VSL, (c) Triangular Fundamental Diagram.

sample time T_{s}), $\bar{v}$ and capacity of link i will be changed according to the following equations (for $t \geq t^*$)

$$\begin{cases} N(x_i^{\mathrm{L}},t+T_s) = N\left(x_i^0,t+T_s-\frac{L_i}{v_{\mathrm{VSL},i}(t^*)}\right),\ \bar{v}_i(t)=v_{\mathrm{VSL},i}(t^*),\ q_{\mathrm{M},i}(t)=\rho_{\max,i}\frac{v_{\mathrm{VSL},i}(t^*)w_i}{v_{\mathrm{VSL},i}(t^*)+w_i} \\ \hfill \text{if } N(x_i^{\mathrm{L}},t)\geq N(x_i^0,t^*) \\ N(x_i^{\mathrm{L}},t+T_s) = N\left(x_i^0,t+T_s-\frac{L_i}{\bar{v}_i(t^*-T_s)}\right),\ \bar{v}_i(t)=\bar{v}_i(t^*-T_s),\ q_{\mathrm{M},i}(t)=\rho_{\max,i}\frac{\bar{v}_i(t^*-T_s)w_i}{\bar{v}_i(t^*-T_s)+w_i} \\ \hfill \text{otherwise} \end{cases}$$

where $v_{\mathrm{VSL},i}(t^*)$ is the value at the time instant t^* of the VSL installed at the upstream boundary of link i. Now the sending number of vehicles for link i can be determined using $N(x_i^{\mathrm{L}},t+T_s)$ and $q_{\mathrm{M},i}(t)$ obtained from the above conditions:

$$S_i(t) = \min\left[N(x_i^{\mathrm{L}},t+T_s) - N(x_i^{\mathrm{L}},t),\ q_{\mathrm{M},i}(t)T_s\right] \tag{13}$$

On the other hand, in order to determine the receiving number of vehicles $R_i(t)$, the capacity $q_{\mathrm{M},i}$ should be altered immediately after the VSL value is changed (this means that the capacity should be calculated using $v_{\mathrm{VSL},i}(t^*)$). This is due to the fact that for predecessor links of link i, the capacity of link i is changed when a new speed limit is introduced. But for the sending number of vehicles at the downstream boundary of link i the capacity remains unchanged until all the vehicles that did not face the new limit reach the end of link i.

3.2.2 Decrease in the Value of VSL

In order to formulate the problem in this case, we note that every vehicle that reaches the downstream end must have entered the link either $\frac{L_i}{\bar{v}_i(t^*-T_s)}$ or $\frac{L_i}{v_{\mathrm{VSL},i}(t^*)}$ time units earlier (note that $v_{\mathrm{VSL},i}(t^*) < \bar{v}_i(t^*-T_s)$). Hence for $t \geq t^*$, $N(x_i^{\mathrm{L}},t+T_s)$ can be equal to $N\big(x_i^0,t+T_s-\frac{L_i}{\bar{v}_i(t^*-T_s)}\big)$, $N\big(x_i^0,t+T_s-\frac{L_i}{v_{\mathrm{VSL},i}(t^*)}\big)$ or $N(x_i^0,t^*)$. In the following, different conditions that may occur and the corresponding changes in the model are presented.

$$
\begin{cases}
N(x_i^{\mathrm{L}}, t+T_s) = N\left(x_i^0, t+T_s - \frac{L_i}{\bar{v}_i(t^*-T_s)}\right), \;\; \bar{v}_i(t) = \bar{v}_i(t^*-T_s), \quad q_{\mathrm{M},i}(t) = \rho_{\max,i}\frac{\bar{v}_i(t^*-T_s)w_i}{\bar{v}_i(t^*-T_s)+w_i} \\
\qquad\qquad\qquad \text{if } N\left(x_i^0, t+T_s - \frac{L_i}{\bar{v}_i(t^*-T_s)}\right) < N(x_i^0, t^*) \\
N(x_i^{\mathrm{L}}, t+T_s) = N(x_i^0, t^*), \;\; \bar{v}_i(t) = \bar{v}_i(t^*-T_s), \;\; q_{\mathrm{M},i}(t) = \rho_{\max,i}\frac{\bar{v}_i(t^*-T_s)w_i}{\bar{v}_i(t^*-T_s)+w_i} \\
\qquad\qquad\qquad \text{if } N\left(x_i^0, t+T_s - \frac{L_i}{\bar{v}_i(t^*-T_s)}\right) \geq N(x_i^0, t^*) \geq N\left(x_i^0, t+T_s - \frac{L_i}{v_{\mathrm{VSL},i}(t^*)}\right) \\
N(x_i^{\mathrm{L}}, t+T_s) = N\left(x_i^0, t+T_s - \frac{L_i}{v_{\mathrm{VSL},i}(t^*)}\right), \;\; \bar{v}_i(t) = v_{\mathrm{VSL},i}(t^*), \;\; q_{\mathrm{M},i}(t) = \rho_{\max,i}\frac{v_{\mathrm{VSL},i}(t^*)w_i}{v_{\mathrm{VSL},i}(t^*)+w_i} \\
\qquad\qquad\qquad \text{if } N\left(x_i^0, t+T_s - \frac{L_i}{v_{\mathrm{VSL},i}(t^*)}\right) > N(x_i^0, t^*)
\end{cases}
$$

The first case above pertains to vehicles reaching the downstream boundary that were not confronted with the new (lower) speed limit. Hence, the update for the cumulative number of vehicles $N(x_i^{\mathrm{L}}, t+T_s)$ should be calculated based on the old speed $\bar{v}_i(t^*-T_s)$. In the second case, due to the lower VSL value for the link, there is no vehicle passing the downstream end for a short period. Thus, $N(x_i^{\mathrm{L}}, t+T_s)$ should be equal to the cumulative number of vehicles at the upstream boundary by the time that VSL sign is changed (t^*). The last case describes the situation that vehicles reach the downstream end while they did encounter the new value of the VSL.

Next, the sending number of vehicles can be calculated using $N(x_i^{\mathrm{L}}, t+T_s)$ and $q_{\mathrm{M},i}(t)$ obtained from the above conditions and (13). However, as mentioned in the previous section,in order to determine the receiving number of vehicles $R_i(t)$ the capacity q_{M} should be altered right after the new speed limit is announced (this means that for $t \geq t^*$, $q_{\mathrm{M},i}(t) = \rho_{\max,i}\frac{v_{\mathrm{VSL},i}(t^*)w_i}{v_{\mathrm{VSL},i}(t^*)+w_i}$ for use in the receiving number of vehicles equation (2)).

Furthermore, all the aforementioned equations in the current section and in the previous section are valid until a new speed limit is introduced. Whenever a new limit is announced, based on the new value (which could be lower or higher than the previous limit), the parameters of the model should be updated as prescribed in this section.

4 CASE STUDIES

In this section, performance of the extended model is evaluated first by simulations for a simple case study and next by real data. Since the data collected from real traffic networks is usually mixed with noise and also traffic demands have stochastic nature, the traffic data (e.g. detected flows and velocities) does not have smooth behavior. Therefore, in order to better observe the effects of VSL on a traffic network and assess the extended LTM, we prefer first to use a simple case study with a few links and a smooth demand profile. Once the performance of the extended LTM is confirmed for this case study, the extended LTM is calibrated for a real network.

4.1 Evaluation of the VSL Extensions on a Simple Case Study

In this subsection we consider a benchmark network depicted in Figure 3a. It consists of four links and two origins (mainstream and on-ramp). A VSL sign is installed at the upstream boundary of the second link. The network is modeled by the extended LTM (we assume that the on-ramp is not metered) with the

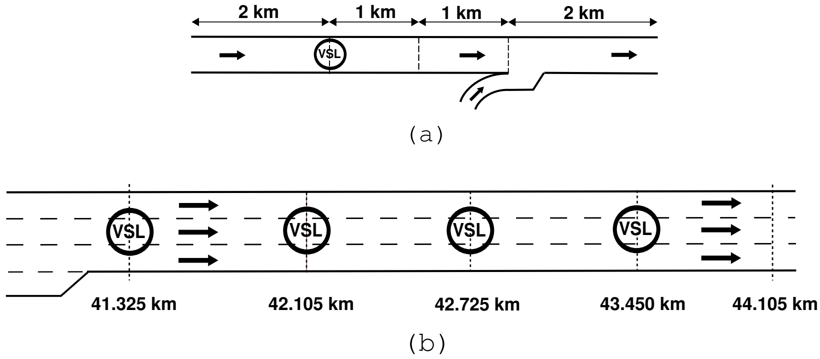


FIGURE 3 (a) Set-up of the simple case study (b) Schematic VSL positions and links of the A12 freeway in The Netherlands.

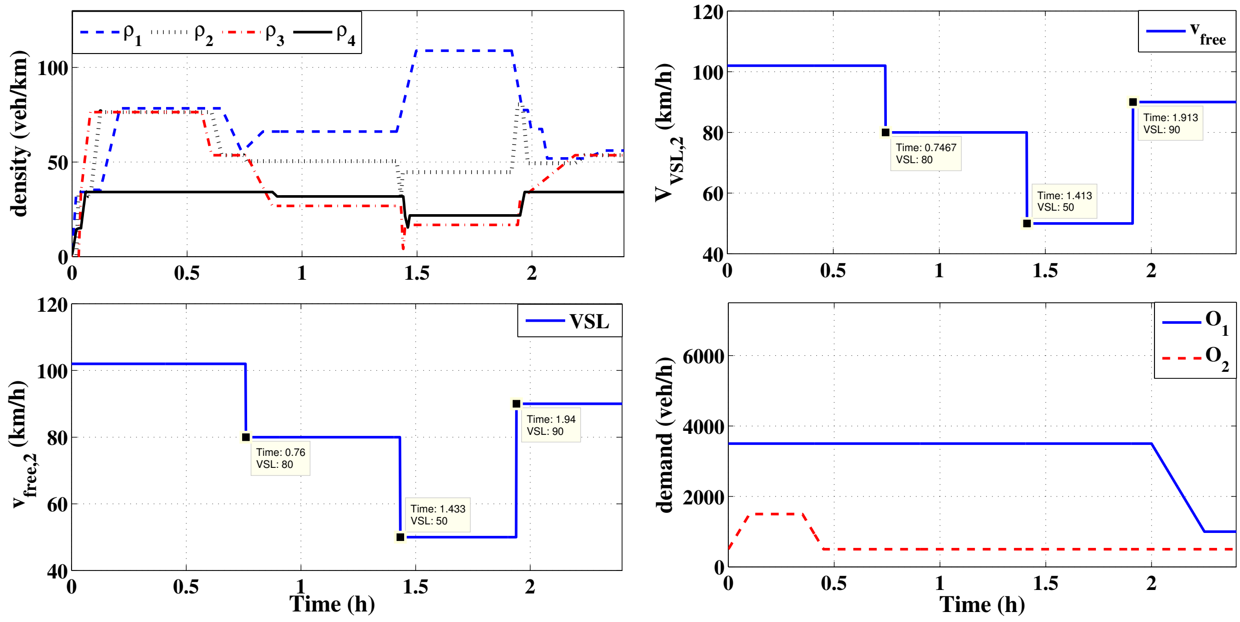
set of parameters $\rho_{\max} = 180$ veh/km, $v_{\text{free}} = 102$ km/h for links 1, 3, and 4, $T_s = 6$ s, and $w = 23$ veh/km. Simulation results for two demand scenarios and different VSL profiles are plotted in Figure 4.

The first case (Figure 4a) corresponds to the situation in which the demand is slightly larger than the capacity at the beginning but after a while traffic demand at the on-ramp goes to a lower level (VSL is inactive in this period, means that vehicles are allowed to travel with the free-flow speed 102 km/h). As a result, densities of the first three links first grow and later start decreasing until a new value for the VSL (80 km/h) is introduced. This creates a new congestion upstream (in Link 1) and helps with resolving the congestion in downstream links. Moreover, as can be seen in the 3rd subplot, the v_{free} used in the LTM takes the new VSL value with a short delay and not immediately after the time instant the VSL value is altered (this is in line with the condition described in Section 3.2; the delay corresponds to the link travel time of the vehicles that did not confront with the new value of VSL). When the VSL value is lowered to 50 km/h, the congestion in link 1 builds up and on the other hand, densities in links 2 to 4 continue to decrease. This proceeds until the VSL value is increased to 90 km/h. From this time on, vehicles can travel with higher speed and can reach the downstream of link 2 in a shorter time. Thus the existing congestion upstream is resolved and therefore the densities in downstream links grow a bit. It should be noted that for a short period the density of link 2 increases. This is due to the fact that more vehicles from link 1 can enter link 2 and this builds up congestion but after the demand from the downstream end of link 1 goes down to a certain level, the density of link 2 starts decreasing until it reaches a steady state.

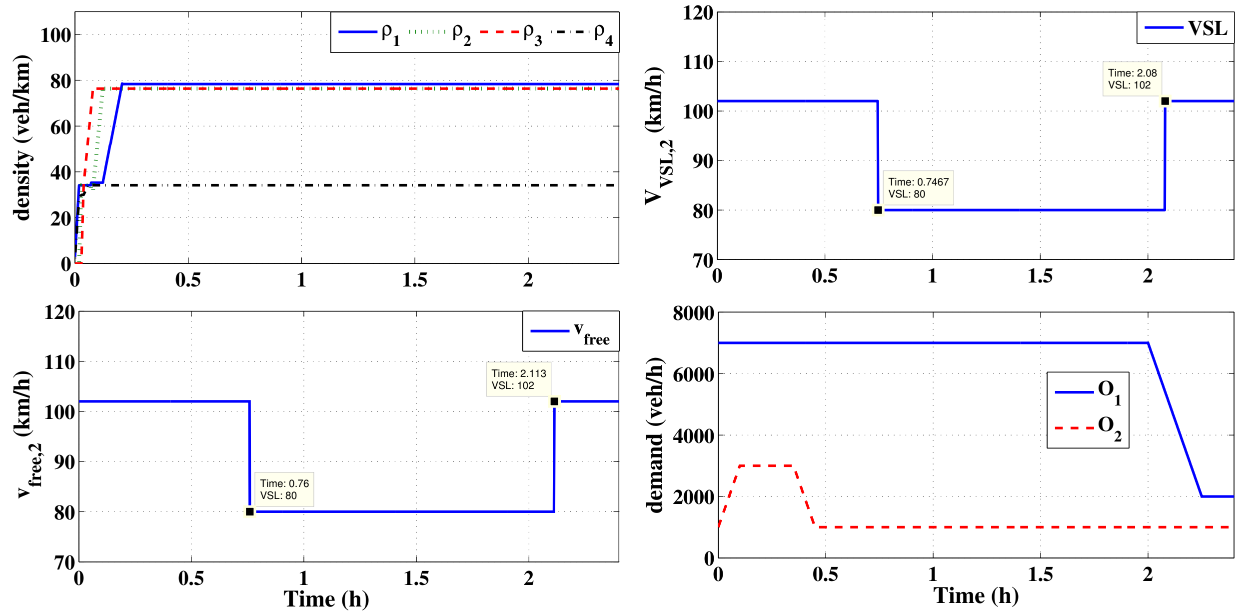
In the second case (Figure 4b), we intentionally double the demand levels at both origins. Consequently, we will have a fully congested situation. As can be observed, the changes in the VSL values do not affect the densities of links. This is in line with reality; in case of congestion in a link, vehicles slow down and the average velocity of the link remains low until the congestion is resolved. During this period, assigning a speed limit higher than the average speed of the link is not effective and has no influence on the density. As illustrated in Figure 4b, the extended LTM is able to model this.

4.2 Identification of the LTM Using Real Data

In this section we identify and calibrate the parameters of the LTM using a set of real data. Thus the calibrated model can be used for study and control of a real network. Furthermore, since data from VSL signs installed on the road are available they can be used to evaluate the performance of the extended LTM. For this purpose a 2.78 km long stretch of the A12 freeway in The Netherlands is used as test field to identify the parameters of the LTM and also to see the performance of the VSL modifications. The freeway connects the city of The Hague to the German border, in Zevenaar. The stretch we use (as depicted in Figure 3b) is from km 41.325 to km 44.105 in the direction of East and it has no off-ramps or on-ramps. Nonetheless,



(a)



(b)

FIGURE 4 Simulation results for (a) uncongested, and (b) fully congested conditions.

Identification of on-ramp/off-ramp cases can be an interesting topic for future research. Sensors installed on the stretch detect flow and velocity and the freeway is equipped with VSL signs as shown in Figure 3b. In the LTM framework, for any change in the physical or other characteristics of the road a node is placed. Thus for this case we have 4 links separated by VSL signs. In order to determine the receiving number of vehicles of the first link, real flow data from the last detector mounted before start of the stretch is integrated over time. Moreover, since the LTM is defined based on cumulative numbers of vehicles at earlier times (delays in the model), vectors of initial values obtained from integrating the flow data of detectors inside the

TABLE 1 Estimated parameters for each link

	v_i (km/h)	w_i (km/h)	$\rho_{\max,i}$ (veh/km)
Link 1	112	38.76	178
Link 2	87	48.62	198
Link 3	96	49.63	198
Link 4	112	29.52	178

stretch is used to initialize the extended LTM. Flow data along with the information about time and value of VSL from two days (21 (Day 1) and 28 (Day 2) of September 2009) are used to identify the parameters of the model. For validation, we use data from 21 of October 2009 (Day 3). In order to have the persistent excitation condition satisfied (*25*), a period of the day is selected in which both free-flow and congested situations exist.

For each link the set of unknown parameters $\theta_i = \{v_{\mathrm{free},i}, w_i, \rho_{\max,i}\}$ is defined (and the capacity $q_{\mathrm{M},i}$ is related to the parameters by (12)). The optimal parameters are identified by minimizing the following objective function:

$$V(\theta) = \frac{1}{N_{\mathrm{d}} N_{\mathrm{l}}} \sum_{i=1}^{N_{\mathrm{l}}} \sum_{k=1}^{N_{\mathrm{d}}} (q_i(k) - \widehat{q}_i(k))^2 \tag{14}$$

where N_{d}, N_{l}, q_i, and $\widehat{q}_i$ denote the number of data samples, the number of links, real flow data and flow prediction provided by the LTM, respectively. Note that once the updates for cumulative numbers of vehicles are determined for the LTM the estimation of the flow can be easily obtained using the following:

$$\widehat{q}_i(k) = \frac{N(x_i^0, k+1) - N(x_i^0, k)}{T_s} \tag{15}$$

The nonlinear optimization problem is solved using the function *patternsearch* from the *Global Optimization* toolbox of Matlab®. The default options are selected and the optimization algorithm is run 10 times for different random initial points in order to prevent reaching a local optimum only. The obtained parameters are presented in Table 1. Further, the flows of two links predicted by the calibrated LTM are plotted in Figure 5. As can be inferred from the results of Day 3 (validation set), predictions from the calibrated LTM can follow the real flows and thus identified parameters of the LTM are verified and the calibrated LTM can be used for modeling the freeway.

5 CONCLUSIONS AND FUTURE RESEARCH

Extension and calibration of the Link Transmission Model (LTM) have been presented in this research. The LTM was extended in order to be used in traffic control. A model for ramp metering was added to the original LTM. Next the possibilities for variable speed limit control based on the LTM were explored and a set of modifications were determined and added to the original LTM. The modified LTM enables us to model the effects of VSL in reality and thus it can be used for model-based control of traffic networks. In order to verify our modifications two case studies were selected. In the first one, a benchmark traffic network was chosen and simulation results for different scenarios have illustrated the accuracy and correctness of our extensions. Next, the extended LTM was calibrated for a part of the A12 freeway in The Netherlands. Optimal parameters of the model were identified using a global optimization method. Furthermore, since the A12 freeway is equipped with VSL, the validity of the modifications in the LTM could be further investigated

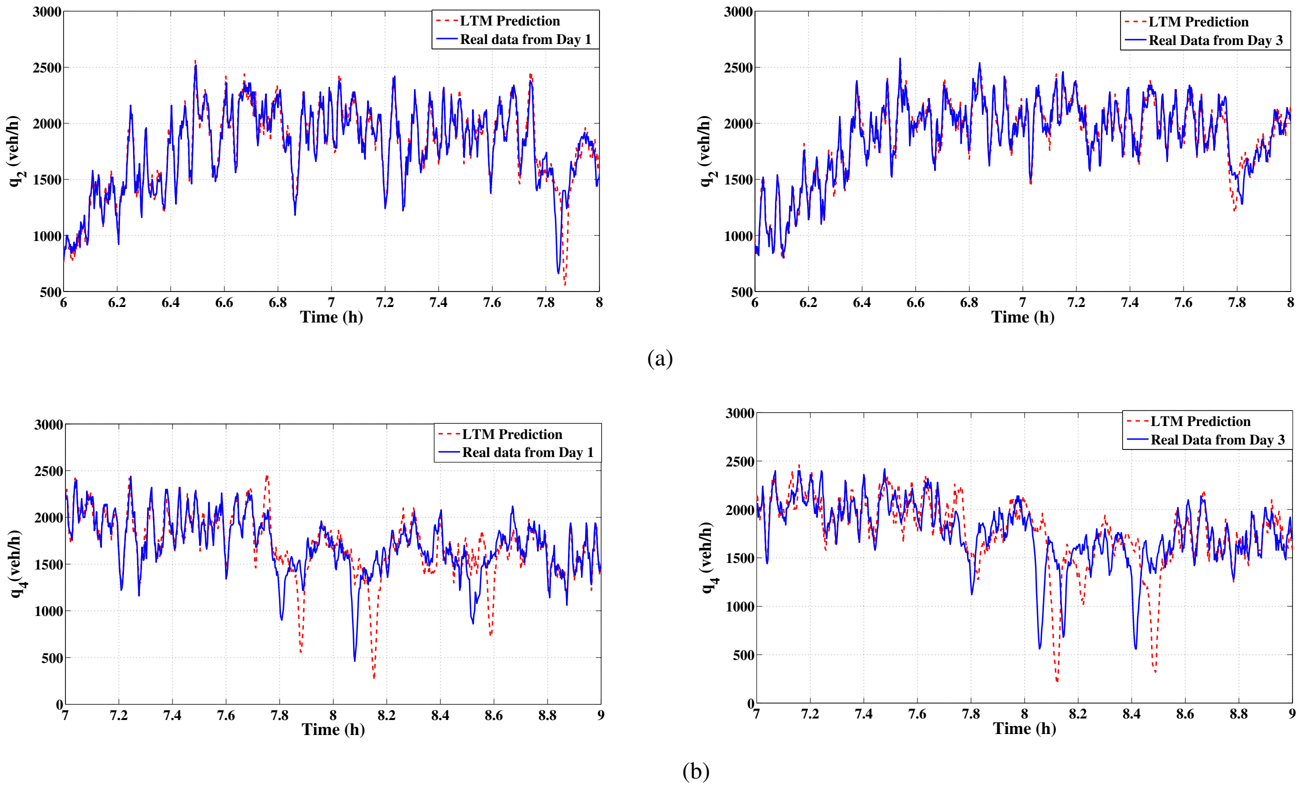


FIGURE 5 Prediction of flow for (a) Link 2 and (b) Link 4 using identified LTM.

using real data. Comparison of the output of the extended LTM with the real data from links in which VSL are installed showed acceptable performance of the modifications.

As an extension to the current work, we aim at identifying and calibrating the LTM for other parts of the A12 freeway including on-ramps and off-ramps. Careful study of traffic phenomena e.g. the negative impacts of stop-and-go traffic and capacity drop in the extended LTM along with sensitivity analysis of the calibrated LTM are interesting topics for future research. Also, the small time lag in the LTM predictions in case of traffic jam needs more investigation. Moreover, the results of this paper can be utilized to develop efficient variable speed limit control based on advanced control methods (e.g. model predictive control).

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